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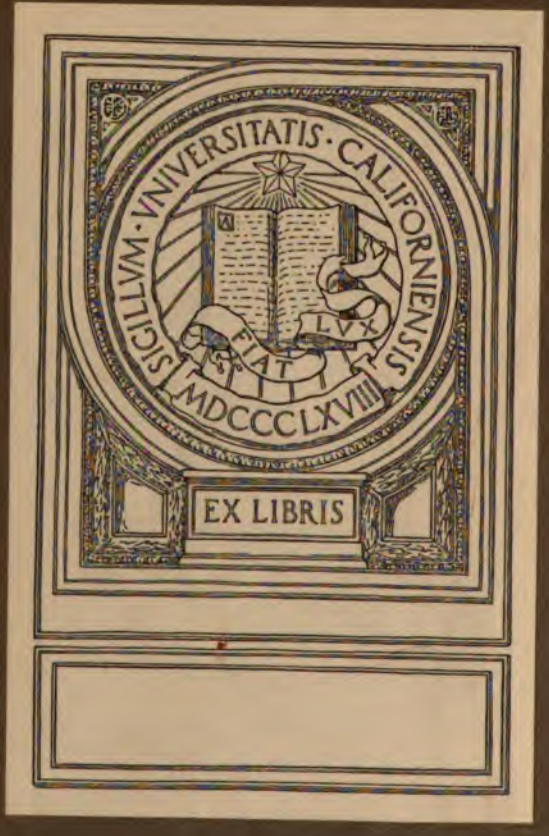
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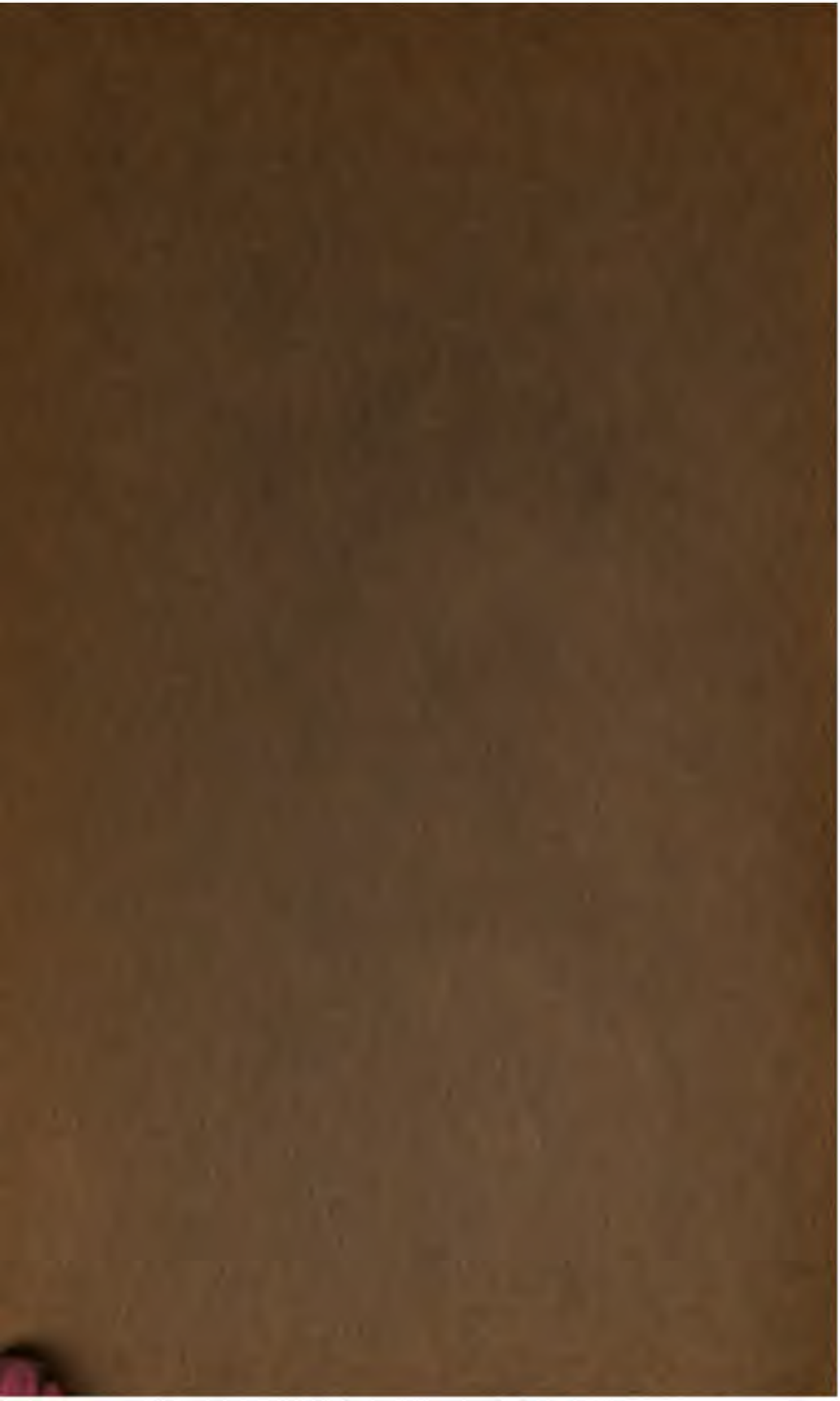

·SURVEYING·

AS PRACTISED BY CIVIL ENGINEERS AND SURVEYORS

INCLUDING

THE SETTING-OUT OF WORKS FOR CONSTRUCTION AND SURVEYS ABROAD, WITH EXAMPLES TAKEN FROM ACTUAL PRACTICE

INTENDED AS A HANDBOOK FOR FIELD AND OFFICE USE
ALSO AS A TEXT-BOOK FOR STUDENTS

BY

JOHN WHITELAW, JUN.

ASSOCIATE MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS
AUTHOR OF "POINTS AND CROSSINGS"

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PREFACE.

IT has appeared to the Author that many of the works on Surveying hitherto published are too much devoted to the elementary part of the subject, and leave the reader almost entirely in ignorance of much of the actual practice of Civil Engineers and Surveyors in the various branches of surveying and setting-out of works. In others, again, the elementary part of the subject is omitted, while they deal with special branches or the higher class of Surveying.

It is hoped that the present treatise will be found to give the student of Engineering sufficiently full details of the work which he may be called upon to carry out in actual practice, while in the earlier chapters it will be seen that simpler classes of Surveying and Levelling are also fully treated.

The aim of the Author has been to make the treatment of the subject as complete and comprehensive as possible, the object kept constantly in view being to make the work a useful text-book of principles and methods for students, as well as a guide to the actual practice of Surveyors and Civil Engineers in the various branches of Surveying.

Tacheometry or Stadia Surveying, which is now coming into general use, has been fully treated in Chapter VI., and full Tables for the reduction of inclined sights have been given.

A chapter has been devoted to Surveys Abroad in

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Undeveloped Country, and as these are conducted under very different conditions from home work, the information thus afforded should prove eminently useful to those who may be called upon to carry out surveys in new countries or undeveloped regions.

A chapter has also been devoted to Astronomical Observations Used in Surveying. These are generally necessary in surveys in new countries, and hitherto when such information has been required search has been necessary in various different works. Being here collected and treated in one chapter, the information will be readily available for reference.

Many examples of surveys, taken from actual practice, have been given throughout the volume.

EDINBURGH,
July 1902.

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SURVEYING

AS PRACTISED BY CIVIL ENGINEERS AND SURVEYORS.

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CHAPTER I.

SURVEYING WITH THE CHAIN ONLY.

Instruments: Chain.—For the purpose of making actual linear measurements on the ground, the chain is most used. It consists of strong links of steel or iron wire of from No. 7 to No. 12 W.G., connected by rings, with a brass handle at each end (Fig. 1). In English-speaking countries there are two different lengths of chain in common use, the Gunter's or 66 ft. chain and the 100 ft. chain. The 66 ft. chain is most used, and possesses the advantage in computing areas that 10 square chains is equal to 1 acre. Both chains consist of 100 links,

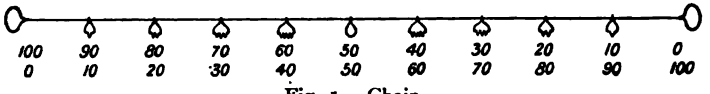


Fig. 1.—Chain.

and every tenth link is distinguished by a brass tablet, as shown in Fig. 1. The first 10 links from the end is marked by a brass tablet with one point; the tablet at 20 links has two points, that at 30 links three points, at 40 links four points, and 50 links or the centre of the chain is marked by a circular tablet. Each 10 links is marked from the other end similarly, so that the chain can be read both ways.

Each link of the 66 ft. chain is therefore $\frac{1}{100}$ part of 66 ft. or 7.92 in., and each link of the 100 ft. chain is 1 ft. Distances are thus measured with the 66 ft. chain in chains and decimal parts of a chain, or links as they are called, as 7.85 chains or

7 chains 85 links; with the 100 ft. chain the measurements are of course in feet only.

In the United Kingdom nearly all railway work and ordinary surveying is executed with the 66 ft. chain, the use of the 100 ft. chain being principally confined to works of water supply, sewerage, and municipal works. The 100 ft. chain is, however, universally used in British India and the United States, and generally in the Colonies.

In countries where the metric system is used, the usual length of chain is 20 metres. This is almost exactly 66 ft., 20 metres being equal to 65.6 ft.

Reading the Chain.—In taking measurements with the chain, we must look for the nearest brass tablet short of the point that is being measured to, and count the number of links from it forward to the point in question. If the nearest tablet indicates

Fig. 2.—Chain and Arrows.

20 links, and the point being measured to is 6 links beyond it, the distance is thus 26 links. Custom soon enables one to read the chain at a glance. Some confusion may arise at first from the fact that the tablet with four points indicates 60 links as well as 40 links, according as we reckon from one end of the chain or the other; in the same way the tablet with two points may either indicate 20 links or 80 links. A little practice, however, soon enables one to tell which is the correct reading. A look at the end of the chain or at the 50-link tablet will always decide the point.

Laying out Chain on Ground.—When not in use, the chain is rolled up in a bundle (see Fig. 2) and fastened with a leather strap. To lay out the chain on the ground, take both handles in one hand, and throw forward the chain, keeping hold of the handles. When fastening up the chain after use, take it up at the centre link and bunch it up double, two links at a time.

Arrows.—Accompanying the chain are ten arrows or marking pins (Fig. 2). These are inserted in the ground to mark the end of each chain. When the ground is too hard to insert the arrow, a scratch may be made with the point for a mark, or the arrow may be simply laid down flat, care being taken that it is not moved while dragging the chain forward.

Ranging Rods and Flags.—These are iron-shod wooden poles, slightly tapering from the bottom, which are used to range out lines and mark points. They are usually 5 or 6 ft. long, and are painted in divisions alternately black and white or red and white, to render them visible at a distance. When they are at a considerable distance, they should have a piece of white or red cotton cloth tied on the top as a flag, to make them conspicuous. A very convenient length is 10 links, painted in ten divisions. These rods come in very useful for measuring short offsets. Three rods are necessary to range out a straight line, but the surveyor should have from six to twelve ranging rods to mark the ends of the chain lines and important points of the survey. For very long lines it is necessary to have poles of greater length, 10 to 15 ft. long or more, according to circumstances. In wet weather for flags use woollen cloth instead of cotton, which clings to the pole when wet.

Laths.—A bundle of laths sharpened to a point sometimes comes in useful in ranging long lines and marking points to be levelled in taking cross sections. They are cheap and easily procurable, and are easily sharpened to a point or cut to any desired length. Being white, they are easily seen, and they may be discarded when done with, without serious loss.

Whites.—When laths are not to be had, small twigs sharpened to a point and cleft in the head, with a piece of paper inserted in the cleft, are good marks.

Steel Tapes.—For accurate measurements a steel tape must be used. This consists of a steel band about $\frac{1}{2}$ in. wide, with feet, inches, and eighth parts of an inch marked on one side, and links or decimals of 66 ft. on the other, in the case of a tape 66 ft. long. Steel tapes may be had of any desired length, and are made to roll up inside a small circular leather case.

Linen Tapes.—For ordinary measurements a linen tape is used. The chief use of the linen tape is the measurement of offsets, the chain being allowed to lie on the ground in the chain line while the offset is measured from it with the tape. Linen interlaced with fine metallic wire has been used as an improvement in point of strength and stretching. It is, however, inferior to a good plain linen tape. New tapes may be bought separately, and are easily inserted in the leather case when the old tape is worn out.

Linen tapes are not to be depended on for accurate or important measurements on account of their stretching, especially if wet, while on a windy day the tape is useless for long measurements, as it catches the wind, sags and pulls. If the tape gets wet and dirty, it should never be rolled up until it has been cleaned and dried. It should be coiled loosely until it can be washed in clean water. It may then be hung up to dry, and afterwards rolled up in the leather case.

Steel Band.—As a substitute for the chain a steel band may be used. This consists of a steel band with a brass handle at each end, similar to the ordinary chain handles. The advantage of the steel band is that it is practically unalterable in length (for ordinary small surveys); while the chain is liable to be stretched or to have its links bent, and must therefore be tested occasionally and adjusted.

The steel band does not accommodate itself to the inequalities of the ground when laid down, as well as the chain. It is, however, no doubt better and more accurate for measuring long lines, especially when not many offsets are required, and it is consequently not necessary to allow the chain to lie on the ground; in such work, for instance, as chaining out the centre line of a railway.

Adjusting the Chain.—This is effected by removing some of the rings connecting the straight links if the chain is too long; or by straightening out the links and inserting some of the additional rings furnished on the chain if it is too short. The chain may be tested by comparing it with a steel tape.

Cross Staff.—This is used for setting out long offsets at right angles to the chain line. The simplest form of cross staff

is that shown in Fig. 3, consisting of four arms with vertical slits for sighting through, at right angles to each other. These are fixed upon an iron-shod ranging rod which is inserted into the ground on the chain line at the point where it is desired to set off an offset line at right angles to the chain line. It is turned round until one pair of opposite slits is in line with the chain line. By then looking through the other pair of slits an offset line at right angles to the chain line may be lined out.

Fig. 4 shows an octagonal form of cross staff with slits on all eight sides. By means of this, lines at an angle of 45° as well as 90° with the chain line may be set off.

Fig. 5 is another form which may be used for setting off or measuring angles approximately. There are graduations round

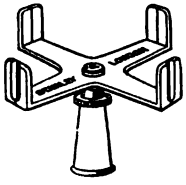


Fig. 3.
Cross Staff.

Fig. 4.
Cross Staff.

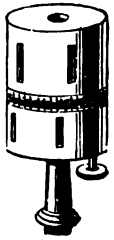


Fig. 5.
Cross Staff.

the circumference; the top part slides over the under part, and may thus be set to any required angle by means of the graduations.

The forms of cross staff shown in Figs. 3, 4, and 5 are those made by Stanley, Great Turnstile, Holborn, London, and any of these may be obtained with compasses fixed on them for taking bearings. The simplest form of cross staff is the best, as the additions only complicate matters, and are better provided for in other instruments specially made for measuring angles. As, in general, long offsets are to be avoided, the cross staff should only be sparingly used.

Optical Square.—This is another instrument for setting out right angles. It consists of a small circular metal box (Fig. 6), which shows the instrument as made by Stanley. It is usually protected by a metal cover as shown in the figure. This slides round so as to cover the openings and protect the mirrors when not in

use. Fig. 7 is a sectional plan of the instrument. A and B are slits for the eye, C is the opening through which the lining rod at D is reflected on to the mirror. E and F are two mirrors placed at 45° to each other, the under half of the mirror E being unsilvered. The instrument is used by placing the eye at the slit A and looking through the slit B along the chain line, at right angles to which it is desired to lay off the offset line. The slits A and B being thus properly placed in line, if a ranging rod be held at D, when it is at right angles to AB it will be reflected in the mirror F, and thence to the mirror E, where its reflection will coincide with the ranging rod at K, as

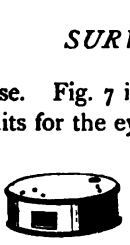


Fig. 6.
Optical Square.

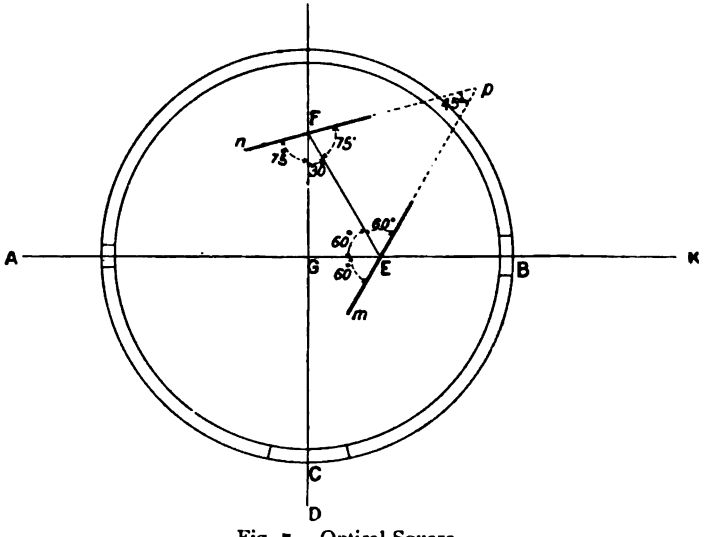


Fig. 7.—Optical Square.

seen through the unsilvered lower half of the mirror E. The chainman being sent out with the ranging rod in the direction of D, must be directed to move it to right or left until its reflection in the mirror E coincides with the ranging rod on the chain line at K.

The theory of the instrument is as follows:—The mirror E is placed at an angle of 60° with the line ABK, and the mirror F is at an angle of 45° with the mirror E. Angle of reflection AE*m* =

angle of incidence $pEF = 60°$. Therefore FEG $= 180° - (2 \times 60°) =$ $60°$ also.

Then in the triangle pEF we have $pEF = 60°$, $EpF = 45°$, and therefore $pFE = 75°$. The angle of incidence nFG is therefore also $75°$, and consequently the angle EFG $= 180° - (2 \times 75°) = 30°$. In the triangle EFG we have then FEG $= 60°$, EFG $= 30°$, and therefore FGE $= 90°$, or the line FGD is at right angles to the line ABK.

It is almost impossible to use the optical square when the ground is not level and one ranging rod is higher than the other, although it may be done by twisting the instrument into a plane approximately parallel to the surface of the ground. The right angle will then, however, be set off in this plane, and its horizontal projection will not be a right angle. Altogether it may be said that the use of the instrument is limited to cases where the ground is practically level. These remarks also apply to the box sextant, page 73, Chapter II.

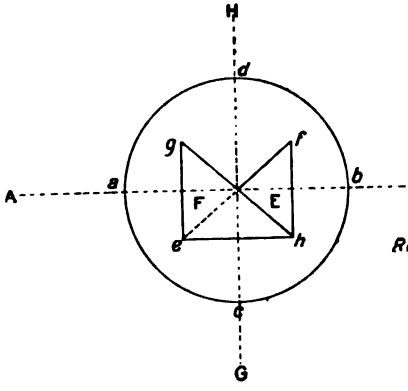


Fig. 8.—Line Ranger.

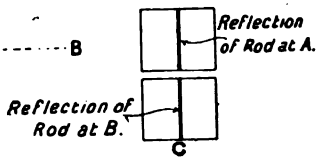


Fig. 9.—Line Ranger.

Line Ranger.—This instrument (Figs. 8 and 9) consists of two square prisms E and F having their hypotenusal sides *ef* and *gh* silvered and at right angles to each other, Fig. 8 being a plan of the instrument. These are placed one above the other in a small box similar to the optical square, having openings at *a* and *b* and slits at *c* and *d*. Upon looking through the slits *c*, *d*, in the direction GH, if the instrument is in the line AB the reflections of the ranging rods at A and B will appear to be in the same vertical line at C

in the mirrors, as shown in Fig. 9. To use the instrument, move it back or forward until the lining rods appear as shown in Fig. 9. A point vertically under the instrument will then be in the straight line AB. It is useful for finding an intermediate point in a long line without the time and labour involved in going to one end to line it out. It is also possible to get into the line by setting up two ranging rods so that they range in with the pole at one extremity of the line. Then try if they also range in with the poles at the other extremity of the line. If they do, they are in the line; and if not, they must be shifted until they range in with the poles at both extremities of the line, their correct position being found by trial and error.

Use of the Chain.—The method of chaining a line is as follows:—Having stretched out the chain, the follower holds his end of it at the ranging rod marking the beginning of the chain line, while the leader moves forward to the full extent of the chain, having with him the ten chaining arrows. The follower, having the ranging rod at the other end of the chain line in view, directs the leader to right or left until he has got the chain into line. The leader then inserts the first arrow to mark the end of the chain. The chain is then moved forward until the follower reaches the arrow, at which he holds the end of the chain, and again directs the leader until he gets him into line. A second arrow is then inserted by the leader to mark the end of the second chain length. The follower picks up and brings with him each arrow after the leader has inserted the one in front, and the operation is repeated in the same way until the ten arrows have been inserted by the leader. This, of course, occurs at the end of 10 chains, at which point the follower delivers up all the ten arrows to the leader, meantime marking the end of the 10 chains by a ranging rod or other temporary mark, until the next arrow has been inserted by the leader. Thus the ten arrows are a valuable check upon the counting of the number of chains.

When offsets (see page 17) are required, the chain is simply allowed to lie stretched out on the ground, in the line, between the chaining arrows until the offsets have been measured to it. When fences, hedges, or boundaries have to be crossed, the chain is pulled through them, and the distance on the chain at which they cross is noted.

Chaining on Slopes.—As the survey when plotted on paper is a horizontal projection of the ground surveyed, all distances must be measured horizontally. When the ground is sloping, if the distance is measured along the sloping surface it will be too great, as may be seen from Fig. 10, where, if the slope is at an angle of 5°, the distance AB as measured on the slope is 10.04 chains, or 10 chains 4 links, whereas the required horizontal distance AC is only 10 chains.

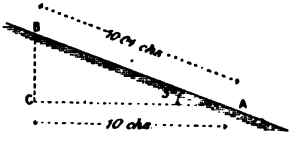


Fig. 10.—Chaining on Slopes.

In all cases, therefore, when the ground slopes there is a certain deduction to be made from the distance as measured on the slope to reduce it to the horizontal distance.

The following table shows the difference per chain of 66 ft. between the hypotenusal or slope measurement and the horizontal measurement:—

TABLE FOR REDUCING MEASUREMENTS ON SLOPE TO HORIZONTAL: 66 FT. CHAIN.

Degrees.	Links.	Degrees.	Links.	Degrees.	Links.	Degrees.	Links.
5	0.4	14	3.0	23	7.9	32	15.2
6	0.6	15	3.4	24	8.6	33	16.1
7	0.7	16	3.9	25	9.4	34	17.1
8	1.0	17	4.4	26	10.1	35	18.1
9	1.2	18	4.9	27	10.9	36	19.1
10	1.5	19	5.4	28	11.7	37	20.1
11	1.8	20	6.0	29	12.5	38	21.2
12	2.2	21	6.6	30	13.4	39	22.3
13	2.6	22	7.3	31	14.3	40	23.4

The following table gives deductions per 100 ft. to be made from measurements taken on slope to reduce them to the horizontal measurements. This table is useful when a 100 ft. chain is used.

TABLE FOR REDUCING MEASUREMENTS ON SLOPE TO HORIZONTAL: 100 FT. CHAIN.

Angle of Slope.	Deduction in Feet.	Angle of Slope.	Deduction in Feet.	Angle of Slope.	Deduction in Feet.	Angle of Slope.	Deduction in Feet.
¼	.001	5¼	.420	10¼	1.596	15¼	3.521
½	.004	5½	.460	10½	1.675	15½	3.637
¾	.009	5¾	.503	10¾	1.755	15¾	3.754
1	.015	6	.548	11	1.837	16	3.874
1¼	.024	6¼	.594	11¼	1.921	16¼	3.995
1½	.034	6½	.643	11½	2.008	16½	4.118
1¾	.047	6¾	.693	11¾	2.095	16¾	4.243
2	.061	7	.745	12	2.185	17	4.370
2¼	.077	7¼	.800	12¼	2.277	17¼	4.498
2½	.095	7½	.856	12½	2.370	17½	4.628
2¾	.115	7¾	.913	12¾	2.466	17¾	4.760
3	.137	8	.973	13	2.563	18	4.894
3¼	.161	8¼	1.035	13¼	2.662	18¼	5.030
3½	.187	8½	1.098	13½	2.763	18½	5.168
3¾	.214	8¾	1.164	13¾	2.866	18¾	5.307
4	.244	9	1.231	14	2.970	19	5.448
4¼	.275	9¼	1.300	14¼	3.077	19¼	5.591
4½	.308	9½	1.371	14½	3.185	19½	5.736
4¾	.343	9¾	1.444	14¾	3.295	19¾	5.882
5	.381	10	1.519	15	3.407	20	6.031

When very great accuracy is not required, the best practical method of chaining on slopes is to do it in steps as shown in Fig. 11, where one end of the chain is raised up until it is horizontal. The raised end of the chain is marked on the ground by holding a plumb bob. If the slope permits of the full chain length being raised to a horizontal position, it is an advantage to have some one to hold it up at the middle to take out the sag. When the slope is very steep, it may be measured by holding up 20 or 30 links of the chain at a time.

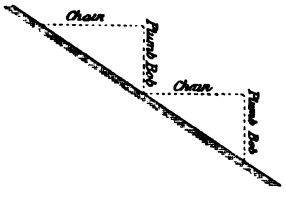


Fig. 11.—Chaining on Slopes.

Accuracy of Ordinary Chaining.—An error in marking the end of each chain with the chain pin of say ¼ in., in a distance of 1 mile, in which there are 80 chain lengths, would, if the

errors were not compensating, result in a total error in a mile, of $80 \times \frac{1}{4}$ in. = 20 in. An error of this kind is, however, compensating, and in such a case the *probable error* is $\sqrt{80} \times \frac{1}{4}$ in. = say $9 \times \frac{1}{4}$ in. = $2\frac{1}{4}$ in.

An error in the length of the chain itself is, however, not compensating, and suppose the chain were $\frac{1}{10}$ in. too long or too short, the error from this cause would be $80 \times \frac{1}{10}$ in. = 8 in. The importance of testing the chain occasionally and adjusting its length is therefore emphasised by the above considerations, which show that the greatest errors in ordinary chaining are more probably due to erroneous length of chain than to any other cause.

An accuracy of 1 in 5,000 may be attained with the steel tape if the following precautions are observed:—The tape held as nearly horizontally as can be estimated; the pull on the tape by the chainmen kept as nearly constant as can be estimated; the ends of the tape marked by plumb bobs on uneven ground; and one or two observations for temperature taken throughout the day. The tape should be tested on a correct standard of length, and its length under the pull proposed to be used in the field and at a given temperature ascertained. The temperature at which its length is *correct* under the proposed pull is then to be calculated, and the field measurements reduced to this temperature.

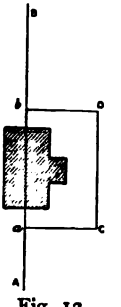


Fig. 12. Obstacle in Chain Line.

In ordinary chaining the error will be about 1 in 3,000 or 1 in 4,000. For the most accurate steel tape measurements see page 443, Chapter XII.

Obstacles to Measuring.—Perhaps the simplest method of passing an obstacle, such as the building shown in Fig. 12, is to "square off" by erecting perpendiculars to the chain line. In Fig. 12, let AB be the chain line, and suppose that on reaching the point *a* the measurement is obstructed by the building shown. At the point *a* set off *a*C perpendicular to AB. At C set off CD perpendicular to *a*C, and again at D set off *b*D perpendicular to CD. CD will then be parallel to AB, and the distance CD will be equal to the distance *ab*. We may therefore chain up to the point *a*, then from C to D, and resume the chaining from *b* to B along the

chain line. As a check on the squaring off, the distance *b*D should of course be equal to *a*c. This method is not recommended *for producing the chain line* when the building cannot be seen over, as *b*D is too small a base to work off.

Setting out a Right Angle with the Chain.—The following simple method of setting out a right angle with the chain is often useful. In Fig. 13, let *bc* be the required perpendicular to the chain line AB at the point *b*. Measure on AB the distance *ba* = 40 links. Then holding one end of the chain at *b*, and 80 links on the chain at *a*, if the chain be taken up at 30 links and pulled tight into the position shown in Fig. 13, the position of the point *c* will be given by the 30-link mark on the chain. This is due to the fact that 30, 40, and 50 represent the sides of a right-angled triangle, and in performing the above operation we make *bc* = 30 links, *ab* = 40 links, and *ac* = 50 links.

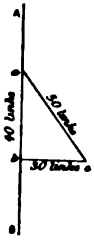


Fig. 13. Setting out a Right Angle with the Chain.

In this connection the following whole numbers representing the sides of right-angled triangles may be useful. Any multiples of these numbers will also give right-angled triangles.

Sides of right-angled triangles—

3, 4, 5
5, 12, 13
7, 24, 25
8, 15, 17
20, 21, 29

Chaining across a River or other Obstacle.—In Fig. 14, let AB be the chain line. At the points *a* and *b* erect perpendiculars *ae* and *bd*. Range the point *d* in line with *e* and *c*. Draw *df* parallel to AB. Then the triangles *def* and *cdb* are similar, and we have—

$$\frac{bc}{bd} = \frac{df}{ef}$$

But $df = ab$, and $ef = ae - bd$, and therefore—

$$\frac{bc}{bd} = \frac{ab}{ae - bd}$$

If *ae* be made equal to twice *bd*, then $ae - bd = bd$, and we get $bc = ab$,

$$\text{or } bc = \frac{bd \times ab}{ae - bd} = \frac{bd \times ab}{bd} = ab.$$

Another method is as follows:—In Fig. 15, at the point *a* in the chain line AB erect the perpendicular *ab* and bisect it in *c*. Lay off *bd* perpendicular to *ab*, and range in the point *d* in line with *c*, and the point *e* on the other side of the river. Then the triangles *bcd* and *ace* are similar and equal.

$$\therefore ae = bd.$$

If the river be at an angle with the chain line as in Fig. 16, lay off *ab* at any convenient angle and bisect it in *c*. Take any point *e* close to the edge of the river, and produce *ec* to *f*, making

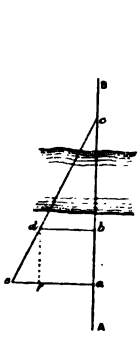


Fig. 14.

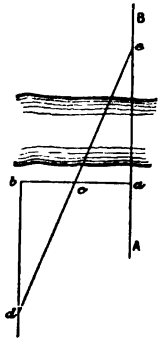


Fig. 15.

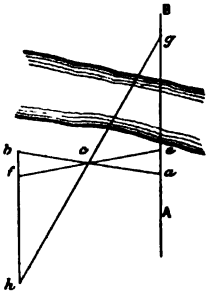


Fig. 16.

Chaining across a River or other Obstacle.

$cf = ce$. Range *h* in line with *bf* and *cg*. Then in the triangles *bcf* and *ace* we have the sides *ac* and *ec* = the sides *bc* and *fc*, and the contained angle *ace* = the contained angle *bcf*. These triangles are therefore equal, and the angle *cbf* = angle *cae*. Also in the triangles *bch* and *acg* the angles *bch* and *acg* are equal, and we have previously the angles *cbh* and *cag* equal. The triangles are therefore similar, and as the sides *bc* and *ac* are equal, the triangles are equal.

$$\therefore ag = bh \text{ or } eg = fh.$$

When the river crosses obliquely, the method in Fig. 17 may also be used. Lay off any line *ab* alongside the river, and produce it to *c*, making $ac = ab$. Lay off *be* perpendicular to *ab*, so as to

reach across the river, and also *cd* perpendicular to *ac*. Then the triangles *abe* and *acd* are similar and equal.

$$\therefore\ ae = ad.$$

Another method is as follows:—In Fig. 18 set off *ab* perpendicular to the chain line AB, and also *bd* perpendicular to *bc*. Then the angle $abd = 90 - abc = acb$. Therefore the triangles *abc* and *abd* are similar. We have then—

$$\frac{ac}{ab} = \frac{ab}{ad} \qquad \therefore\ ac = \frac{ab^2}{ad}$$

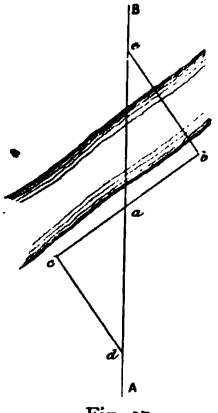


Fig. 17.

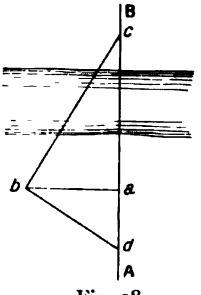


Fig. 18.

Chaining across a River or other Obstacle.

The obstacle shown in Fig. 19 may be got round as follows:—On *ag* lay off any triangle *acf*, and produce the sides to *d* and *e*, making *ad* and *ae* proportional to *ac* and *af*, so that the triangle *aed* is similar to the triangle *afc*.

Then $\frac{bg}{ag} = \frac{cd}{ac}$ or $bg = \frac{ag \times cd}{ac}$

Or in the case of Fig. 20, lay off any line *ac*, and produce it to *d*, making $cd = ac$. Take any convenient point *b* and line out *bc* to *e*, making $ce = bc$. Then the two sides *ce* and *cd* of the triangle *ced* are equal to the sides *bc* and *ac* of the triangle *abc*, and the contained angles *acb* and *dce* are equal. These triangles are therefore equal, and $ab = de$.

Many other methods similar to those shown in Figs. 14 to 20 may also be used.

Surveying with the Chain only.—Small surveys are very often made by means of the chain, although as a rule the use of an instrument for measuring angles greatly facilitates operations. In this chapter we shall confine ourselves to surveys made with the chain only.

Reconnaissance.—The first thing to do in executing a survey is to make a thorough reconnaissance by going carefully over the ground. By this means the surveyor obtains a clear knowledge of the boundaries, streams, buildings, and the various obstacles and difficulties that may be in the way. He is thus enabled to decide upon the main chain lines to the best advantage. The time thus spent may be more than repaid in having

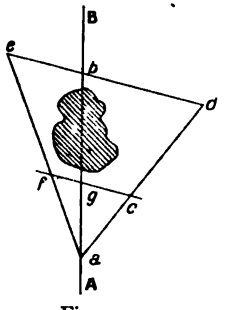


Fig. 19.

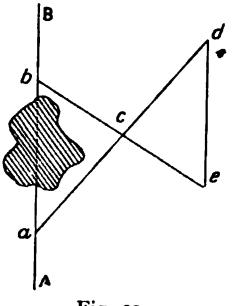


Fig. 20.

Chaining past an Obstacle.

the main lines well laid out, and much more time might be lost in having to run subsidiary lines to take up boundaries or in having to alter and rechain lines on account of unexpected obstacles encountered.

Sketching.—After going over the ground, next make a rough sketch of the survey, marking on it the chain lines, which should be lettered or numbered consecutively. Ranging rods or poles may then be put up at all the stations and the chaining begun.

Field Book.—The usual form of field book is oblong, 8 in. by $4\frac{1}{2}$ in., opening lengthwise. There are two red lines about $\frac{3}{4}$ in. apart ruled in the centre of each page, which form a column for the longitudinal distances on the chain lines. The spaces to

right and left of this column are used for marking the offsets, sketching in details, or making notes, &c. See the field book on pages 21 to 39.

Testing the Chain.—Before commencing a survey, if a chain is used, it should be tested, as some of the links often become bent in pulling through hedges or fences, and require straightening out or the insertion of some of the spare rings. In an extensive survey the chain should be tested daily. This may be done by comparing it with a steel tape.

Chainmen.—Two men are required for the chain, a leader and a follower. The more intelligent of the two should be selected for the follower, as he has to put the leader in line. It is bad policy for the surveyor to take one end of the chain himself, as he should be free to take notes and keep the field book. It is very important to have intelligent, trustworthy, and willing men for chainmen.

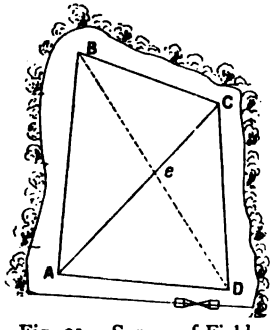


Fig. 21.—Survey of Field.

Examples of Surveying with the Chain only.—In surveying with the chain only, and excluding for the present theodolite and compass measurements, in order to determine the relative positions of any points it is necessary to divide the ground into triangles. To take the simplest case, the four-sided field shown in Fig. 21, we divide the field into the two triangles ABC, ADC, and measure the lines AC, AB, BC, CD, and DA. If we now lay down upon paper to scale a line representing AC, and then take in the compasses the length AB to the same scale, and with centre A and radius AB describe part of a circle in the neighbourhood of the point B, and similarly with centre C and radius CB describe part of another circle near the same point, the intersection of these circles will fix the point B. Similarly the lengths AD and CD determine the point D.

Tie Lines.—Although the measurement of the four sides and the one diagonal is thus sufficient to enable us to plot the

survey to scale, there is no check upon the accuracy of the work. In order to effect this we must also measure the dotted line BD, noting the distances *be* and *eD*, which, when the survey is plotted, should scale the same as measured on the ground. BD is called a "tie line," because it "ties in" the apices of the triangles B and D. Every triangle should have a tie line as a check upon the measurements, see *bf*, Fig. 22.

Offsets.—The actual boundaries of the field, which consist of a hedge on three sides and on the fourth side a fence with a gate in it, are fixed by means of distances or "offsets" as they are called, measured from and at right angles to the chain lines AB, BC, CD, and DA. These offsets are measured with an offset staff or a tape, from the chain, which is laid on the ground in the line of AB, BC, &c., during the measurement of these lines, in order to enable the offsets to be taken. It is only necessary to take offsets to salient points where the direction of the boundary changes. The distances on the chain lines at which the offsets are taken are of course also noted.

Field Book.—The field book of the detailed example of chain survey shown in Fig. 24 is given on pages 21 to 39, and is fully described on pages 40 and 41.

It will be seen that each line is begun at the bottom of the page, and that the distances proceed upwards on the page, so that in holding up the book the distances run *from* one in the direction of the line being measured. It is a good plan to begin a new page at the commencement of each line.

Sketch and Fixing Stations.—In making the survey shown in Fig. 21, the first thing to do after walking over the ground is to make a rough sketch of it, marking the chain lines AC, AB, BC, &c.

Ranging rods are then placed at the stations A, B, C, and D, and the chaining begun, the offsets to the boundaries being taken with the tape as the chaining proceeds. The perpendicularity of short offsets to the chain line may as a rule be judged by the eye. In the case of longer offsets they should be laid out by setting out a right angle with the chain, cross staff, or optical square.

Long Offsets.—Long offsets are to be avoided where much accuracy is expected. When the offsets become long, it is better

to run a subsidiary triangle to take up the boundary, as shown in Fig. 22, where in place of long offsets such as *de* the boundary is taken up by the subsidiary triangle *abc*. Short offsets from *ab* and *bc* then give the hedge shown. *bf* is the "tie line" of the triangle *abc*.

In the field book the beginning of each line should be marked, as shown on pages 21 to 39, "Line AB," "Line BC," &c. The end of each chain line as well as all other stations formed by the intersection of chain lines should be distinguished by an oval drawn round the figures representing the distance, and should be marked "End of line AB," "End of line BC," &c. The boundaries should be sketched in the spaces to right and left of the central column. In the case of Fig. 21, the boundaries will be all to the left if we proceed in the direction AB, BC, &c., and *vice versa*. The offsets are also marked in these spaces opposite the distances on the chain line, see field book, pages 21 to 39.

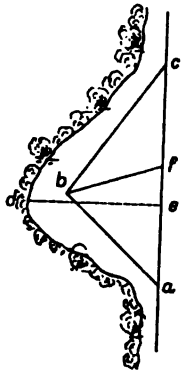


Fig. 22.
Long Offsets.

All chain surveys are conducted on the same principles as the above simple example. In the case of larger surveys, more triangles and more chain lines are required, and there may be more details to take up, such as roads, streams, buildings, ponds, timber plantations, enclosures, &c.; but in every case the same principles apply, and the skill of the surveyor is chiefly exercised in laying out the triangles and chain lines judiciously, so as to take in the greatest amount of detail with the smallest number of lines and as short offsets as possible, while avoiding obstructions and unnecessary crossing and recrossing of hedges, &c.

Best Form of Triangles.—In laying out the triangles, it may be observed, as a rule, that the longer the chain lines and the fewer triangles the better. Also, the nearer the chain lines approach to the boundaries the better, as the offsets are thereby rendered shorter. The triangles should always be as "well conditioned" as possible, and it would be bad work to spoil a well-conditioned triangle for the sake of running a line alongside a fence.

A well-conditioned triangle is one in which the angles are neither very acute nor very obtuse—that is, the best-conditioned triangle is one approaching the equiangular and equilateral shape. The advantage of having the triangles nearly equilateral is that if some of the dimensions are a little out, the resulting error will be less than in the case of an ill-conditioned triangle. The angles of the triangles for this reason should not be less than 30° or more than 120°. If the angle ABC, Fig. 21, for example, is less than 30°, the error in the position of the point B will be more than double any error in one of the lengths AB, BC. It is, however, not always possible to avoid using angles less than 30° either owing to the configuration of the ground or to the limited time at the surveyor's disposal.

General Principles to be observed in laying out the Chain Lines.—It may also be observed as a general rule that the best plan in laying out the chain lines is to run a line throughout the whole length of the ground, as nearly as possible through the centre, as AC, Fig. 21. Upon this construct as many triangles as may be necessary, taking care to check each triangle by a tie line.

There are, however, cases where to run a long line throughout the centre of the ground would be inconvenient and laborious, and in these cases the surveyor must dispose the triangles so as to suit the ground to the best advantage. Another point to be remarked is that, as the chief object in laying down the main lines is to get a good series of triangles, it would be very bad policy to alter, divert, or break up a good line merely to avoid some obstacle which by a little ingenuity may be overcome, as shown on pages 11 to 14.

Surveying a Pond or Plantation with the Chain only.—Very often in the case of a wood or plantation of thick timber or a pond it is impossible to run the chain lines through the ground to be surveyed. In these cases the lines must be run round the outside of the boundaries, as shown in Fig. 23. The angles at A, D, and C are measured by producing the sides and measuring the lines a, a_2, d, d_2, c, c_2. In the case of the angle at B, b, b_2 may be measured without producing the sides.

The distances a_1a_3, d_2d_3, c_1c_3, and b_2b_3 should also be measured as a check. By means of these measurements the lines

AB, BC, AD, DC, may be plotted at the proper angles with each other. The boundaries of the pond are surveyed in the usual manner by offsets from the chain lines AB, BC, &c. Of course such a survey would be much better executed by using a theodolite to measure the angles at A, B, C, and D, but the above is given to show how it may be done with the chain only, if necessary.

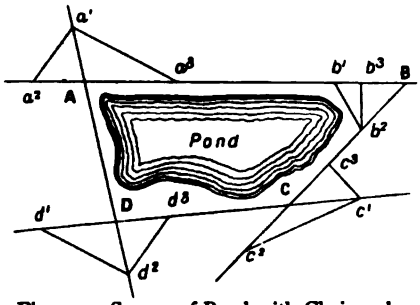


Fig. 23.—Survey of Pond with Chain only.

Fully detailed Example of Chain Survey.—Fig. 24, Plate I., is an example of a larger survey measured with a 66 ft. chain. In this case it will be observed that a main chain line AB, 69.85 chains long, has been run through the centre of the survey. Upon it the triangles AFO, BEO, BCO, and ADO have been laid off, the remainder of the boundaries being taken up by the triangles OEF and OCD. The subsidiary chain line *a*o has been ranged in line with co in order to take in the road between *a* and o. Similarly the subsidiary chain line *hc* takes up the longitudinal and cross fences shown. The chain line *de* is run for the same purpose, while the other lines *fg* and *hk* take up a footpath and a cross road respectively. It will be seen that upon plotting the four main triangles AFO, BEO, BCO, and ADO, the lines EF and CD which complete the other two triangles in this case act as tie lines to the previous four triangles. That is, the distances EF and CD should scale the same as measured on the ground. Each triangle should, however, have a separate tie line, so that any mistake may be traced to the triangle in which it occurs, otherwise in the event of an error the whole work might have to be rechained before the error was located.

In this case the subsidiary chain lines *a*o, *bc*, *de*, *fg*, and *hk* act as tie lines to all the triangles except the last, EOF. This triangle, however, should "check in" by the distance EF agreeing with that measured on the ground, when the other triangles have been plotted and checked.

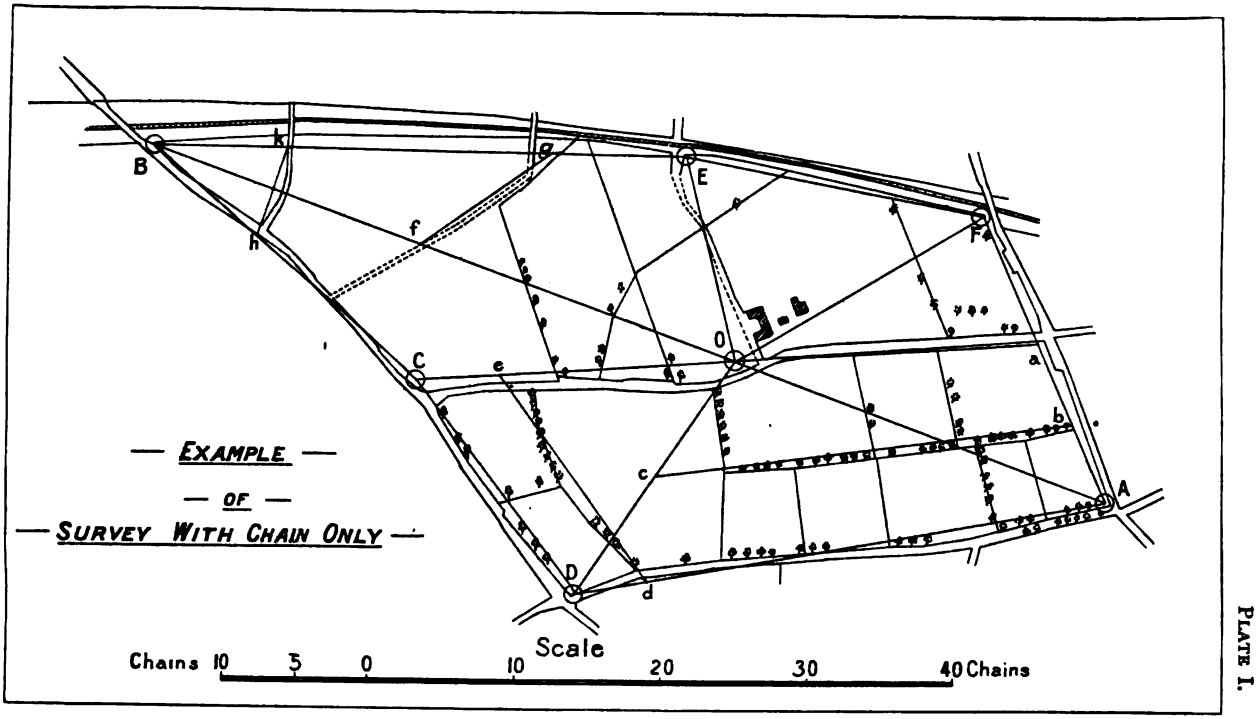
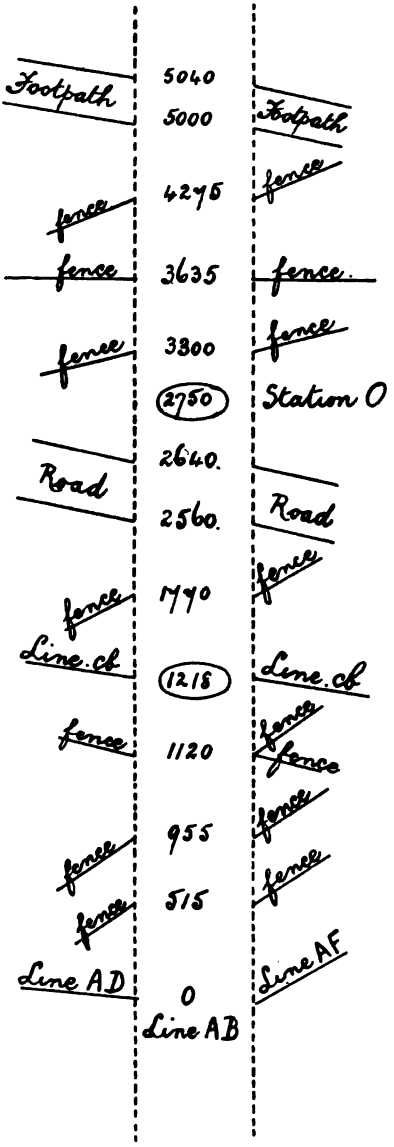
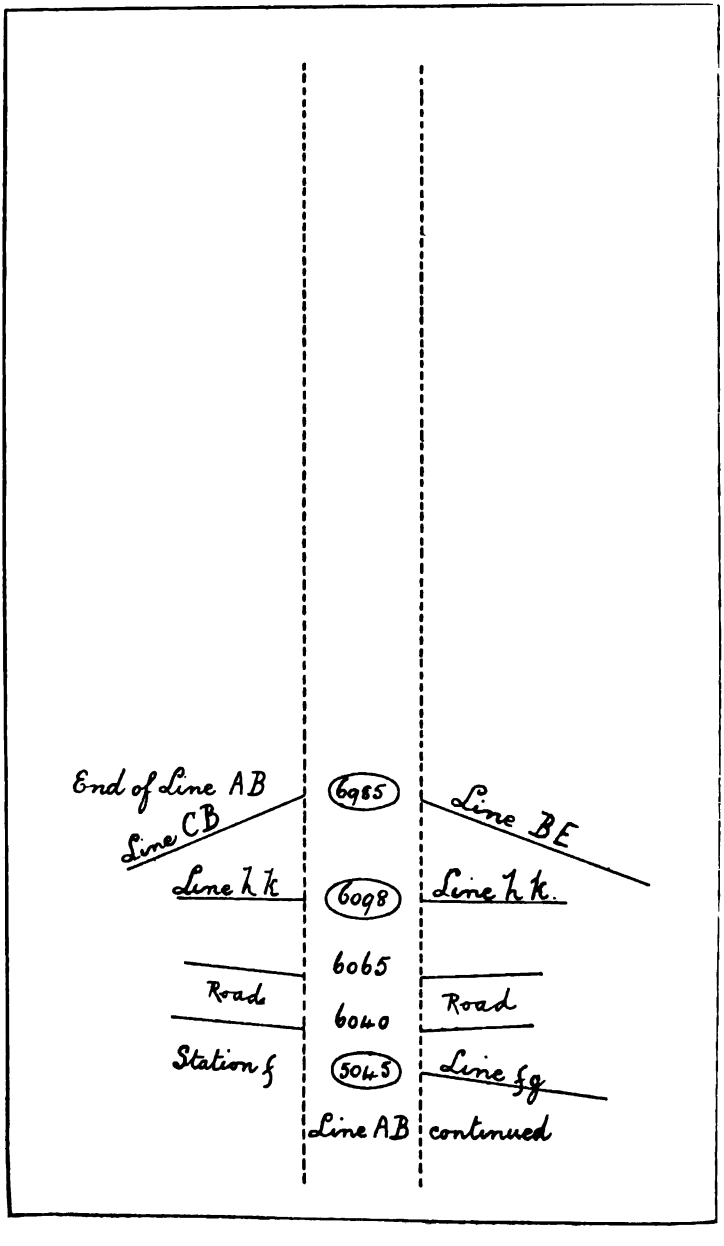


Fig. 24.

PLATE I.

FIELD BOOK OF CHAIN SURVEY.

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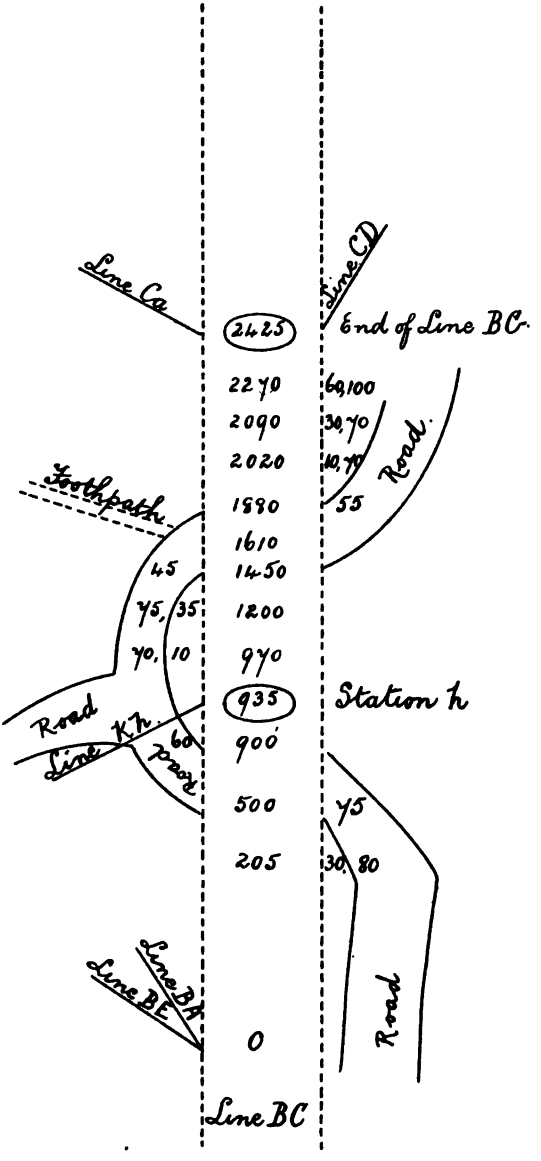
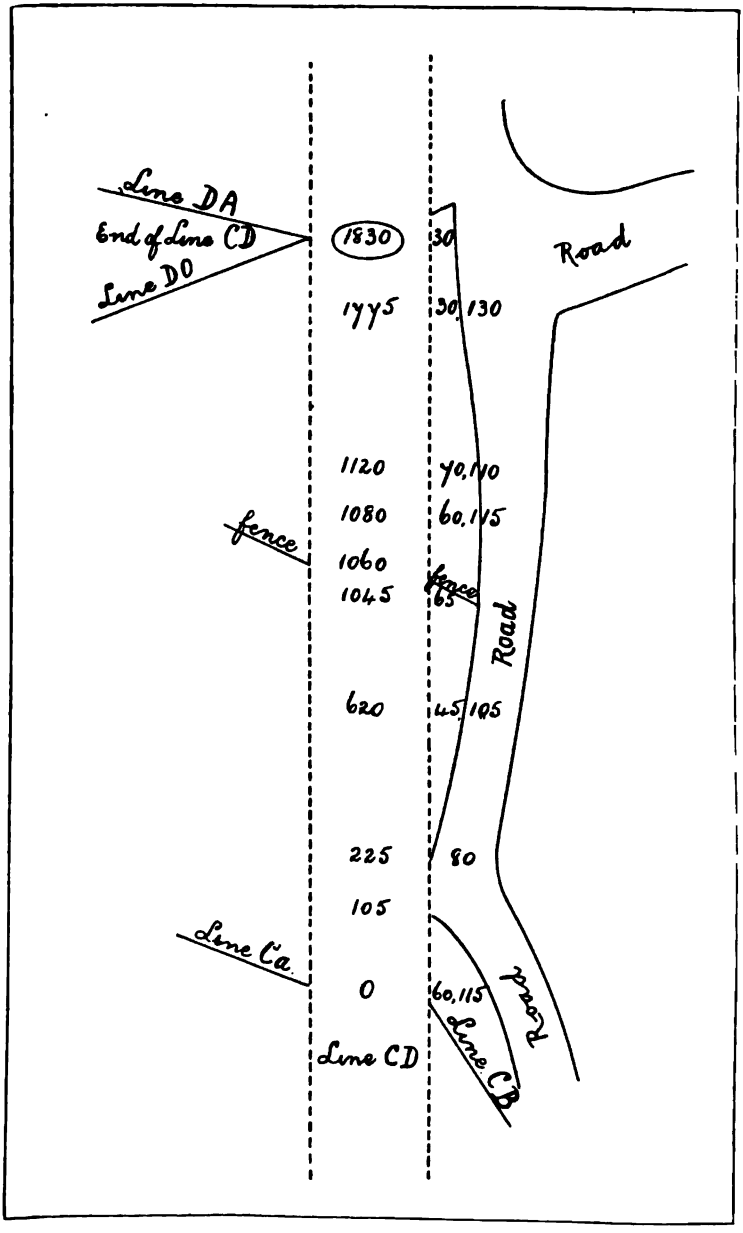
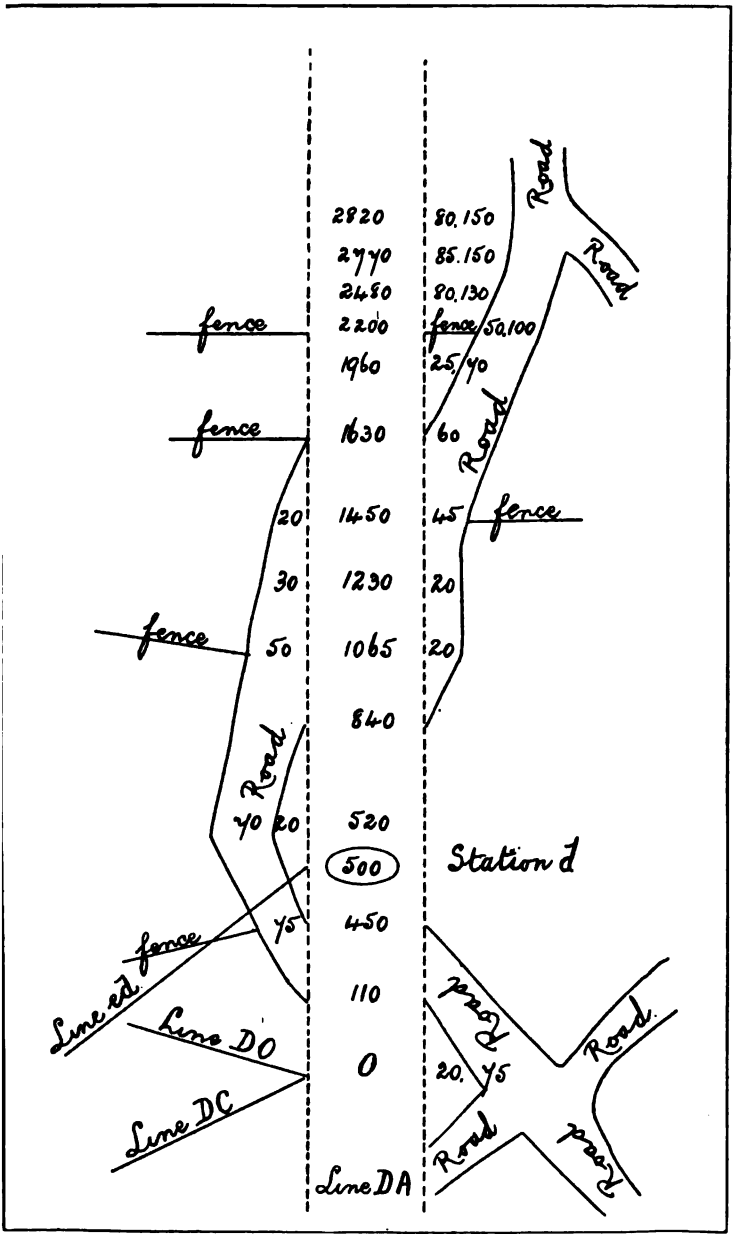
Line CB
6985
Line BE
Line h k
6098
Line h k.
6065
Road
6040
Road
Station f
5045
Line fg
Line AB continued


Line Ca
Line CD
2425
End of Line BC.
2270
2090
2020
1880
1610
1450
1200
970
935
900
500
205
0
60,100
30,70
10,70
55
45
75, 35
70, 10
Road.
Footpath
Road
Line
K.R.
60
Road
Station h
75
30, 80
Road
Line BA
Line BE
Line BC


Line DA
End of Line CD
Line DO
1830
30
Road
1995
30, 130
1120
70, 140
1080
60, 145
fence
1060
fence
1045
65
Road
620
45, 105
225
80
105
Line Ca.
0
60, 115
Line CB
Road
Line CD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2740
2480
2200
1960
1630
1450
1230
1065
840
520
500
450
110
0
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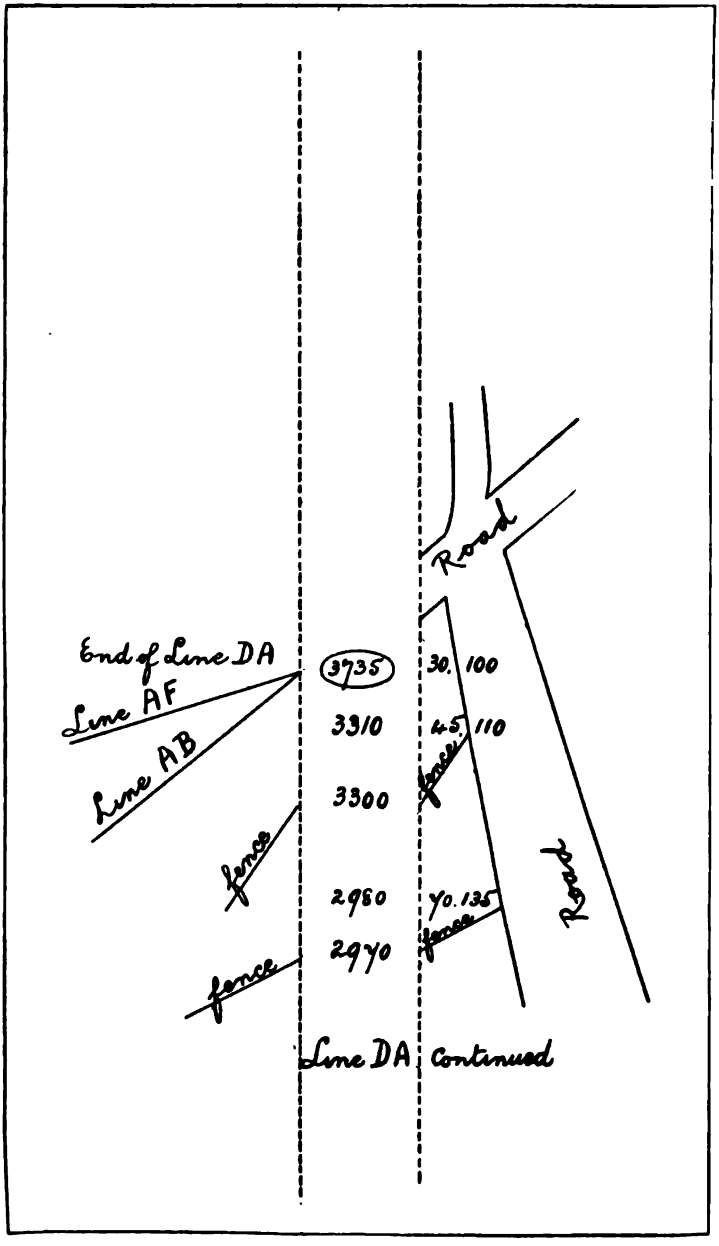
80.130
fence 50.100
25.40
60
45
20
20
20
20
15
20.15
fence
fence
fence
fence
Road
Road
Road
Road
Road
Road
Road
Road
Road
Station d
Line d1
Line DO
Line DC
Line DA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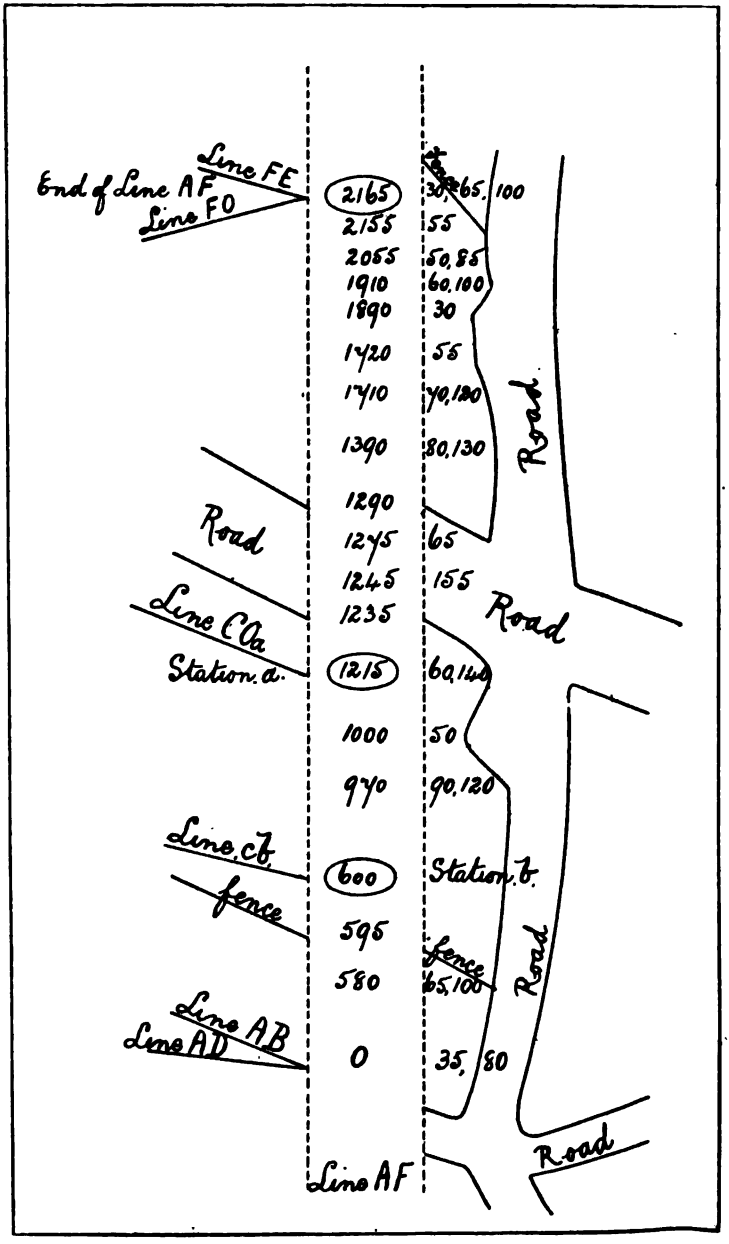
3735
30. 100
Line AF
3310
45. 110
Line AB
fence
3300
fence
fence
2980
40. 135
fence
2940
fence
Road
Line DA Continued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Line FE
Line FO
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2155
2055
1910
1890
1420
1410
1390
1290
1245
1245
1235
Station a.
1215
1000
940
Line Cb
Station b.
600
fence
595
580
Line AB
Line AD
0
Line AF
30, 65, 100
55
50, 85
60, 100
30
55
90, 120
80, 130
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fence
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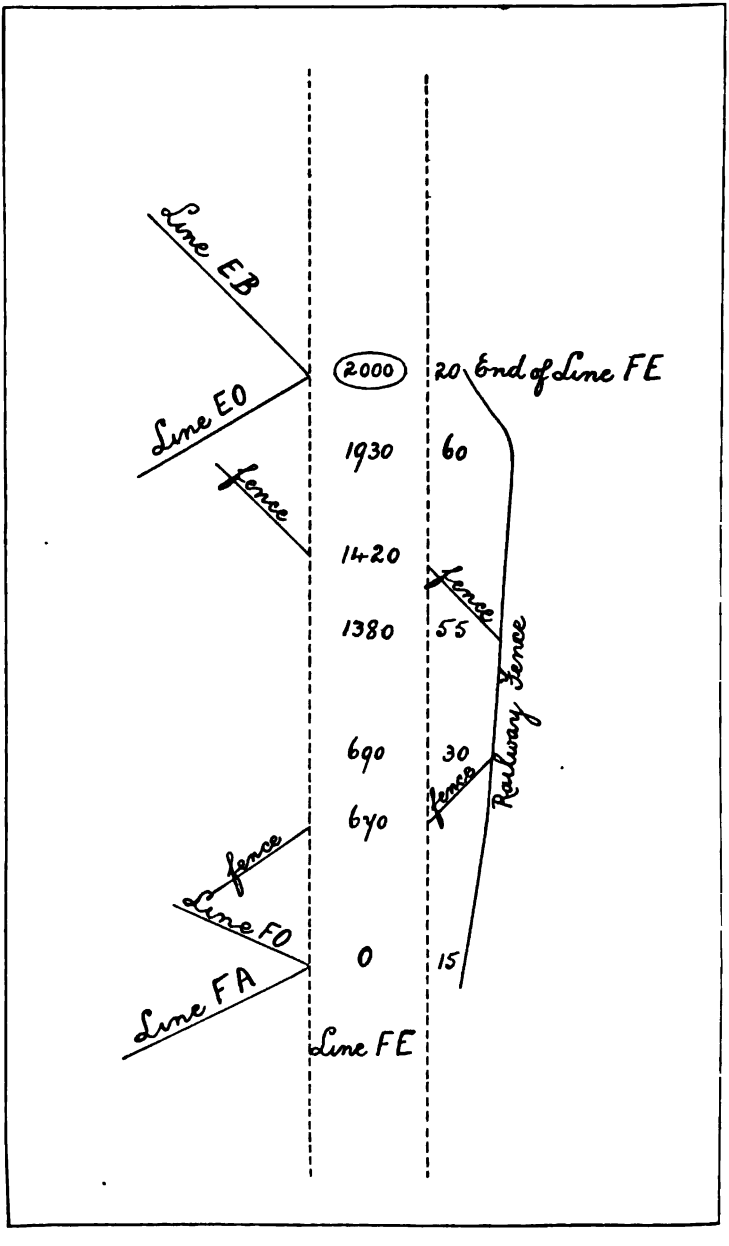
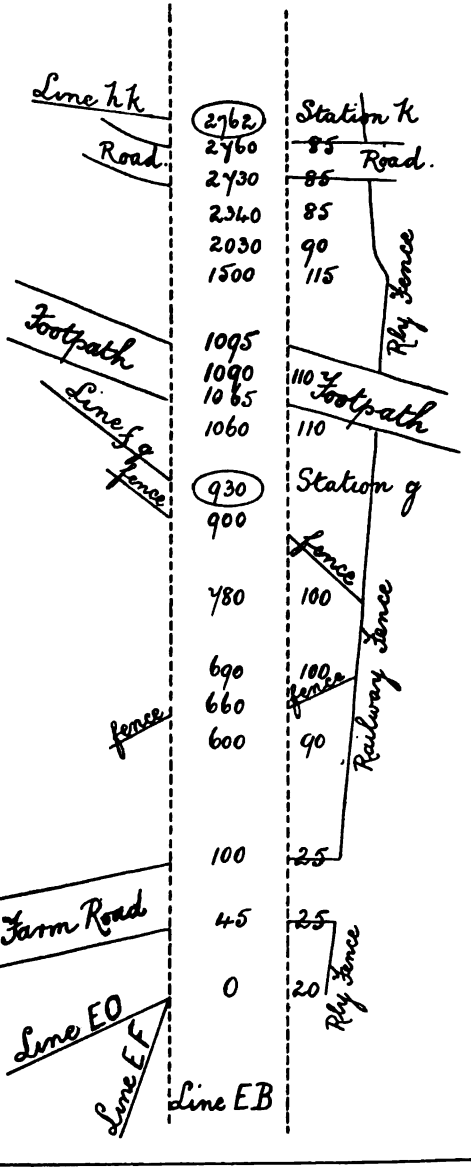
35, 80
Road
Road
Road
Road


Line EB
Line EO
Fence
Fence
Fence
Fence
Fence
Line FO
Line FA
Line FE
2000
1930
1420
1380
690
640
0
20
60
55
30
15
End of Line FE
Railway Fence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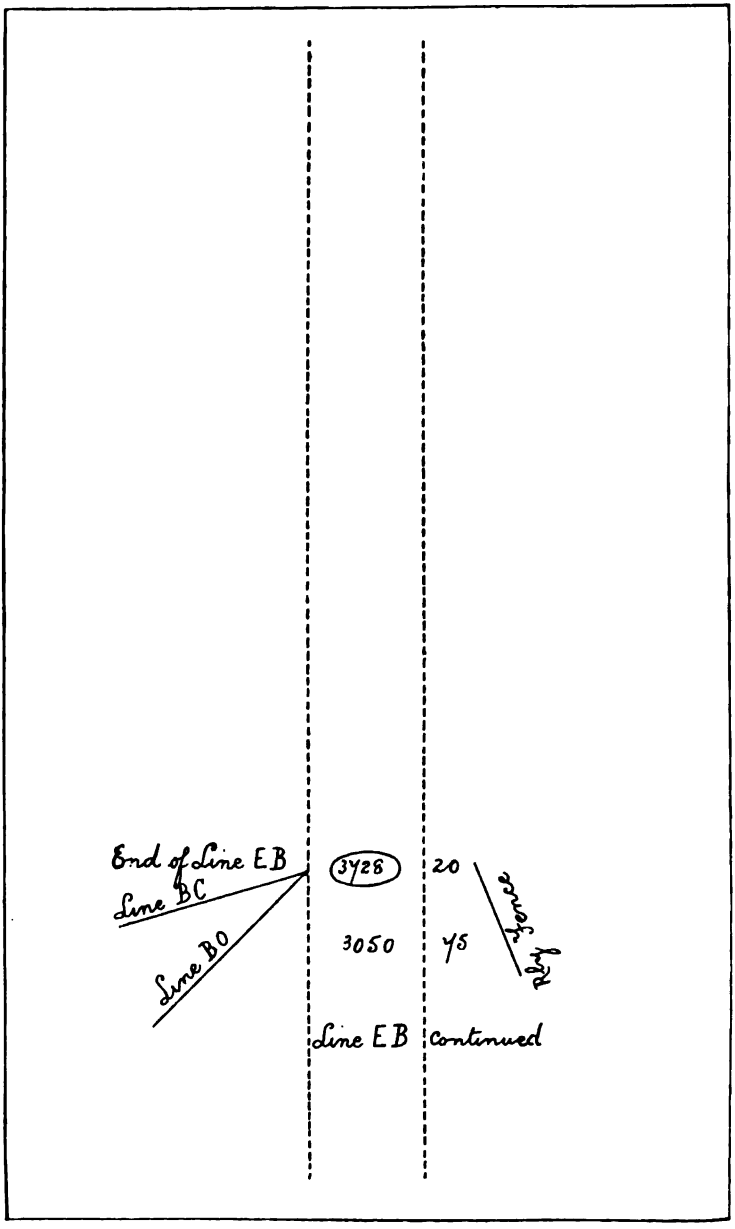
Road.
2762
2760
2730
2340
2030
1500
Station k
85
85
85
90
115
Road.
Rly Fence
Footpath
1095
1090
1085
1060
110
Footpath
Line fg
Fence
930
900
Station g
Fence
980
100
690
660
600
100
Fence
Fence
Railway Fence
90
100
25
Farm Road
45
25
0
20
Rly Fence
Line EO
Line EF
Line EB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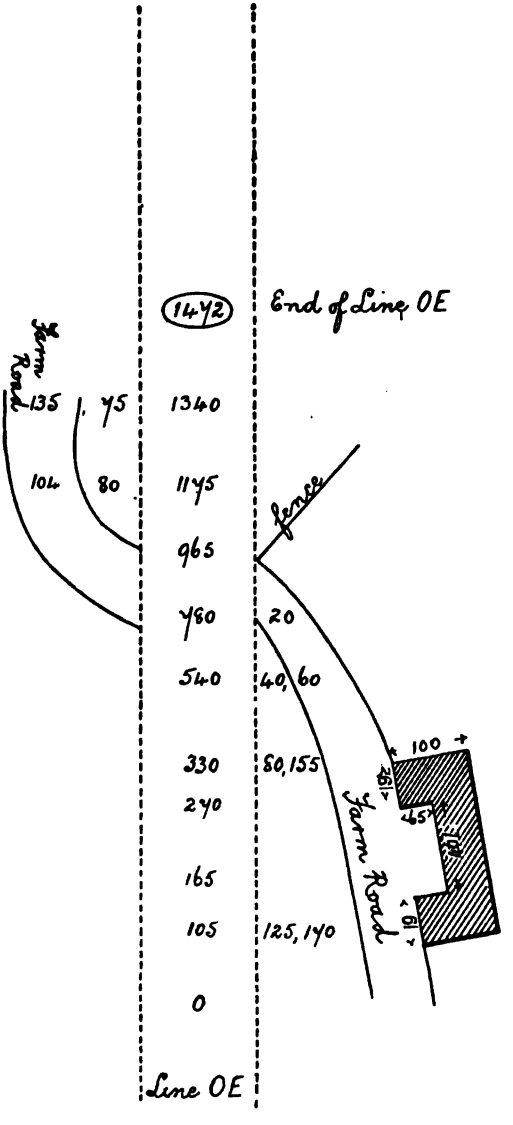
Line BC
Line BO
3428
3050
20
45
Ref. to mee
Line EB continued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End of Line OE
Farm Road
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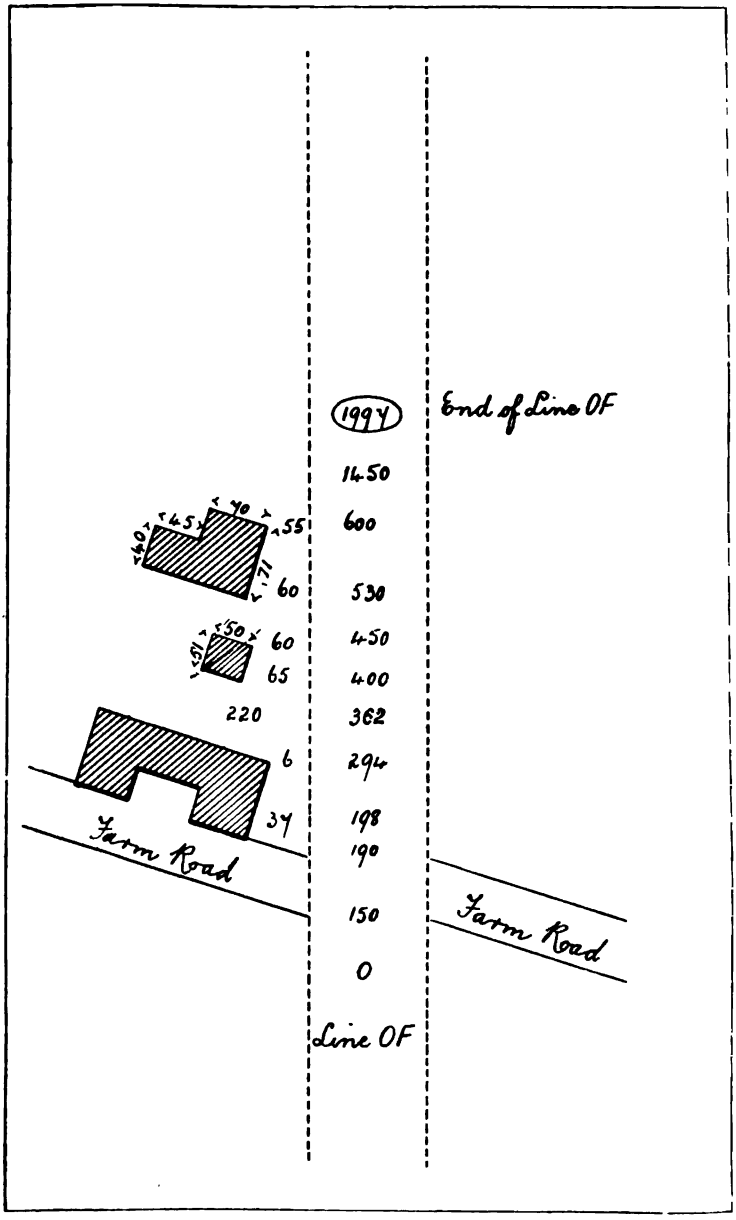
145
104
80
1340
1145
965
960
540
330
240
165
105
0
Line OE
fence
20
40, 60
80, 155
125, 140
100
25
45
Farm Road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End of Line OF
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600
530
450
400
362
294
198
190
150
0
Line OF
40
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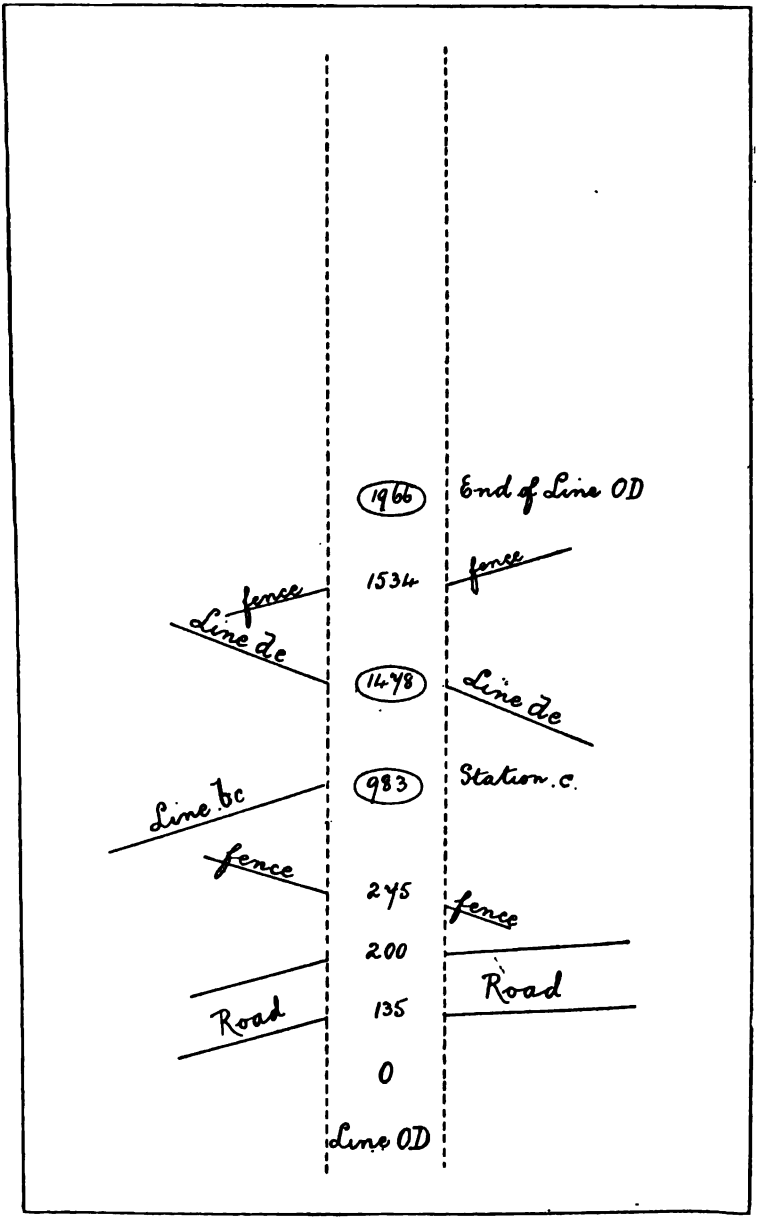
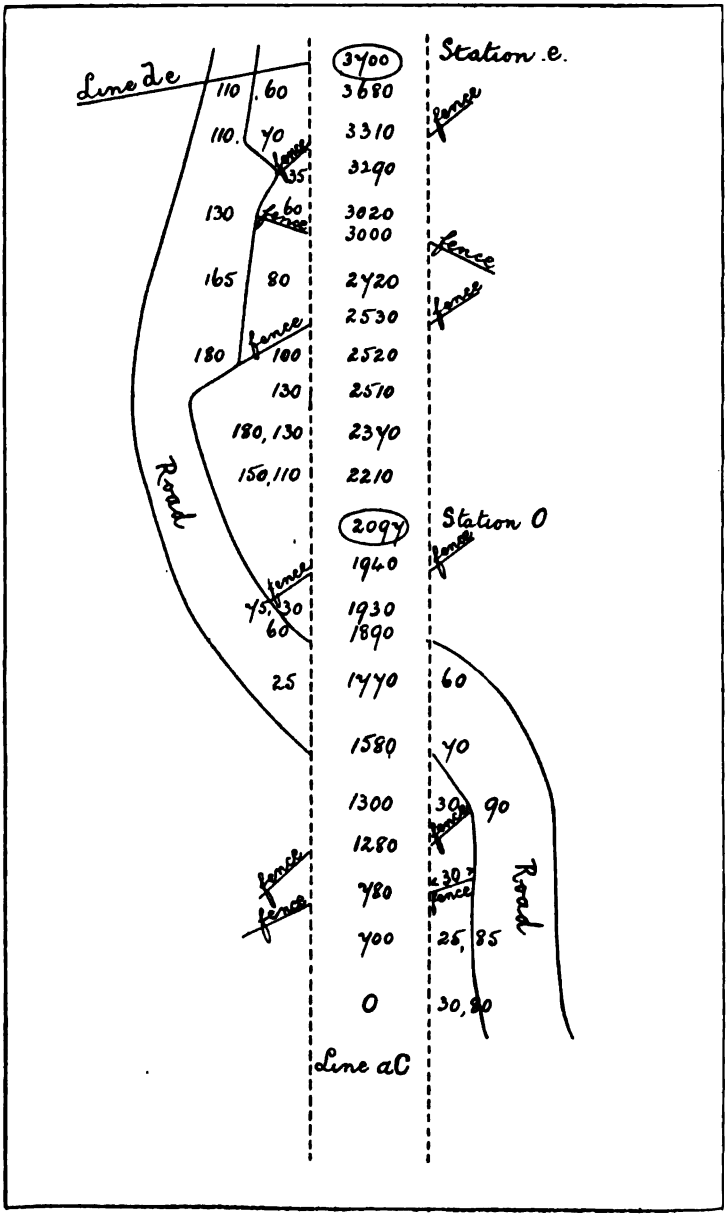
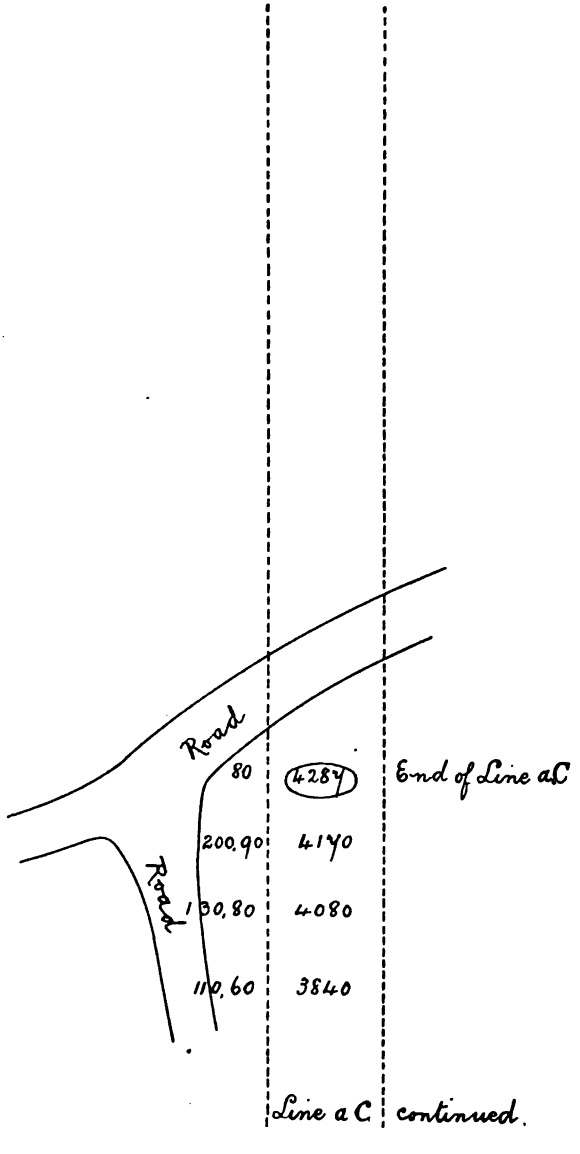
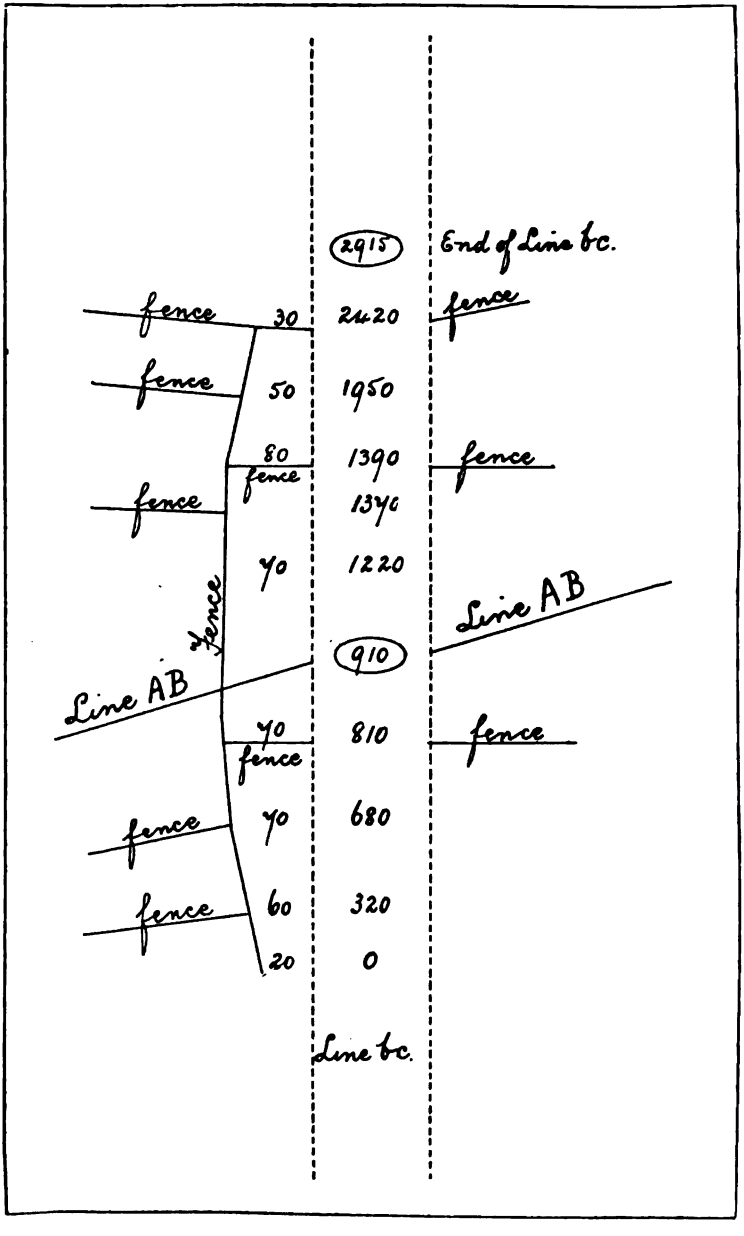
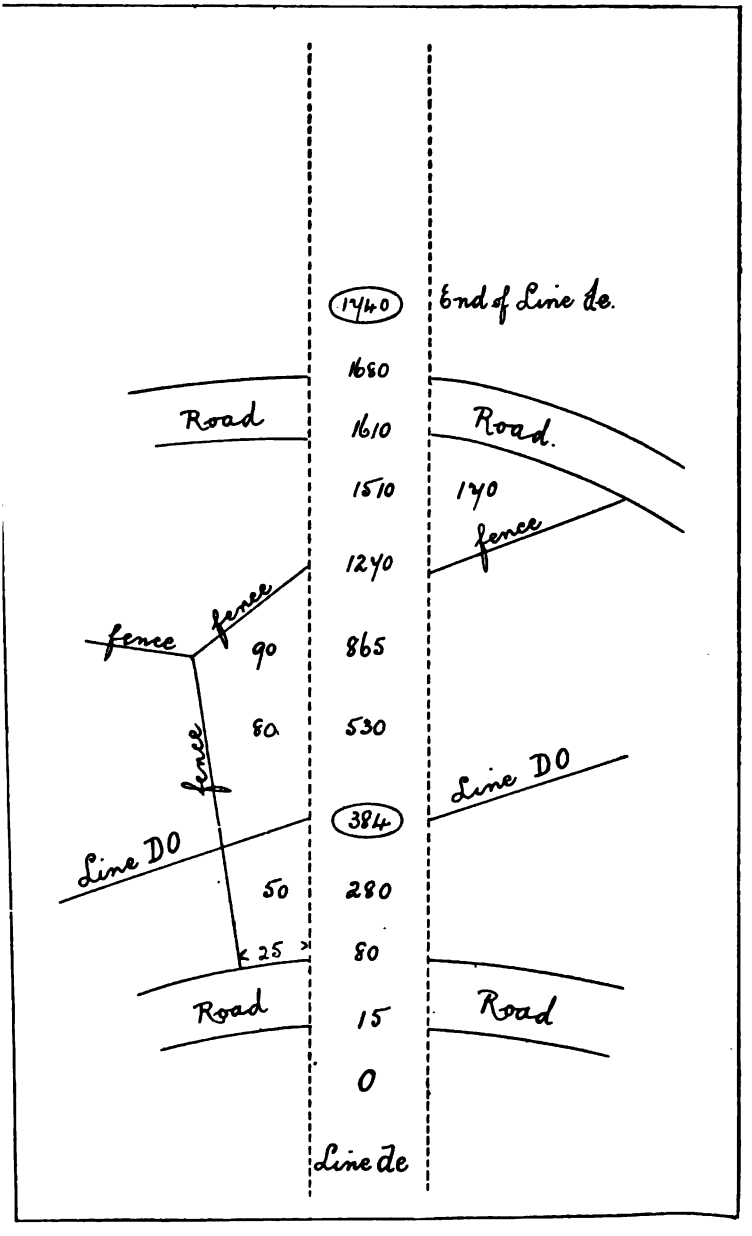
40
71
60
50
60
65
220
6
34
Farm Road
Farm Road

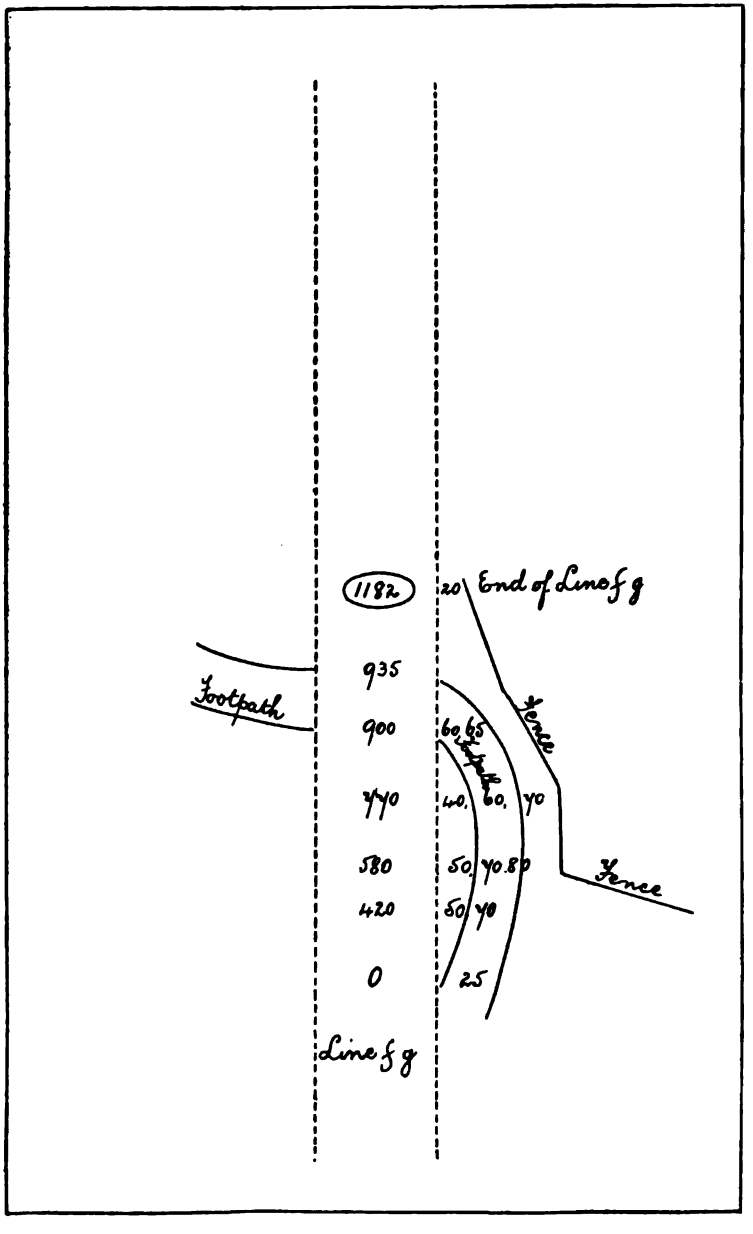

1966
End of Line OD
1534
fence
fence
Line de
1478
Line de
983
Station. c.
Line bc
fence
245
fence
200
Road
135
Road
0
Line OD

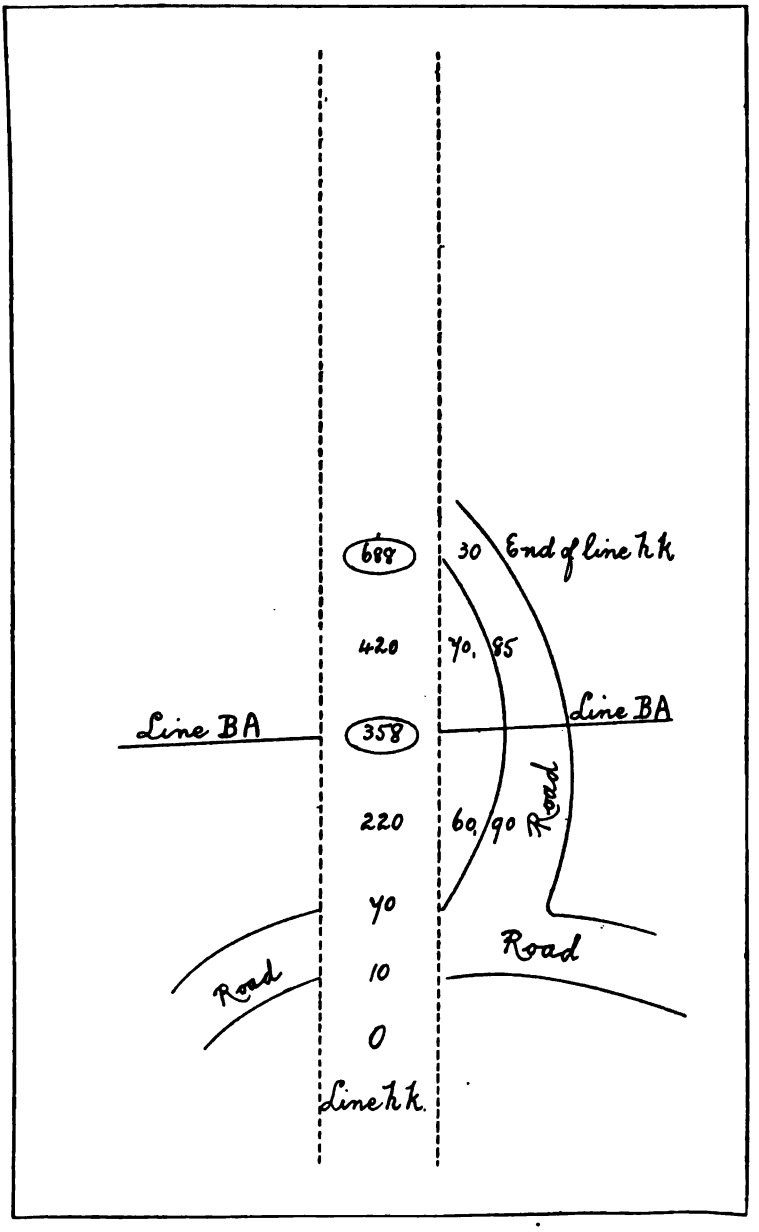

Line de
Station e.
3400
3680
3310
3290
3020
3000
2720
2530
2520
2510
2370
2210
2094
Station O
1940
1930
1890
1770
1580
1300
1280
980
900
0
Line aC
110
60
110
70
35
130
60
165
80
180
100
130
180, 130
150, 110
75, 30
60
25
60
70
30
90
30
25, 85
30, 80
Fence
Road


Road
Road
80
4284
End of Line aC
200.90
4190
130.80
4080
110.60
3840
Line aC continued.


2915
End of Line bc.
fence
30
2420
fence
fence
50
1950
80
1390
fence
fence
fence
1340
70
1220
fence
Line AB
Line AB
910
70
810
fence
fence
fence
70
680
fence
60
320
20
0
Line bc.


1740
End of Line de.
1680
Road
1610
Road.
1510
140
fence
1240
fence
fence
90
865
80
530
fence
Line DO
384
Line DO
50
280
25
80
Road
15
Road
0
Line de


1182
20
End of Line f g
935
Footpath
900
60 65
Fence
740
40.
60.
70
580
50.
70. 80
Fence
420
50
70
0
25
Line f g


688
30
End of line h k
420
40. 85
Line BA
358
Line BA
220
60. 90
Road
40
Road
Road
10
0
Line h k.

The full details of this survey are given in the field book, pages 21 to 39, and it will be a good exercise for the student to plot the survey from the field book to a scale of say 2 chains to 1 inch. The line AB was first measured. Referring to the field book, it will be seen that we start at the foot of the page with the words "Line AB." Then the beginning of the line is marked zero, while the chain line AD is shown to the left and the chain line AF to the right, both converging to the commencement of the line AB.

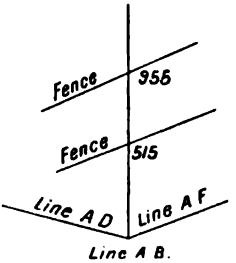


Fig. 25.—Field Book.

The first fence is crossed at 5.15 chains or 515 links (it is usual to note all distances in links), and is denoted by the lines to right and left of 515 with the word fence written along them.

It will be noticed that the use of two red lines to make a central column makes it appear as if there were a break in the continuity of the fence, which may perhaps be a little confusing at first, but a little practice in keeping the field book will soon accustom one to this. Some surveyors prefer to use only a single red line to represent the chain line, as shown in Fig. 25.

In this case the fences, boundaries, &c., crossed are represented by continuous unbroken lines crossing the chain line, as shown at 515 and 955, Fig. 25. The two red lines forming a central column are, however, almost universally used, as this system has the advantage that it keeps the distances on the chain line distinctly separate from the offsets, which are confined to the spaces to right or left of the central column. In the system shown in Fig. 25, the figures representing the distances on the chain line are apt to get confused with those representing the offsets.

At 955 again a fence is crossed and noted, as shown in the field book. At 1120 a fence crosses the chain line, while also at this point another fence runs off towards the right hand, as shown. At 1218 the subsidiary chain line *cb* is crossed. As previously stated, this and all other stations should be marked, as shown, by an oval drawn round the figures representing the distance of the station on the chain line.

A road is crossed at 2560 and 2640, these being the distances at which the chain line cuts the fences of the road, and is indi-

cated in the field book, as shown. Similarly a footpath is crossed at 5000 and 5040, and again a road at 6040 and 6065.

The stations o and *f* and the intersection of the line *hk* are marked by ovals round the figures representing the distance on the chain line, as shown. The end of the line is at 6985, and is marked "End of line AB," where also the lines CB and BE are shown converging to the end of the line AB.

It so happens that there are no offsets on this line, but there are plenty of offsets on the other lines, as for instance on line BC, which crosses the road twice, as shown on page 23 of field book. The first offsets are at 205, where we have 30, 80 to the right. This means that it is 30 links to the nearer fence of the road and 80 links to the further side. At 500 the line sketched to represent the nearer side of the road is run into the central column, which means that this fence of the road is crossed at 500 on the chain line, while the offset 75 refers to the distance from the chain line to the further side of the road. This side of the road crosses in turn at 900, as shown by the line being run into the central column. The sketch of the road is continued again, by the lines starting from 500 and 900, on the other side of the central column, and the offsets 70, 10 at 970 and 75, 35 at 1200, &c., refer to the boundaries of the road as before.

The remainder of the survey is conducted in the same manner, there being no special difficulties calling for remark.

On the line OE we have to take offsets to the building shown. We have 125, 170 at 105; 125 being the distance to the farm road, while 170 is to the corner of the building. The next angle of the building is opposite 165 on the chain line, and again the other angle is opposite 270 on the chain line. The other end of the building is opposite 330, the distance to the corner being 155. The width of the building is 100 links, as shown, and the set-back in front 65 links, as shown on sketch. The building being square at all the angles, these are all the offsets and dimensions necessary to fix it, but as old farm buildings are often not square built, it is usually wise to measure all round them and take plenty of offsets to their angles. Again on line OF this same building is taken, and also two others, the offsets being as shown.

Fixing Positions of Buildings.—When a building is lying at an angle to the chain line, the most expeditious way to survey

it is to find the points *b* and *c* on the chain line, in line with the sides of the house AF and BC respectively (Fig. 26). The corner

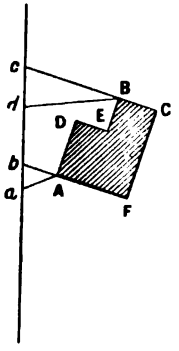


Fig. 26.

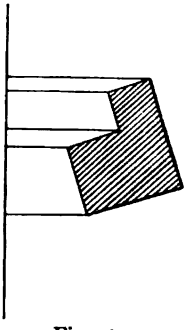


Fig. 27.

Offsets to Buildings.

A is then fixed by the triangle A*ab* and the corner B by the triangle *c*B*d*, while the lines *b*A and *c*B being produced, give the directions of the sides of the house, which should then be measured all round.

This is much quicker than taking offsets perpendicularly opposite all the angles, as shown in Fig. 27.

Ranging Line across Elevations or Depressions.—In ranging out a long line, there may be depressions or elevations in the ground at intermediate points in it, at which places the ranging rods marking the ends of the line may not be visible. In order to be able to chain across these it will be necessary to send forward a man and range in rods on each side of the depression and on the elevation. Thus in Fig. 28 the ranging rods C, D, and E being lined in from A upon reaching the low ground between C and D, the chain may be kept in line with D and E, although B is not visible.

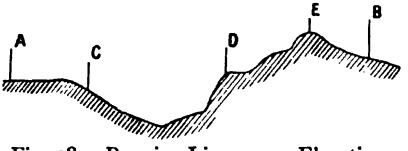


Fig. 28.—Ranging Line across Elevations or Depressions.

Marking Stations with Pegs.—In a large and important survey the stations should be permanently marked by means of

pegs driven in to within an inch or two of the ground. These pegs should be about 12 to 15 in. long by about 1 to 1½ in. square, one end sharpened to a point, and may be readily obtained in hundreds from the nearest sawmill. Failing this, the local carpenter will supply them.

Ranging Intermediate Points in Long Lines.—In ranging long lines it is a good plan to send a man forward with ranging rods or laths, with instructions to range one in and leave it at each fence or hedge crossed, as it often happens that the line is obscured at intermediate points by high hedges, although it may be all visible from the extremities.

Cutting Down Hedges and Fences.—The greatest prudence and caution should be exercised in cutting down hedges, fences, or trees, or in causing unnecessary damage of any description. There have been cases where much unpleasantness has arisen, and even strong opposition in Parliament to important projects, through careless and needless damage to property.

Clearing-up Ground after Survey.—The ground should be cleared of all ranging rods, poles, laths, whites, and litter of every sort, and left as much as possible in the same condition as it was previous to the survey.

Plotting the Survey: Scales.—Common scales for ordinary small chain surveys are 1 chain to 1 in. and 2 chains to 1 in.; 3, 4, 5, 6, 8, and 10 chains to 1 in. are also often used. The surveyor should be provided with a box of ivory or boxwood scales, consisting of the 10, 20, 30, 40, 50, 60, 80, and 100 scales as they are called. These are scales of 1, 2, 3, 4, &c., chains to 1 in. They are divided into chains and tenth parts of chains or 10 links on one side, and into corresponding divisions of 100 ft. and 10 ft. on the other side. These scales may also be used as scales of 10 ft. to 1 in., 20 ft. to 1 in., and so on, which is the reason that they are usually called the 10, 20, 30, &c., scales; they may also be used as scales of 10 chains to 1 in., 20 chains to 1 in., &c. In each box there is usually a short piece of each scale about 2 in. long in addition to the scale itself. These are used for scaling the short offsets, and are called offset scales, see Plotting Offsets, page 47. Stanley, Great Turnstile, Holborn, London, makes good scales.

In most foreign countries the scale is expressed by the ratio of the linear dimensions of the plotted survey to the actual linear dimensions of the ground surveyed, as $\frac{1}{500}$, $\frac{1}{10000}$, &c. The following are the scales of the Ordnance Survey of the United Kingdom:—25.344 in. to 1 mile or $\frac{1}{2500}$; 6 in. to 1 mile or $\frac{1}{10560}$; 1 in. to 1 mile or $\frac{1}{63360}$. Ordnance Sheets of most large towns are also published on a scale of 10 ft. to 1 mile or $\frac{1}{528}$, and 5 ft. to 1 mile or $\frac{1}{1056}$. The 25 in. Ordnance Scale is the one most used for the general *working* plans and sections of railways and other large public works, the 6 in. scale being used for the *Parliamentary* plans. Ordnance sheets on the scale of 4 miles to 1 in. are also published.

Drawing Instruments.—Probably the most satisfactory way of getting the ordinary drawing instruments is to purchase a case, which may be had at prices ranging from £1. 1s. to £30;

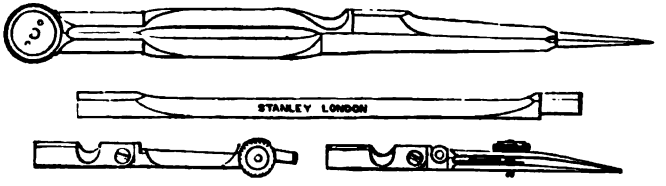


Fig. 29.—Compasses, Lengthening Bar, and Pencil and Ink Points.

but loose drawing instruments may also be purchased as required. At all events the surveyor should have a *pair of compasses* (Fig. 29), with *lengthening bar* and *pencil and ink points*, also a small pair of *pencil and ink bows* (Fig. 30), and a set of *spring*

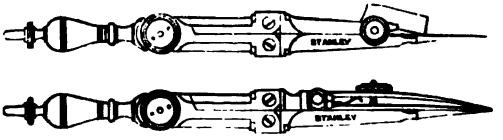


Fig. 30.—Pencil and Ink Bows.

bows for very small circles (Fig. 31). For plotting the triangulation of a survey to a large scale, *beam compasses* (Fig. 32) are required. These are clamped on to a straight-edge, and are used for sweeping out arcs, to get by their intersections

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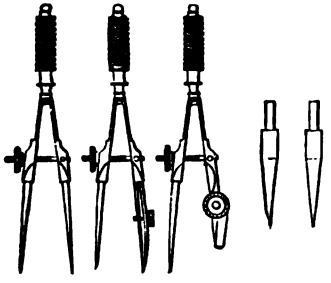

Fig. 31.—Spring Bows.

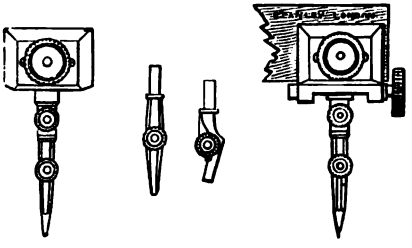

Fig. 32.—Beam Compasses.

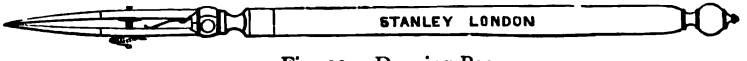

Fig. 33.—Drawing Pen.

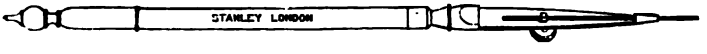

Fig. 34.—Pricker.

Fig. 35.—Parallel Ruler.

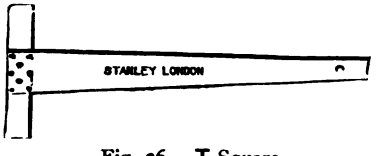

Fig. 36.—T Square.

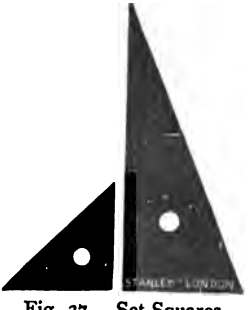

Fig. 37.—Set Squares.

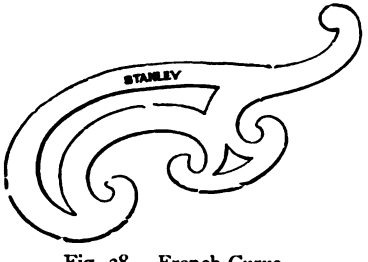

Fig. 38.—French Curve.

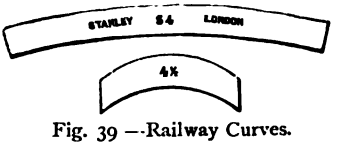

Fig. 39.—Railway Curves.

the apices of the triangles. A couple of good *drawing pens* (Fig. 33) and a *pricker* (Fig. 34) are required. These, with a good *brass parallel ruler* (Fig. 35), a T *square* (Fig. 36), a *steel straight-edge* and a selection of *set squares* (Fig. 37), are about all that are absolutely necessary. *Transparent set squares* have been introduced, and will be found very convenient to use. One or two *French curves* (Fig. 38) are useful for drawing irregular boundaries, and a box of *railway curves* (Fig.

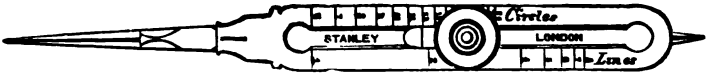

Fig. 40.—Proportional Compasses.

39) is also often required. *Proportional compasses* (Fig. 40) will be found useful for enlarging and reducing plans.

Drawing Paper.—For drawing paper, Whatman's rough or smooth double elephant should be used either unmounted or mounted on cloth. Unmounted paper should be stretched by

Fig. 41.—Drawing Board.

being damped and glued to the *drawing board* (Fig. 41). This makes a better drawing surface than if the paper is merely fastened down with drawing pins.

Plotting.—Before commencing to plot, a scale should always be drawn on the paper, so that if the paper shrinks the scale will shrink equally, and will still apply to the plotted survey. The first thing to do is to plot all the main chain lines and ink them in with a fine blue or red line. These are to be plotted as described for survey of field on page 16, using beam compasses for long lines. The details of the survey may then be plotted. It is a bad plan to attempt to plot the survey from pencil chain lines, as any rubbing out necessitates redrawing bits of the chain line which may be rubbed out, and therefore conduces to inaccuracy and unnecessary trouble. Never commence to plot the details until all the chain

lines have been plotted, together with the tie lines, and the accuracy of the work has been tested by the lines "tieing in" properly. In some cases inked in chain lines may be objected to, and there is then no alternative but to plot from pencil chain lines.

A survey should always be plotted looking north, so that the top and bottom, right and left of the sheet are respectively to N., S., E., and W. It will sometimes be found a good plan to roughly plot the survey lines to a small scale to get the shape of the survey, and so facilitate the laying of it down on the sheet.

Very long lines should not be produced with the straight-edge, but should be laid down by means of a fine thread stretched tight. Intermediate points are marked with a pricker beneath the thread, and the straight-edge is then applied to these and the line drawn in between them.

Plotting Offsets.—In plotting offsets, a quick method is to lay down the long scale along the chain line and keep it down with two or three lead weights. Then by sliding the short offset scale along at right angles to the long scale, the offset at any required distance as read on the long scale may be pricked off on the offset scale.

Colouring.—As a rule a survey should be coloured by flat washes of colour to indicate either the different classes of land, as arable, pasture, &c., or the different owners, &c. The best colours are in cakes, which are rubbed down as required in dishes or palettes (Fig. 42). The following colours are the most useful:—Burnt sienna, burnt umber, sepia, neutral tint, gamboge yellow, Prussian blue, and crimson lake. With these, by mixing, nearly all other tints may be readily got. Always mix *plenty* of colour, as it is often difficult to mix again exactly the same strength of tint. Sable brushes are the best to colour with, and it is useful to have a brush at each end of the stick, one for clean water for shading off edges, &c. Always keep plenty of colour in the brush, and colour towards you. Only the best Indian or Chinese ink should be used for inking in the lines. Inferior qualities run when coloured over.

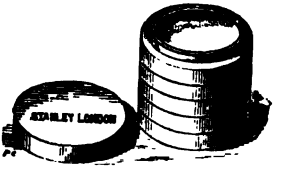

Fig. 42.—Colour Dishes.

Conventional Signs for delineating various objects on Plans.—Some of these are shown in Fig. 43.

Buildings are usually drawn in outline and hatched with lines or coloured with Indian ink or neutral tint. Shade lines should always be used for the outlines of buildings, the light being taken

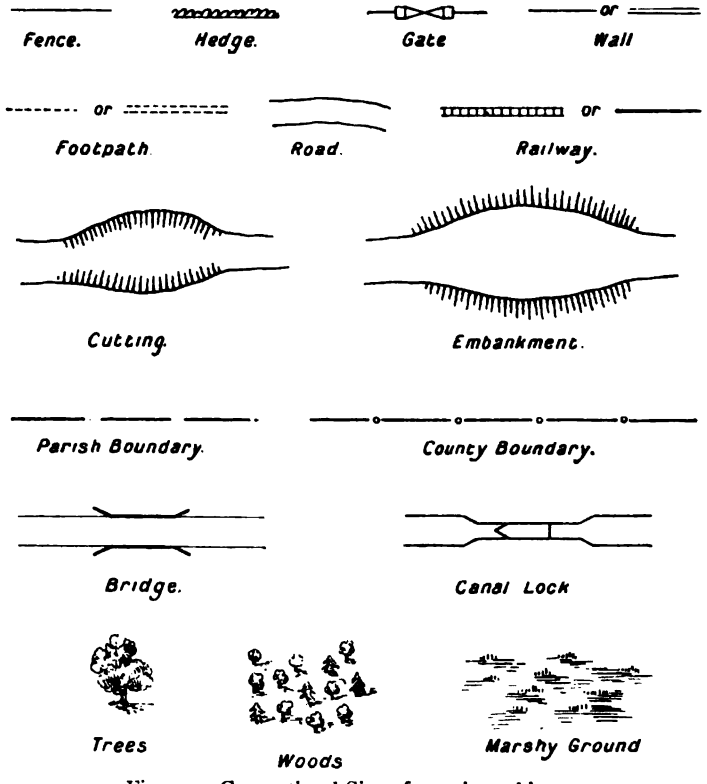

Fig. 43.—Conventional Signs for various objects.

as coming from the left hand top corner of the sheet at an angle of 45°.

North Point.—North points are usually drawn in an ornamental fashion, and should always be shown on a survey; the magnetic north with the date marked on it being drawn with a

dotted line. The direction of magnetic north may be found by the compass (see Chapter II.).

Margin.—As a rule a plain border of a good strong black line looks best, but often ornamental designs are used at the corners. The margin should be an inch or two in width; a very narrow margin spoils the appearance of a plan.

Printing.—Neat printing and italic writing are a great help to the appearance of a plan. Nothing looks worse than bad printing. Plain vertical and sloping block and italics are most used. The set square (Fig. 44) will be found useful for giving the correct slopes of various letters.

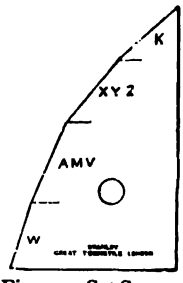

Fig. 44.—Set Square for Printing.

Survey executed with Incorrect Chain.—If a survey has been executed with an incorrect chain, then the true length of any line is given by

True length of any line

$$= \text{measured length of that line} \times \frac{\text{length of incorrect chain}}{\text{length of true chain}}$$

Measures of Length.—The standard British measure of length is the yard, and is the distance between two points on a bar kept in the office of the Exchequer at Westminster, at a temperature of 62° F. and mean atmospheric pressure.

The following are the subdivisions and multiples of the yard used for the purposes of measuring:—

The inch $= \frac{1}{36}$ yd., with binary or decimal subdivisions.
The foot $= \frac{1}{3}$ yd. $= 12$ in.
The fathom $= 2$ yds. $= 6$ ft.
The chain $= 22$ yds. $= 66$ ft.; divided into 4 poles of $5\frac{1}{2}$ yds. or $16\frac{1}{2}$ ft. each, and into 100 links of 7.92 in. each.
The statute mile $= 1{,}760$ yds. $= 5{,}280$ ft.; 1 mile $= 80$ chains $= 8$ furlongs.

The Scottish mile and the Irish mile are obsolete. The Scottish mile was 5,929.6 ft., and the Irish mile 6,720 ft.

The French standard measure of length is the *metre*, which is approximately $\frac{1}{10000000}$ of the distance from the equator to the

pole measured on the meridian of Paris. This was measured at the time of the French Revolution, and recorded in 1799 by two platinum rods of that length at 0° C. Subsequent measurements have, however, proved the *metre* thus established to be considerably too small. It does not therefore accurately represent the above fraction of the meridian.

A *metre* = 3.2808693 ft. = 39.370432 in. British measure, and is divided decimally into *decimetre* = $\frac{1}{10}$ *metre*; *centimetre* = $\frac{1}{100}$ *metre*; and *millimetre* = $\frac{1}{1000}$ *metre*. A *decametre* = 10 *metres*; a *hectometre* = 100 *metres*; a *kilometre* = 1,000 *metres*. A *kilometre* = .6213750 of a British statute mile.

The French measures of the "*systeme usuel*" were in use from about 1812 to 1840, and are occasionally met with in books published during that period. They are as follows:—The *ligne usuel* or line = .09113 in.; the *pouce usuel* or inch = 12 *lignes* = 1.09362 in.; the *pied usuel* or foot = 12 *pouces* = 13.12344 in.; the *aune usuel* or ell = 47.245 in.; the *toise usuel* = 6 *pieds* = 78.74172 in.

The Russian measures of length are as follows:—The *foot* = the British foot; the *sachine* = 7 ft.; the *verst* = 500 *sachine* = 3,500 ft. = .6629 mile.

The Spanish *vara* of Castile = 32.8748 in. = 2 ft. $8\frac{7}{8}$ in. nearly.

In California by law the *vara* = 33.372 in., and the *legua* = 5,000 *varas* = 2.6335 miles.

The United States measures of length as well as weight are by law the same as the British.

Duodecimal subdivisions of the inch are "*the point*" = $\frac{1}{12}$ in.; "*the line*" = 6 *points* = $\frac{1}{12}$ in. These are, however, rarely used.

Measures of Area.—The standard measures of area are as follows:—

The square inch.
The square foot = 144 sq. in.
The square yard = 9 sq. ft.
The acre = 10 sq. chains. The acre is divided into 4 roods = 160 poles, and is = 4,840 sq. yds. and 43,560 sq. ft.; 1 rood = 40 poles; 1 pole = $30\frac{1}{4}$ sq. yds.
The square mile = 640 acres = 3,097,600 sq. yds. = 27,878,400 sq. ft.

It is useful to bear in mind in estimating acreages by the eye, that a strip of ground 10 chains long and 1 chain wide = 1 acre; also a strip 1 mile long and 1 chain wide = 8 acres.

The French measures of area are the *square millimetre*, *square centimetre*, *square decimetre*, *square metre* or *centiare*, *square decametre* or *are*, *square hectometre* or *hectare*, and the *square kilometre*.

The Spanish *fanegada* = 1.5871 acres.

Computation of Areas.—The area of a survey should be calculated in square chains and decimals. It is then reduced to acres and decimals by simply moving the decimal point one place to the left. If required, it is then reduced to roods and poles by multiplying the decimal fraction of an acre by 4 and 40 successively.

For example, suppose the area as measured from the plotted survey is 437.85 sq. chains.

Thus we have area = 43.785 acres

$$\underline{\quad 4}$$

3.140 roods

$$\underline{\quad 40}$$

5.600 poles

or area = 43 acres 3 roods 5.60 poles.

The area of a survey may of course be computed in square feet, square yards, square metres or any other unit, either by using a 100 ft., 20 metre, &c., chain for making the survey, and taking the measurements in feet, metres, &c., when the computed area will be in square feet, square metres, &c.; or by multiplying the area as computed in any given unit of measurement by the proper number to reduce it to the area in the required unit of measurement.

Give and Take Lines.—In order to calculate the area of a survey it should be completely divided into triangles, the irregular boundaries being included in these by means of "give and take lines," as shown in Fig. 45, where the boundary is equalised by the "give and take line" AB. The part above the line AB which is omitted is made equal to the part below it which is included in the area.

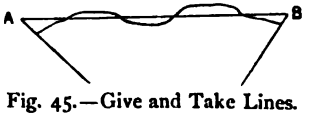

Fig. 45.—Give and Take Lines.

The best way to determine the line AB is to stretch a fine thread from A to B. The parts included and omitted then present themselves to the eye simultaneously, and may be very nearly equalised by moving the thread up or down.*

Area of Triangles: Ordinary Method.—The areas of the triangles are calculated from the formula, area triangle = ½ base × perpendicular height, the base and perpendicular height of each triangle being scaled from the plotted survey. The measurements being scaled in chains and decimals, the resulting area will be in square chains and decimals, which is easily reduced to acres, or acres, roods, and poles, as above described. As a rule it will be found necessary to redivide the survey into triangles by give and take lines for the purpose of calculating the area, as the triangles formed by the chain lines will not in general include the true area. For this purpose, to avoid disfiguring the plotted survey, it will be found a good plan to pin a sheet of tracing paper over it and draw the triangles on the tracing paper.

Calculation of Area from Figures in Field Book.—When more accuracy is required, the area may be calculated from the figures in field book alone. The area of each triangle is then calculated from the formula, area = $\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$, where $s = \frac{1}{2}$ sum of sides, and a, b, c each denote one of the sides of the triangle; a, b, and c are the measured lengths taken from the field book. The part between the triangles and any irregular boundary is then calculated from the offsets, and added or deducted as the case may be. The calculation of this part of the area is as follows:—Each portion included between two adjacent offsets, the chain line and the irregular boundary, is considered to be a trapezoid, and its area = ½ sum of offsets × distance between them on the chain line; the total being the sum of all these trapezoids. The offsets and distances between them on the chain lines are taken from the figures in the field book. The method of calculation from figures in field book only is very laborious, and only adopted when great accuracy is required.

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* Give and take lines may be drawn geometrically correct with a couple of set squares, but the above method is usually near enough if care and judgment is used.

Area of Irregular Strip of Ground.—A convenient method of calculating the area of an irregular strip of ground, as shown in Fig. 46, is to run a line AB about the middle of the plot. Divide

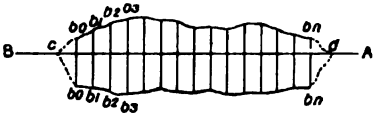

Fig. 46.—Area of Irregular Strip of Ground.

it into any number of equal parts by the ordinates b_0, b_1, b_2, &c. . . . b_n.

Then area = $\frac{1}{2}$ sum of first and last ordinates plus sum of intermediate ordinates × common distance apart d, or

$$\text{Area} = d\left(\frac{b_0 b_0}{2} + b_1 b_1 + b_2 b_2 + \text{\&c.} + \frac{b_n b_n}{2}\right).$$

If the ends converge to a point as at c and d, then

$$\text{Area} = d\,(b_0 b_0 + b_1 b_1 + b_2 b_2 + \text{\&c. . . . } + b_n b_n).$$

Simpson's Rule for Calculation of Areas.—For greater accuracy Simpson's rule may be used. This is based on the assumption that the curved boundary consists of short parabolic arcs. In this case an *even* number of divisions of the line AB must be made, *i.e.*, an odd number of ordinates. The area is then equal to sum of first and last ordinates plus twice sum of even ordinates plus four times sum of odd ordinates, multiplied by one-third of the common distance apart d, or

$$\text{Area} = \left\{ b_0 b_0 + b_n b_n + 2\,(b_2 b_2 + b_4 b_4 + \text{\&c. . . . }) + 4\,(b_1 b_1 + b_3 b_3 + \text{\&c. . . . }) \right\} \frac{d}{3}.$$

Instruments for Measuring Areas: Amsler's Planimeter.—By far the most convenient and accurate instrument for measuring areas is Amsler's planimeter (Fig. 47). It consists of two arms having a rolling wheel with index and vernier scale and also a revolution counter, at the junction of the two arms. At the end of one arm is a needle point with weight which is fixed into the paper. At the end of the other arm there is a tracing point. The instrument when in use rests on the roller,

the weighted needle, and the tracing point. In order to get the area of any figure with the planimeter it is only necessary to pass completely round the boundary with the tracing point. This causes the rolling wheel to revolve and thus indicate the number of square inches of area on the plan included in the boundary passed over by the tracing point. A simple multiplication by the number of units of area per square inch of plan then gives the actual area included within the boundary passed round. Another form of the instrument (that shown in Fig. 47) is made so that the arm on which the tracing point is, can be slid out and in and so lengthened or shortened. On this arm there are marks corre-

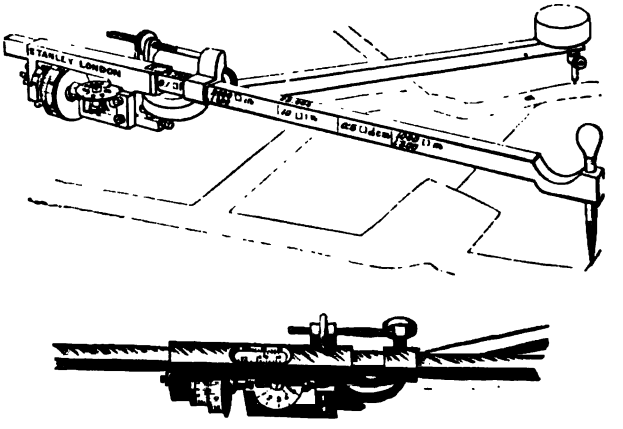

Fig. 47.—Amsler's Planimeter.

sponding to different scales, as for instance $\frac{1}{2500}$ or the 25 in. Ordnance Scale. By setting the arm to this mark and clamping it, the actual area in square feet, or acres, square yards, &c., will be indicated, in place of square inches of plan, when used on a 25 in. Ordnance Sheet.

When very great accuracy is required, the rolling wheel travels on a special disc furnished on the instrument, which corrects any irregularities of the paper.

Unless on very old and crumpled tracings or plans, the ordinary planimeter will be found to give very accurate results. In the case of very irregular boundaries, the planimeter will give the area

more accurately than any other method, even if the ordinates are taken very close together, and the area calculated by any of the methods given above.

Amsler's planimeter may be had at prices ranging from £2. 8s. to £39. For a very detailed description of the planimeter see Heather's "Mathematical Instruments."

The Hatchet Planimeter.—The hatchet planimeter was invented by Captain Prytz of the Danish Army. It is merely a rod 300 millimetres (11.81 in.) long, bent at one end to a tracing point, and at the other to a hatchet-shaped edge. If any given figure is traced round with the point, and the point of contact of the hatchet with the paper marked at the beginning and at the end of the operation, the distance between these two points multiplied by the length of the rod gives the area of the figure. *The greatest dimension of the figure must not exceed half the length of the rod*, but very large figures may be divided into parts, and the area of each taken out separately. Three hundred millimetres has been found in practice the most convenient length, but for English work an even number of inches, say 10, would be most suitable. A soft medium, as blotting paper, should be placed between the paper and the drawing board, and the rod should be weighted with small weights to obviate side slip. With these precautions excellent results may be obtained.

Stanley's Computing Scale.—Next to the planimeter, Stanley's computing scale is the most useful instrument for measuring areas (Fig. 48). It consists of a plainly divided scale reading to acres and decimals on one side, and acres, roods, and

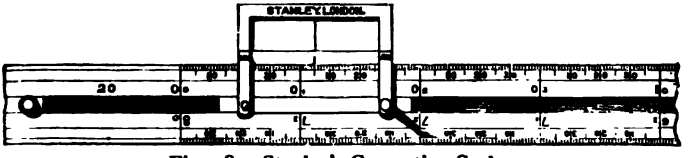

Fig. 48.—Stanley's Computing Scale.

poles on the other. It has a groove throughout its length with a sliding indicator frame having a fine wire stretched on it, the space being open to allow the plan to be seen underneath. To

use the scale, a sheet of tracing paper, having lines ruled on it at an equal distance apart, is spread over the area to be measured (see Fig. 49). The process of measuring the area simply consists in measuring the lengths of the successive horizontal strips *abcd*, *efgh*, &c., until the whole area has been gone over. To do this, place the scale so that the wire on the slide is in the position *ab*, and the scale itself is parallel to the horizontal lines on the tracing paper. The slide is then moved along to the position *cd*. The scale itself is then bodily moved until the wire of the slide is in the position *ef*, when the slide is moved along to *gh*, care being taken while moving the scale that the position of the slide is not changed. By this means the length of each strip is measured in succession, and the total length of the sum of all the strips is read off the scale at the conclusion of the operation in units of area.

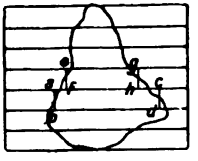

Fig. 49.—Areas by Computing Scale.

The width of the strips is usually about $\frac{1}{4}$ in., and the scale is so constructed as to read so many acres per lineal inch, $\frac{1}{4}$ in. wide, according to the scale for which it is intended. It is thus obvious that the computing scale only applies to plans drawn to the scale for which it is made. It may, however, be used for other scales by multiplying the result as scaled by a constant which will vary for each scale and is easily calculated. The tracing paper has to be twisted about until the whole area is included between two of the parallel lines as shown in Fig. 49.

Area of Survey executed with Incorrect Chain.—The true area of a survey which has been executed with an incorrect chain may be found as follows:—

True area of survey

$$= \frac{(\text{incorrect length of chain})^2}{(\text{correct length of chain})^2} \times \text{calculated area of survey.}$$

Copying Plans.—The most common method of copying plans is to trace them on tracing paper or tracing linen. When a tracing is very much crumpled or dirty, the use of a "copying glass" facilitates operations. This consists of a strong sheet of plate glass in a wooden frame with sometimes the addition of a mirror underneath to reflect light upwards. The plan to be

copied is placed on the glass, and the light underneath enables the tracing to be done more readily. Tracing even on ordinary drawing paper can be done with a copying glass. A plan may be copied by pricking through the salient points on to another sheet placed underneath it. These points are then joined, and the minor details plotted in. Pricking through is apt to spoil a plan, and is most useful when the plan consists of straight lines principally. For copying *tracings* various photographic processes are now employed. The ferro-prussiate process gives white lines on a blue ground. Specially prepared papers may also be had which give black lines on a white ground. The tracing to be copied is stretched in a glass frame over the prepared paper, and the face of the tracing exposed to the sun. The prepared paper is then simply washed in clean water immediately on being taken out of the frame, and the parts which were under the lines on the tracing and were unacted upon by the sun come out in white or black as the case may be. If the tracing is fairly clean and not crumpled, these processes give very good copies.

Another method of copying a plan is to trace it and transfer it to another sheet of paper by placing the tracing on it with a sheet of transfer paper between them. By going over the lines of the tracing with a hard pencil or style we get a copy on the sheet of paper underneath, which has then to be inked in.

Enlarging and Reducing Plans.—The only really accurate method of enlarging and reducing plans is to replot them from the field books to the required scale. For ordinary purposes, however, a fairly accurate copy, enlargement, or reduction may be made by ruling the whole plot into squares 1 or 2 in. in the side. The same number of squares is ruled on another sheet to the required scale, and the copy is made by transferring the part included in each square separately. Each point is fixed by scaling its rectangular co-ordinates, one of the corners of a square being taken as the origin. For this purpose the proportional compasses shown in Fig. 40 are useful.

To avoid spoiling the plotted survey, it is a good plan to cover it with a sheet of tracing paper, and draw the squares upon it.

The Pantagraph.—This is an instrument for copying, en-

larging, or reducing plans by simply going over the lines with a tracing point. It is shown in Fig. 50.

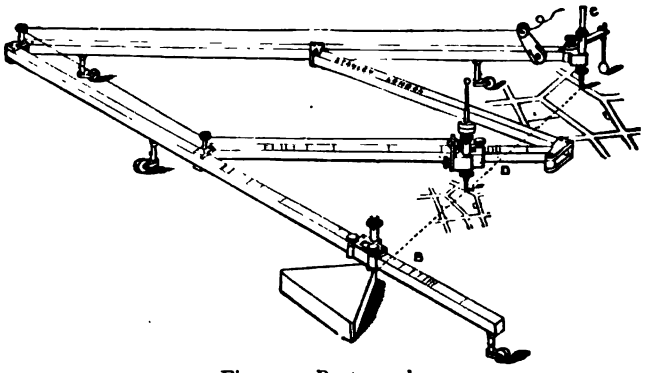

Fig. 50.—Pantagraph.

The Eidograph.—This is a similar instrument, and was invented by Professor Wallace to remedy the unsteadiness of the pantagraph, which arises from its numerous joints and supports upon the paper.

As a rule, neither the pantagraph nor the eidograph is a satisfactory instrument. They require a great deal of room on the drawing table, and for copying, enlarging, or reducing plans, one of the methods already described is recommended. Full descriptions of both the pantagraph and the eidograph are to be found in Heather's "Mathematical Instruments."*

Lithographing Plans—When a great many copies of plans are required, it is usual to have them lithographed. The process of lithographing consists of first making a tracing of the plan in specially prepared lithographic ink. This is laid face downwards on the stone, and the copy transferred to it. From the stone as many copies as are required are printed off. As the tracing is damped before it is laid on the stone, it expands, and sometimes gets stretched unequally. The impression is therefore usually larger than the original. The paper on which the copies are taken off the

* To get really satisfactory results with the pantagraph and eidograph, very well made and expensive instruments are necessary, and an extra large table to use them on.

stone being also damp during the process, shrinks after drying, which to some extent counteracts the expansion and stretching of the tracing. As a rule, however, lithographic copies are not to be depended on when much accuracy is required. All drawings to be lithographed should have a scale drawn on them so that it may be affected by the expansion and shrinkage along with the drawing. Even then the lithographs will often be found not to scale.

Reducing Plans by Photography.—The 6 in. Ordnance Sheets are reduced from the 25 in. sheets by photography. The details are then traced from the photograph, and the tracing is placed on a copper plate. The lines are then gone over with a sharp instrument which cuts through the tracing and indents the copper underneath it. When great accuracy is required, as in the case of the main stations, the points are *plotted* on the copper. Engravings from copper usually shrink, as they are taken off the copper when damp, and all measurements should therefore be taken with the scale drawn on the sheet. A process for photographing the details directly on to copper or zinc has also been introduced.

CHAPTER II.

SURVEYING WITH THE AID OF ANGULAR INSTRUMENTS.

Instruments: Theodolite.—By far the most useful instrument to the surveyor is the theodolite. The chief use of the theodolite is for measuring and setting out horizontal and vertical angles and for ranging out lines. Fig. 51 shows the instrument, as made by W. F. Stanley, Great Turnstile, Holborn, London. Fig. 52 shows Cooke's transit theodolite, as made by Messrs T. Cooke & Sons, York, and 8 Victoria Street, Westminster. Very good instruments are also made by Messrs Troughton & Simms, 138 Fleet Street, London (see Fig. 53). Each maker's instrument varies in small details of construction, but the main features in all are the same.

ab is the *telescope*, which is simply an ordinary telescope with the addition of a *diaphragm c*. The *diaphragm* is either a glass with fine lines etched on it, or a brass ring having fine spider's hairs or wires stretched across it, as shown in Fig. 54. Stadia hairs (see Chapter VI.) should be added as shown at *s*, *s*. The diaphragm is held in place by the four capstan screws *d*, *d*, *d*, *d*, Figs. 52 and 54, and is adjusted either horizontally or vertically by means of these screws. When properly adjusted, the intersection of the hairs at the centre of the diaphragm is in the axis or "collimation line" of the telescope. The telescope is focussed by moving the object glass at *a* out or in by means of a milled headed screw at the side of the telescope near the eyepiece (not shown in Fig. 52). When properly focussed, the image of the object observed coincides with the diaphragm. The image is viewed through the *eyepiece b*, which magnifies it. The eyepiece is adjusted by pulling it out or in until a clear view of the cross hairs of the diaphragm is obtained. By the addition of extra glasses an erect image in place of the usual inverted image may be obtained, but at the expense of the power of the telescope. There is no special advantage in having

an erect image, and one soon gets accustomed to the inverted image after a little practice.

The *spirit level e* is usually set on the top of the telescope, see Stanley's instrument (Fig. 51). In Cooke's instrument (Fig. 52) it is attached near the vertical circle. It is fixed by the capstan screws *f, f,* by means of which it is made parallel to the collimation

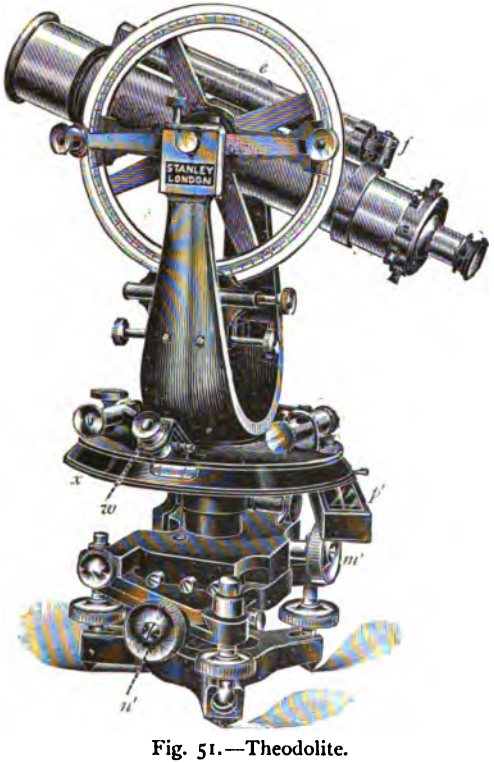

Fig. 51.—Theodolite.

line of the telescope, so that when the bubble is in the centre of its run the collimation line of the telescope is horizontal.

The *vertical circle g* is fixed to the *horizontal axis* upon which the telescope revolves, and rotates with the telescope. It is usually divided into four quadrants, and is read by the *verniers h, h.* When the line of collimation is horizontal, the verniers

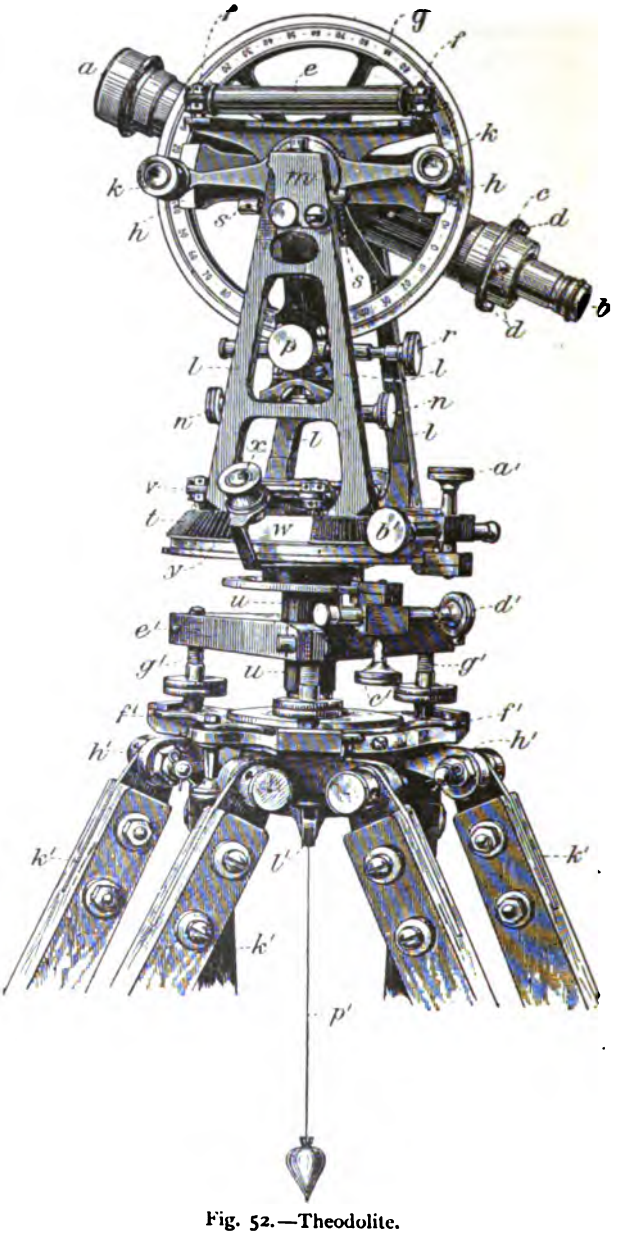

Fig. 52.—Theodolite.

should each read zero. By means of the vertical circle vertical angles are measured. The verniers are read by the microscopes *k*, *k*.

The horizontal axis of the telescope rests in the supports *ll*, *ll*, and is secured by the clips *m* at the head of each support, the vertical arc being clamped in position by the screws *n*, *n* after the telescope is placed in its supports. The *screw p* clamps the verniers *h*, *h* to the vertical circle, and the *tangent screw r* is then used to adjust the vertical circle to any given angle, or to adjust the telescope exactly upon any object after it has been approximately directed upon it by hand. The supports *ll*, *ll* are made high enough to permit of the telescope being revolved in a complete vertical circle upon its horizontal axis. In Stanley's instrument (Fig. 51) the supports are in one piece, which makes them lighter and not so top heavy. Messrs Cooke have also a design with supports of somewhat similar shape.

Fig. 53.—Theodolite.

The *horizontal axis* of the telescope should be at right angles to the line of collimation of the telescope and perfectly level, so that the telescope when revolved may move in a truly vertical plane. To effect this, the supports at the ends of the *horizontal axis* are raised or depressed by means of the adjusting screws *s*, *s*.

The *upper plate* or *vernier plate t* is a circular plate rotating on the *vertical axis u*, *u*, and perpendicular to it. It carries the supports *ll*, *ll* of the horizontal axis of the telescope. In the centre of the *upper plate* and between the supports there is a compass for taking magnetic bearings. Stanley's instrument (Fig. 51) is provided with a trough compass *p'* in place of the ordinary circular compass between the supports.

There are two *spirit levels v* at right angles to each other, attached to the *upper plate* by capstan screws, by which they are

adjusted so as to be parallel to it. On this plate are also the verniers *w*, read by the microscopes *x*.

The *lower plate* or *horizontal circle y* also rotates on the *vertical axis u*, and is perpendicular to it. The edge is bevelled and divided into 360° and smaller divisions, depending on the diameter of the circle. The *upper* and *lower plates* are also called *limbs*.

For ordinary railway and surveying purposes, a 5 in. diameter theodolite, divided into degrees and half degrees, and reading by two verniers to single minutes and half minutes, is the most useful size. A 6 in. instrument is usually divided into degrees and third parts of a degree or 20 minutes, and reads by the verniers to 20 seconds. In Cooke's instruments the graduations on the bevelled edge are covered all round by the projecting piece *t* of the *upper plate*, except at the verniers *w*. In Stanley's instrument the graduations are also covered. It is an advantage to have the graduations covered except at the verniers, otherwise they are liable to get scratched and dirty.

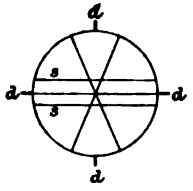

Fig. 54.
Diaphragm.

The *upper plate* is clamped to the *lower plate* by the screw *a'*, and is set to any given angle by the *slow motion* or *tangent screw b'* after being approximately adjusted by hand.

The *lower plate* is clamped to the axis by the screw *c'*, and is finally adjusted by the *slow motion* or *tangent screw d'*.

On the vertical axis *u* is fixed the *upper parallel plate e'*, through which pass the three *levelling screws g', g', g'* which rest on the *lower parallel plate f'*. By means of these screws and the spirit levels *v* on the *upper plate t* the instrument is "levelled up," making the *vertical axis u* truly vertical, and the upper and lower plates *t* and *y* horizontal.

In the older instruments there are four levelling screws. These are liable to jam when screwed up far, and require to be adjusted with both hands, two screws at a time. With three screws the instrument can be levelled up with one hand only, and the screws do not jam.

The lower parallel plate *f'* is made to slide bodily within certain limits on the *plate h'* to which the *legs k', k', k'* are fastened, and is clamped by the *screw plate l'* underneath. By this arrangement the instrument can be pushed bodily by hand into any desired

position after being approximately set up over the point by the legs k', k', k'. In some instruments there are adjusting screws at right angles to the *levelling screws* g', g', g' provided for this purpose by which a nicer adjustment may be made. In Stanley's instrument (Fig. 51) these are shown at m' and n'.

To the under side of the vertical axis u, and passing through a hole in the plate f', is attached the *plumb line* p' and plumb bob, by means of which the instrument is set vertically over the peg or other mark. Fig. 54A shows Messrs Troughton & Simms' arrangement for adjusting the instrument bodily over any desired point.

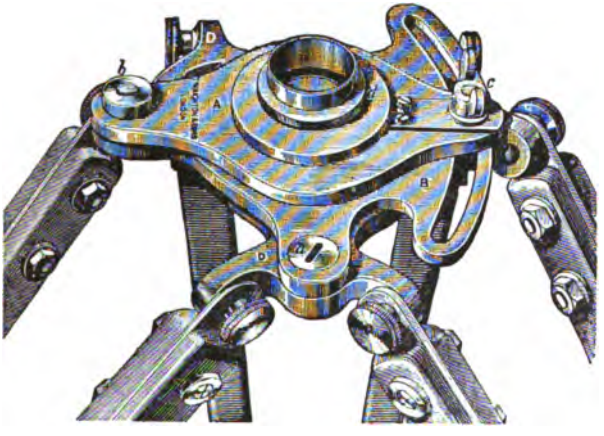

Fig. 54A.—Arrangement for setting Theodolite over Point.

The plate A rotates horizontally round the pivot b, and is clamped to the plate B by the screw c, which moves in the slot shown in B. The plates A and B thus clamped together may be moved again round the pivot a, and are clamped by another screw which moves in a slot in the lowermost plate D to which the legs are fixed. The arrangement thus permits of two motions at right angles to each other, and allows of a movement of about 3 in. each way.

Everest Theodolite.—Fig. 55 shows the Everest theodolite, as made by Messrs Cooke & Sons. This instrument takes its name from the designer, the late Sir Geo. Everest, of the Indian

Trigonometrical Survey. In it the supports of the horizontal axis of the telescope are made low for the sake of compactness, and the telescope cannot be revolved completely on its horizontal axis as it can in the ordinary transit theodolite. It may, however, be reversed by lifting it completely out of its bearings, and replacing

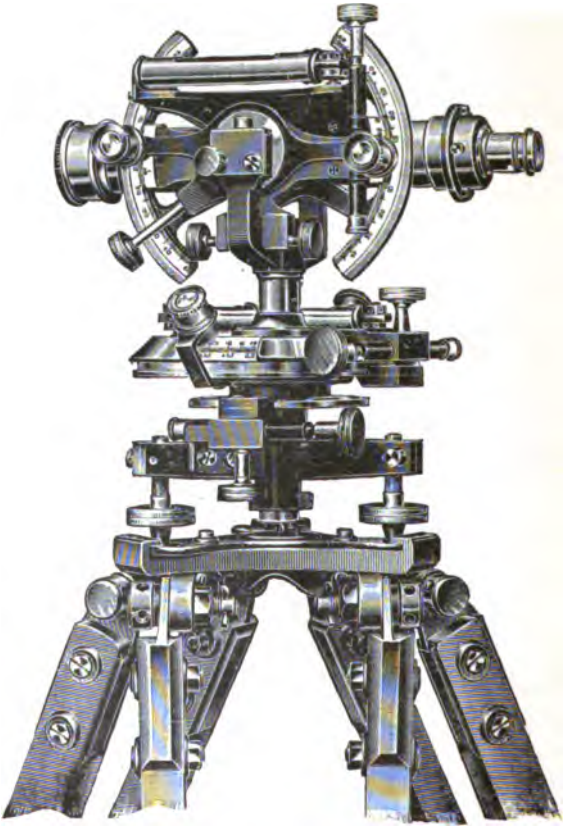

Fig. 55.—Everest Theodolite.

it after turning it end for end. In place of a complete vertical circle there are two opposite arcs of about 90° each, by means of which angles of elevation and depression up to 45° may be measured. In other respects the instrument is the same as the ordinary transit theodolite.

Plain Theodolite.—Fig. 56 shows the plain theodolite as made by Stanley. In this instrument the vertical arc is a semicircle only, and the telescope cannot be revolved completely on its horizontal axis. It may, however, be reversed by opening the clips *a*, *a*, Fig. 56, taking out the telescope, and replacing it with the ends reversed. In the Everest theodolite and the plain theodolite it is a great disadvantage not to be able to revolve the telescope vertically through a complete circle, as when producing a line from a back point say, as in taking it out of its bearings and replacing it with the ends reversed the instrument is liable to be disturbed. This may be avoided by turning the *upper or vernier plate* through 180° as read by the vernier, but these instruments are of course not nearly so useful as the ordinary transit theodolite.

Fig. 56.—Plain Theodolite.

Vernier.—Before describing the measurement of angles with the theodolite, it is necessary to describe the vernier. The vernier is a device by means of which angles may be read or set off to minutes and seconds, although the graduated lower limb of the theodolite may be divided into degrees and half degrees or third parts of a degree only. It is of course also applicable to the reading of any plain scale, and the principle of its use is the same in all cases.

In the case of a theodolite divided into degrees and half degrees, in order to read to single minutes the vernier may be as shown in Fig. 57. The total length of the vernier in this case will be equal to 14° 30′ of the graduated lower limb, and this

space is divided into thirty equal parts. Each division of the vernier is therefore equal to

$$\frac{14^\circ\ 30'}{30} = \frac{870'}{30} = 29'.$$

To put the matter otherwise, in the space of 14° 30′ on the graduated lower limb there are 29 divisions. This same space on the vernier is divided into 30 divisions, and each division is therefore equal to $\frac{29}{30}$ of a division on the graduated lower limb, *i.e.*, $\frac{29}{30}$ of 30′ = 29′ as before. If now the zero of the vernier be made to coincide with the zero of the graduated lower limb, then because the space between the zero and the first division of the vernier is 29′, while the space between the zero and the first division of the graduated lower limb is 30′, therefore the first division of the vernier falls behind the first division of the graduated lower limb by an amount equal to the difference of 30′ and 29′ or 1′. Similarly the second

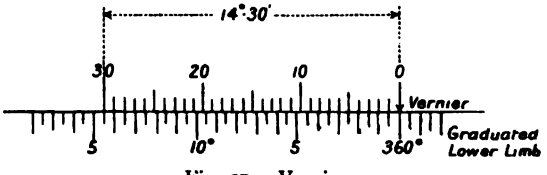

Fig. 57.—Vernier.

division of the vernier falls 2′ behind the second division of the graduated lower limb, the third division 3′ behind, and so on up to the thirtieth division, which is 30′ behind. *It is therefore obvious that in order to set off any desired number of degrees and minutes we have only to set the zero of the vernier to the given number of degrees on the graduated lower limb, and then bring the division on the vernier corresponding to the given number of minutes, to coincide with the next division in advance of it on the graduated lower limb.* For instance, suppose we wish to set the instrument to 29° 14′, we set the zero of the vernier to 29° on the graduated lower limb, and clamp the upper and lower plates together by the screw *a′*, Fig. 52, then by the tangent screw *b′*, Fig. 52, bring the fourteenth division of the vernier to coincide with the next division in advance of it on the graduated lower limb. The minuteness to which angles may be read or set off depends on the *length* of the vernier and the minuteness

of the graduation of the lower limb. In some instruments which have the graduated lower limb divided into degrees and half degrees, the length of the vernier is 29° 30′, as shown in Fig. 58. This space is divided into sixty parts, and the length of each division of the vernier is therefore

$$\frac{29^\circ\ 30'}{60} = \frac{1770'}{60} = 29\tfrac{1}{2}'.$$

The divisions of the graduated lower limb being = 30′, the difference is $30' - 29\frac{1}{2}' = \frac{1}{2}'$ or 30″. In this case, therefore, the vernier

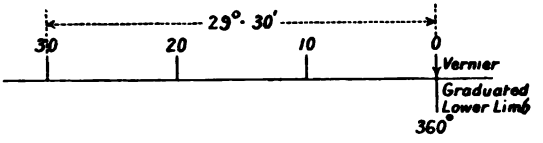

Fig. 58.—Vernier.

reads to 30″ in place of to 1′ as in Fig. 57. For instance, suppose the given angle is 41° 28′ 30″, to set the vernier to this angle the zero is first made to coincide with 41° on the graduated lower limb. The division of the vernier corresponding to 28′ 30″ is then made to coincide with the next division in advance of it on the graduated lower limb; the zero of the vernier will then be at 41° 28′ 30″. If the odd minutes and seconds exceed 30′, for example if the angle is 41° 38′ 30″, the zero is first set at 41° 30′ on the graduated lower limb, and the division of the vernier corresponding to 8′ 30″ is then made to coincide with the next division in advance of it on the graduated lower limb. In other words, when the minutes and seconds exceed 30′, the number of degrees + 30′

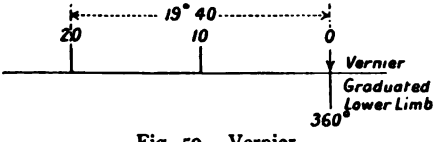

Fig. 59.—Vernier.

is set off by simply setting the zero mark to that division on the graduated lower limb, and only the *odd* minutes and seconds *in excess* of 30′ are set off by means of the vernier. The reason of this is of course because the total extent of the vernier does

not exceed 30′, and it is unnecessary that it should exceed that amount.

Similarly when the graduated lower limb is divided to third parts of a degree or 20′, the extent of the vernier may be = 19° 40′, and this space is divided into sixty parts, as shown in Fig. 59. In this case each division of the vernier

$$= \frac{19^\circ\ 40'}{60} = \frac{1180'}{60} = 19\tfrac{2}{3}'.$$

The divisions on the graduated lower limb being each 20′, the

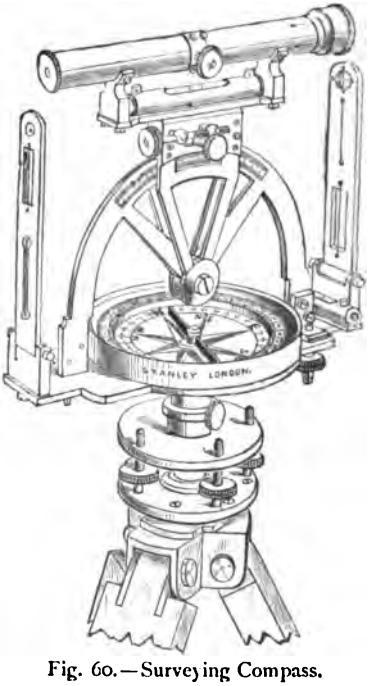

Fig. 60.—Surveying Compass.

difference is $20' - 19\frac{2}{3}' = \frac{1}{3}' = 20''$, and the vernier therefore reads to 20″. The angles are to be set off and read precisely as described for the vernier reading to 30″.

For convenience the mark of each division representing 1′ on the vernier is made longer than the intermediate ones, and the divisions representing 10, 20, and 30 minutes are marked 10,

20, 30 (see Fig. 57). Each intermediate division then in the case of Fig. 58 represents ½′ or 30″, and in the case of Fig. 59 ⅓′ or 20″, and the reading is facilitated. This somewhat lengthy explanation has been given as beginners often experience some difficulty in learning to use the vernier. The process of reading off angles is simply the converse of the operation of setting off a given angle. The degrees and half degrees or thirds of a degree, as the case may be, are read directly from the graduated lower limb, and are indicated by the zero point of the vernier. The division of the vernier which *coincides* with one of the divisions of the graduated lower limb is next searched for, and this division indicates the odd minutes and seconds.

Circumferentor or Surveying Compass.—Fig. 60 shows the circumferentor as made by Stanley. The instrument consists essentially of a large magnetic compass, with telescope and sights and a vertical arc. The chief use of a circumferentor is for taking magnetic bearings, which may be readily done with the large compass and needle on these instruments. It is much used by mine surveyors under the name of the mining dial, and may be provided with either telescope or plain sights, or both, as shown in Fig. 60. A pocket magnifying glass should be used to read the bearings. For magnetic compass surveys these instruments are very handy, and good work may be done with them. Owing to the large diameter of the compass and long needle, magnetic bearings may be taken with greater facility than with the small compass usually attached to the theodolite. A plainer form of surveying compass is shown in Fig. 231, Chapter XI., page 398.

Whitelaw's Theodolite and Mining Dial.—Fig. 61 shows Whitelaw's theodolite and mining dial, invented by the author's father, and made by Messrs John Davis & Son, Derby. The special feature of this instrument is that the telescope and sights are supported by a movable semicircular arc at right angles to the graduated vertical arc. The standards carrying the sights are thus dispensed with, and the compass graduation is not obstructed. Angles of elevation or depression may be taken up to 90°. In the ordinary transit theodolite a diagonal eyepiece must be used when the vertical angles are nearly 90°. The instrument is made either with sights or with telescope, or with both. The sights are made to fold down when not in use, and the telescope

supports are constructed so that the plain sights may be used *with the telescope on.* The telescope supports also fold down when the instrument is being used without the telescope. A graduated limb with external verniers may also be had on this instrument if desired, as shown at v, Fig. 61. A full

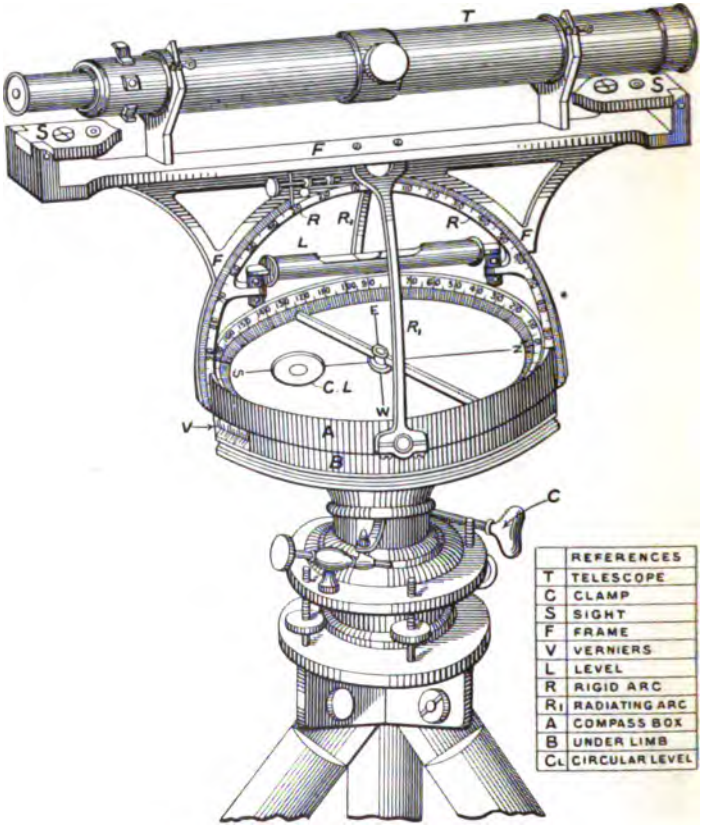

Fig. 61.—Whitelaw's Theodolite and Mining Dial.

description of the instrument by the inventor is given in Chapter XI., page 398. The compass is of large diameter, and for ordinary small surveys, such as described in this chapter, and route surveys abroad, as described in Chapter XI., this will be found a very suitable instrument, and will answer all the

purposes of a theodolite when furnished with external graduated limb and verniers, while it is lighter and cheaper.

Sextant.—For description and adjustments of sextant, see Chapter X., page 331.

Box Sextant.—This is a very handy instrument for approximate measurement of angles, and as made by Stanley, is shown in Fig. 62. It consists of a circular brass box 3 or $3\frac{1}{2}$ in. in diameter, and about $1\frac{1}{2}$ in. deep. It is protected by a lid or cap which is screwed on the top when not in use, and is also screwed on to the bottom to serve as a handle when using the instrument. It may be used either with or without the telescope T, which for convenience in carrying is fixed by the screw S so that it may be unfastened when not in use. In the upper part of

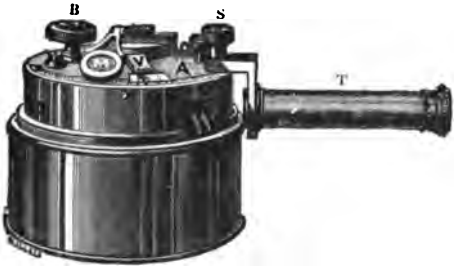

Fig. 62.—Box Sextant.

the instrument there is an arc of a circle A usually about 70° in extent and divided into 140°. The angle is read off by the vernier V and the microscope M. Opposite the slit for the eye or telescope T and inside the box there is a mirror having its upper half only silvered, so as to allow of one of the two objects between which the angle is to be measured to be seen directly through an opening in the side of the box beside this mirror. This mirror is called the *horizon* glass, and is *fixed* perpendicular to the plane of the instrument. The other side of the box opposite the side of the instrument shown in the figure is left open, and there is placed there inside the box another *movable* mirror which moves on the same axis as the vernier arm V. This mirror is called the *index* glass, and it and the vernier V are both moved simultaneously by the screw B which is connected to a rack and pinion

arrangement inside the box. When the two mirrors are parallel to each other the vernier should read zero. The angle actually read on the divided arc is the inclination of the two mirrors to each other. As shown below, this angle is half the actual angle between the two objects, and the arc of 70° is therefore divided into 140°. This is a source of error, as of course any error of observation is doubled. The vernier reads to 1 minute.

Use of Box Sextant.—To use the instrument sight directly on one of the objects through the opening opposite the lower unsilvered half of the fixed mirror, and then rotate the other mirror by means of the screw B until the second object is seen reflected in the upper silvered half of the fixed mirror directly over the first object as seen through the lower unsilvered half as shown in Fig. 63. The angle between the two objects is then to be read off by means of the vernier.

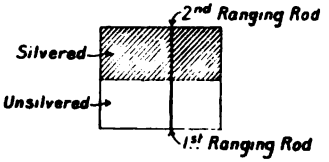

Fig. 63.—Observing with Box Sextant.

Theory of the Sextant.—The principle on which the instrument is based is as follows:—"If there are two plane mirrors whose reflecting planes make a given angle with each other, and a ray of light in a plane perpendicular to the planes of both mirrors is reflected from both successively, its direction after the second reflection makes with its original direction an angle which is twice the angle made by the mirrors with each other." That this is so may be proved as follows:—In Fig. 64, let A be the fixed or horizon mirror, and B the movable or index mirror. Then ar^1 is the line of sight, and the first object r^1 is viewed directly through the

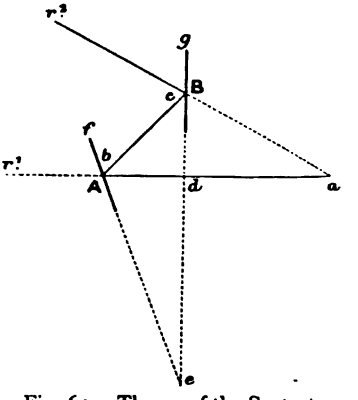

Fig. 64.—Theory of the Sextant.

aperture in the box and the lower unsilvered half of the fixed mirror A. The second object r^2 is reflected at c from the movable mirror B on to the fixed mirror A at b and thence to the eye at a, the mirror B being rotated until the reflection at b is in the line of sight ar^1. The mirrors being in this position, the angle actually measured is the angle bec between the two mirrors. It is now to be shown that this angle is equal to half the actual angle r^1ar^2 between the two objects r^1 and r^2.

In the triangle bca the exterior angle cbr^1 = sum of interior angles = $bca + bac$.

Therefore $cbr^1 - bca = bac$ . . . . . (1)

But $cbr^1 = 2cbf$, because $cbf = dbe$, and $dbe = fbr^1$, therefore $cbf = fbr^1$, or $cbr^1 = 2cbf$ . . . . . (2)

Similarly $bca = 2bce$, because $bce = r^2cg$, and $r^2cg = dca$, therefore $bce = dca$, or $bca = 2bce$ . . . . . (3)

Therefore substituting in (1) the values of cbr^1 and bca given by (2) and (3), we get $2cbf - 2bce = bac$ . . . . . (4)

That is, in the triangle cbe twice the exterior angle cbf minus twice the interior angle bce is equal to bac.

But twice the exterior angle of a triangle minus twice one of the interior angles is equal to twice the other interior angle, which in this case is bec or $2cbf - 2bce = 2bec$; but from (4), $2cbf - 2bce = bac$.

Therefore $2bec = bac = r^1ar^2$, or $bec = \frac{1}{2}r^1ar^2$.

Measuring Angles with Box Sextant when Ground is not Level.—In using the box sextant, unless the two objects between which the angle is measured are on a level with the eye, the angle observed will not be measured in a horizontal plane. As it is the horizontal projection of the angle that is required, we may observe this by placing two ranging rods in line with the objects so that these ranging rods are as nearly as possible on a level with the eye. The angle between the ranging rods may then be taken. If the ground does not permit of this, then the actual angle may be measured and afterwards reduced to its horizontal projection by the proper formula (see Chapter X., page 337).

Prismatic Compass.—This instrument is much used for rough surveys, and is especially useful for a rough traverse. Fig. 65 shows the instrument as made by Messrs T. Cooke & Sons. It consists of a compass card fixed to a magnetic needle in the

same manner as the ordinary ship's compass. At A there is a prism and a vertical slit for the eye, and at B there is an upright with vertical hair for sighting upon the end of the line whose bearing is required. C is a mirror which slides up and down the upright B, and may be set at any angle so as to reflect the sun or other object at a considerable altitude. Upon placing the eye at the slit at A and directing the vertical hair in B upon any object, the magnetic bearing is simultaneously seen reflected in the prism at A, and is thus read off at once. The instrument may be used either mounted on the light tripod as shown in the figure or held in the hand. When not in use, the sight and mirror at B, as also the prism at A, are folded down, and the whole may be compactly packed in a leather case. For surveying in the details in jungle or dense forest where traversing is the only method available, the prismatic is extensively used. It is best to use the prismatic on the tripod, as the needle comes to rest quicker and the work is altogether more satisfactory. A good pocket prismatic is shown in Fig. 232, page 408, Chapter XI.

Fig. 65.—Prismatic Compass.

Pocket Compass.—No surveyor should go abroad to survey in undeveloped country without an ordinary pocket compass. In working through jungle and dense forest it will be found useful,

and should always be carried. In case of accident it will prevent one from losing one's way, besides being useful even for making a rough traverse upon occasion. The best kind is one protected in a case similar to the hunting case of a watch.

Plane Table.—This instrument is much used by native surveyors on Government work in India for surveying detail. The plane table as made by Stanley is shown in Fig. 66. It consists of a drawing board varying in size from about 16 × 13 in. to 26 × 22 in., mounted on a tripod. A sheet of drawing paper is stretched on the board, on which is placed the "alidade," which may have either plain sights or a telescope as shown. There

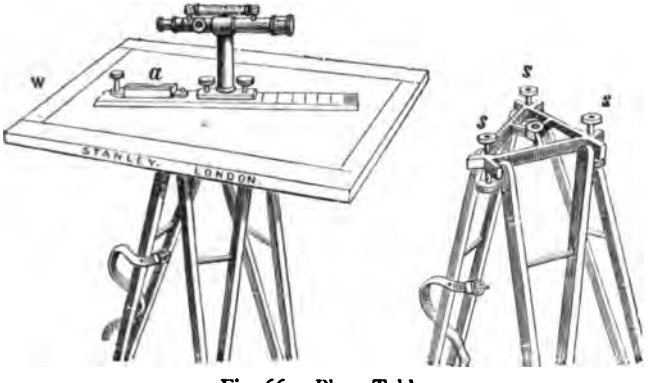

Fig. 66.—Plane Table.

is also shown in the figure a trough compass attached to the alidade, and a level is attached to the telescope. In its plainest form the alidade is simply a metal rule with bevelled edge, having plain sights at its ends. A trough compass and a spirit level are then carried separately, and laid on the table when required. The table is attached to the tripod by a butterfly nut, and there are usually three levelling screws for levelling it up. The best form has also two rollers underneath worked by a ratchet motion for rolling and unrolling a continuous roll of paper and keeping it stretched tightly. One of the chief advantages of the plane table is that the work is plotted at once in the field, but its usefulness is greatly impaired in a wet climate, as it is of course impossible to work with it in wet or even showery weather.

Range Finders: The Labbez Telemeter.—Various small hand instruments called range finders have been devised to give approximate distances. Fig. 67 shows the Labbez telemeter,

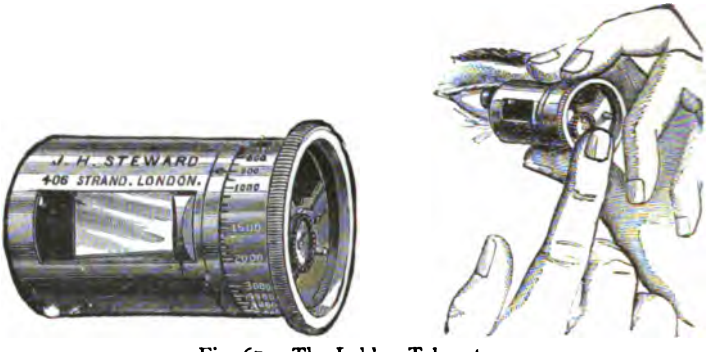

Fig. 67.—The Labbez Telemeter.

which gives without calculation distances from 250 to 3,000 yds. To use the instrument, select an object B, Fig. 68, at a considerable distance, and as nearly as possible at right angles to the line AC whose distance is required, and fix a point D in line with B at a distance of 30 yds. from A. Having set the zero of the revolving cylinder to the zero mark on the fixed cylinder, and also the toothed wheel at the end of the cylinder to zero, standing at A, look through the instrument at B, and revolve the toothed wheel with the forefinger until the reflection of C coincides with B. Now go to D, and again looking at B, revolve the toothed wheel until the reflection of C again coincides with B. The distance AC is then that indicated on the revolving cylinder opposite the zero mark on the fixed cylinder.

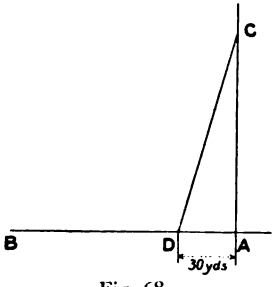

Fig. 68.
Use of the Labbez Telemeter.

The Weldon Range Finder.—The Weldon range finder is the patent of Colonel Weldon, R.A. It consists of three prisms of crystal ground to 90°, 88° 51′ 15″, and 74° 53′ 15″. In Fig. 69, let the distance AB be required. With the 90° prism, standing at

A, set off the point C by making it coincide with the reflected image of B. AC is then perpendicular to AB. Now walk backwards from A, keeping in the line AC until the point D is reached, where the reflected image of B as seen in the 88° 51′ 15″ prism coincides with the point A. The angle ADB is then 88° 51′ 15″, and the distance AB is equal to 50 times the distance AD, which has to be measured. Somewhat greater accuracy is attained by using the 88° 51′ 15″ prism only with a base twice the length of AD in Fig. 69 (see Fig. 70). Here CD = 2AD, and AB = 25 times CD. When the length of the base AD or CD is considerable, in order to save time in measurement retreat along the line DB until the point E is reached, where the reflected image of D as seen in the 74° 53′ 15″ prism coincides with A or C. The length of the

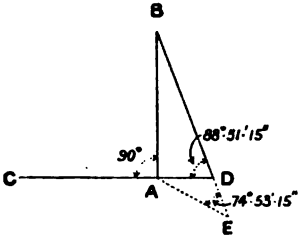

Fig. 69.

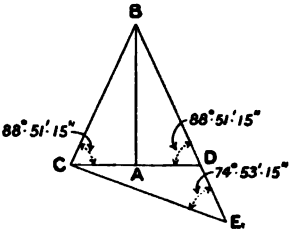

Fig. 70.

Use of the Weldon Range Finder.

base AD or CD is then 4 times DE, and AB is equal to 200 times DE in Fig. 69, and in Fig. 70, AB = 100 times DE. In the official trials at Aldershot the mean error of the Weldon range finder was 34 yds. for each distance, and in India the average error was found to be 35 yds. for each distance.

The Bate Range Finder.—The Bate range finder consists of a binocular field glass with two graduated limbs for measuring the angles. It is used in the same way as the Weldon and similar instruments. The angle subtended by the base is measured by putting in a gauge between the two graduated limbs, and the distance is expressed as a multiple of the base. The distance is equal to the measured base multiplied by the number indicated by the gauge.

Care of Instruments.—All instruments should be protected from rain, dust, and damp as much as possible. A waterproof bag should be used to cover the theodolite, level, or compass, &c., when mounted on the tripod and not in use. No part open to dust should be oiled, as this keeps all the dust that falls on it. The axes should be cleaned occasionally with chamois leather and a very little pure watch oil. Plumbago may be used instead of oil, as a soft lead pencil cut and a little put on the axis with the finger. The verniers and glasses should be cleaned with a soft camel's hair brush. Very dirty glasses may be cleaned with alcohol. If dust lies on the cross hairs, it may be removed by taking off both eyepiece and object glass and blowing through. Unless the cross hairs are wire, this is apt to break them if great care is not taken. If moisture gets inside the telescope tube, take off the eyepiece and let it evaporate. If moisture gets in between the two parts of the object glass (if composed of two glasses), remove the object glass and dry it by gentle heat at a fire or lamp, but do not take the glasses apart.

Trigonometrical Formulæ for the Solution of Plane Triangles:—*Right-angled Triangles.*—In Fig. 70A, let ABC be a right-angled triangle, right-angled at C. Then

$$\text{Sin A} = \frac{a}{c} \quad \text{Cosec A} = \frac{c}{a}$$

$$\text{Cos A} = \frac{b}{c} \quad \text{Sec A} = \frac{c}{b}$$

$$\text{Tan A} = \frac{a}{b} \quad \text{Cot A} = \frac{b}{a}$$

Given.	Required.	Formulæ.
a, b	A, B, c	$\text{Tan A} = \frac{a}{b}$, $\cot \text{B} = \frac{a}{b}$, $c = \sqrt{a^2 + b^2}$
a, c	A, B, b	$\text{Sin A} = \frac{a}{c}$, $\cos \text{B} = \frac{a}{c}$, $b = \sqrt{(c+a)\,(c-a)}$
A, a	B, b, c	$\text{B} = 90° - \text{A}$, $b = a \cot \text{A}$, $c = \frac{a}{\sin \text{A}}$
A, b	B, a, c	$\text{B} = 90° - \text{A}$, $a = b \tan \text{A}$, $c = \frac{b}{\cos \text{A}}$
A, c	B, a, b	$\text{B} = 90° - \text{A}$, $a = c \sin \text{A}$, $b = c \cos \text{A}$.

Oblique Triangles (Fig. 70B)—

Given.	Required.	Formulæ.
a, b, c	A	If $s = \frac{1}{2}(a+b+c)$, $\sin \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}}$; $\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}$, $\tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$; $\sin A = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc}$
A, a, b	B	$\sin B = \frac{b \sin A}{a}$ (ambiguous case)
a, b, C	A − B	$\tan \frac{1}{2}(A-B) = \frac{(a-b)}{a+b} \tan \frac{1}{2}(A+B)$
A, B, a	b	$b = \frac{a \sin B}{\sin A}$
a, b, c	Area	If $s = \frac{1}{2}(a+b+c)$, area $= \sqrt{s(s-a)(s-b)(s-c)}$
A, b, c	Area	Area $= \frac{1}{2} bc \sin A$
A, B, C, a	Area	Area $= \frac{a^2 \sin B \sin C}{2 \sin A}$

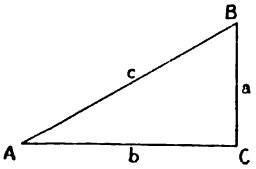

Fig. 70A.—Right-angled Triangles.

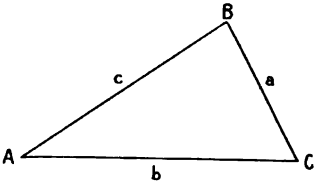

Fig. 70B.—Oblique Triangles.

General Trigonometrical Formulæ—

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B) \sin(A-B)$

$\cos^2 A - \sin^2 B = \cos(A+B) \cos(A-B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$

$\sin A - \sin B = 2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$

General Trigonometrical Formulæ—

$$\text{Cos A} + \text{cos B} = 2 \cos \tfrac{1}{2} (\text{A} + \text{B}) \cos \tfrac{1}{2} (\text{A} - \text{B})$$

$$\text{Cos B} - \text{cos A} = 2 \sin \tfrac{1}{2} (\text{A} + \text{B}) \sin \tfrac{1}{2} (\text{A} - \text{B})$$

$$\text{Tan A} = \frac{\sin \text{A}}{\cos \text{A}}$$

$$\text{Cot A} = \frac{\cos \text{A}}{\sin \text{A}}$$

$$\text{Tan} (\text{A} \pm \text{B}) = \frac{\tan \text{A} \pm \tan \text{B}}{1 \mp \tan \text{A} \tan \text{B}}$$

$$\text{Tan A} \pm \text{tan B} = \frac{\sin (\text{A} \pm \text{B})}{\cos \text{A} \cos \text{B}}$$

$$\text{Cot A} \pm \text{cot B} = \pm \frac{\sin (\text{A} \pm \text{B})}{\sin \text{A} \sin \text{B}}$$

$$\frac{\text{Sin A} + \text{sin B}}{\text{sin A} - \text{sin B}} = \frac{\tan \tfrac{1}{2} (\text{A} + \text{B})}{\tan \tfrac{1}{2} (\text{A} - \text{B})}$$

$$\frac{\text{Sin A} + \text{sin B}}{\cos \text{A} + \cos \text{B}} = \tan \tfrac{1}{2} (\text{A} + \text{B})$$

$$\frac{\text{Sin A} + \text{sin B}}{\cos \text{B} - \cos \text{A}} = \cot \tfrac{1}{2} (\text{A} - \text{B})$$

$$\frac{\text{Sin A} - \text{sin B}}{\cos \text{A} + \cos \text{B}} = \tan \tfrac{1}{2} (\text{A} - \text{B})$$

$$\frac{\text{Sin A} - \text{sin B}}{\cos \text{B} - \cos \text{A}} = \cot \tfrac{1}{2} (\text{A} + \text{B})$$

$$\text{Tan } \tfrac{1}{2} \text{ A} = \frac{\sin \text{A}}{1 + \cos \text{A}}$$

$$\text{Cot } \tfrac{1}{2} \text{ A} = \frac{\sin \text{A}}{1 - \cos \text{A}}$$

Measuring Angles with the Theodolite.—Suppose the angle BAC, Fig. 71, is to be measured. Set up the theodolite exactly over the station at A (see page 202), and level up the instrument by means of the *levels v* on the upper *plate t*, Fig. 52. Set one of the verniers to zero or 360° by means of the clamp *a'* and tangent screw *b'*. Unclamp the screw *c'* and direct the telescope approximately on to the point B by hand. Then clamp *c'* and direct the cross hairs exactly on B by the tangent screw *d'*. Now unclamp *a'*, and direct the telescope approximately on to the point C by hand. Clamp *a'* again, and adjust the cross hairs exactly on C by the tangent screw *b'*. The number of degrees, minutes, and seconds as now read on the vernier is then the required angle BAC. By

reading both verniers "errors of eccentricity" arising from the plates *t* and *y* and the vertical axis *u*, not being exactly concentric, are eliminated.

If one of the verniers is adjusted to 360° or zero, the others should also be read before measuring the angle, as in most instruments the second vernier does not read exactly 180° when the first reads 360°, owing to errors of graduation and position of verniers, &c. Similarly when there are three verniers the second and third do not generally read exactly 120° and 240° when the first is set at zero.

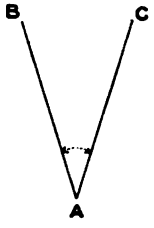

Fig. 71.
To Measure an Angle with the Theodolite.

Method of Repetition.*—When great accuracy is required, errors of graduation may be reduced to any required extent by repetition. This process consists in repeating the observation of the angle any number of times, according to the degree of accuracy aimed at, the verniers not being read until the last observation. Thus if the angle is observed six times, the total angle, as read at the end of the six observations, divided by six, is the required angle. Be careful to count 360° for each complete revolution of the horizontal circle. By the process of repetition the errors of graduation are diminished, but errors of observation are as a rule accumulated, as an observer tends to make the same error of observation each time the telescope is directed upon the same object. Errors due to instability of clamping and tangent screws are also introduced (see also page 459, Chapter XII.). For the most accurate methods of measuring angles, see pages 460, 461, Chapter XII.

Using both Faces of the Instrument. — In making important observations both "faces" of the theodolite should be used in order to eliminate errors of adjustment of the instrument. For instance, errors arise from the horizontal axis of the telescope not being exactly level, either owing to the spirit levels *v* not being exactly parallel to the plate *t*, or owing to the bearings of the horizontal axis in the supports *ll*, *ll* not being exactly level (adjustment 1, Chapter IV.). The telescope consequently does not move in a vertical plane from either of these causes, and the

* See also page 459.

accuracy of the observation is thereby affected. This error affects the observation, whether the telescope during the observation is revolved vertically through 180° or not. When the telescope is revolved vertically through 180°, another error arises, if the collimation line is not exactly perpendicular to the horizontal axis of the telescope (adjustment 2, Chapter IV.). All errors are eliminated by using both "faces" of the instrument.* This consists in turning the whole of the upper part of the instrument by hand through 180° and repeating the observation. The mean of the two observations will then be the correct result. By turning the whole of the instrument through 180°, the errors are reversed. For instance, suppose the telescope, instead of revolving in a vertical plane, revolves in a plane which slopes upwards to the *left* of the observer. If now the instrument is moved bodily through 180°, and the telescope again directed *from* the observer, it will be found that it now lies in a plane sloping upward the same amount to the *right* of the observer. Similarly if the error of the collimation line was to the *left* of the observer, it will now be the same amount to the *right*. The mean of the two observations is therefore the correct result.

Of course the spirit levels *v*, and the bearings of the supports of the horizontal axis of the telescope, as well as the collimation line, may be adjusted by means of the screws provided for the purpose, as explained in Chapter IV., but as the instrument is always liable to get out of adjustment, in all important observations both "faces" should be used.

Fig. 72.
Accurate Method of getting in Line.

Accurate Method of setting Instrument in Line.—Sometimes an error arises from the plumb bob not being exactly in the vertical axis of the instrument. In order to avoid this error, range in a point at a little distance back from the point over which the instrument is to be set up, and set the instrument exactly in line with the two back points by trial and error. If the cross hairs bisect the two back points, the instrument will be in the line. For instance, suppose the

* With the exception of errors due to eccentricity, which are eliminated by reading both verniers, and errors of graduation, which are eliminated by measuring the angle several times on different parts of the graduated limb.

instrument to be set exactly in the line at A, and it is required to set it up at B, Fig. 72. Line in the point C a little way back from B, and measure BC. The instrument is now to be set up at B, so that the cross hairs are directed exactly on both A and C, while the axis is at the measured distance BC from C. This is a more accurate way of getting into line than setting up by the plumb bob. Sometimes the operation is a little troublesome, but with the aid of adjusting screws (*m'*,*n'*, Fig. 51) on the instrument and by getting first approximately into line by the plumb bob it is usually easy enough. This method is of course not necessary for ordinary work, and is only used when great accuracy is required, as for instance in setting out tunnels, &c.

Method of conducting an Ordinary Small Survey with the Theodolite.—The chief advantage of using a theodolite is that the measurement of tie lines is unnecessary, and also the measurement of those lines which in a chain survey are measured only to enable the work to be plotted. With the theodolite a sufficient number of angles is taken to enable the work to be plotted and also to give a check, in the same way that the tie lines act as checks in a chain survey.

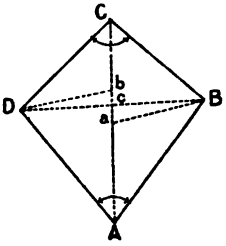

Fig. 73.
Small Theodolite Survey.

For example, in Fig. 73, if the four sides ABCD are measured, and also the angle DAB, the survey may be plotted. Thus we may lay down the line AB to scale on the paper, and then AD, making the angle BAD equal to the observed angle. By now taking the lengths CD and BC in the compasses, and sweeping out arcs from D and B as centres, we get by their intersection the point C. As a check on the work one at least of the other angles must be measured, say the angle at C, and it is preferable to measure *all* the angles. Suppose the angle at C to be measured, then if there is any error either in the measurement of any of the sides or in the measurement of the angle at A, the angle at C when measured on the paper with the protractor will not agree with the angle observed at C on the ground. The advantage of measuring all four angles at A, B, C, and D is that we can check the accuracy of

the angular measurements before leaving the ground, as "the sum of the interior angles of any rectilinear figure is equal to twice as many right angles as the figure has sides, less four right angles." In the case of Fig. 73, as the figure is four-sided the sum of the interior angles is equal to $2 \times 4 \times 90^\circ - 4 \times 90^\circ = 360^\circ$. If then the sum of the four interior angles as measured on the ground is equal to 360°, or, as perfect accuracy is impossible, to 360° plus or minus a fair allowable amount of error, we are satisfied before leaving the ground that the angular measurements at all events are correct. The accuracy of the work is then proved by testing the angles with the protractor after plotting. If now the same figure be surveyed with the chain only, in order to plot the work it is necessary to measure one of the diagonals AC or DB, say AC, while the two tie lines *a*B and *b*D or the other diagonal BCD are necessary to check the two triangles ABC, ACD. By the use of the theodolite, therefore, the measurement of the dotted lines shown in Fig. 73 is rendered unnecessary. This is a not inconsiderable saving of labour, as it is much easier to measure a few angles with the theodolite than to chain several lines. At the same time it must be admitted that in many cases in surveying with the chain only it is possible to select the main and subsidiary chain lines, so that those lines which must in any case be chained to take up interior fences and other details, themselves act as tie lines. Thus in a chain survey, lines which are run for no other purpose than to act as ties or to enable the work to be plotted, are avoided where possible. For example, referring to the chain survey in Fig. 24, it will be seen that all the chain lines shown are required to take up the fences, &c. On the other hand, however, if all the interior angles of the boundary lines at A, B, C, D, E, and F had been measured, by a rearrangement of the chain lines, the measurement of some of the lines OA, OB, OC, OD, OE, and OF might have been avoided.

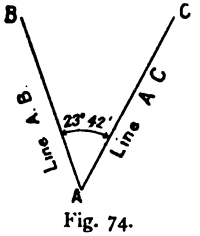

Fig. 74.
Entering Angles in Field Book.

Field Book.—The field book is kept in the same manner as the field book of the chain survey (Fig. 24, pages 21 to 39), and the angles may either be entered in the field book at the beginning of each line, or marked on the rough sketch of the survey which is

made before commencing to chain the lines. If preferred, the angles may be entered all together on separate pages of the field book, as in Fig. 74. As a rule it is better to take all the angles last after the chaining and offsets are finished. They may then be entered in their proper places in the field book.

Reconnaissance of Ground.—The same remarks as to reconnaissance of ground as made on page 15 apply to theodolite surveys.

Practical Hints.—See that the instrument is firmly planted in the ground by pressing in the legs, and get it as nearly level as possible by means of the legs before proceeding to level it up with the levelling screws. In sighting on to a point on which a ranging rod is being held, always take the iron-shod point at the lower end of the ranging rod, or, if that is not visible, see that the ranging rod is carefully plumbed, and sight on to the lowest visible part of the ranging rod.*

Obstacles to Measuring.—In passing an obstacle by "squaring off," as in Fig. 12, Chapter I., the operation is much facilitated by having the theodolite to set out the right angles at *a*, C, D, and *b*, and greater accuracy is attained.

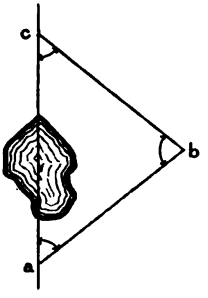

Fig. 75.
Triangulating round an Obstacle.

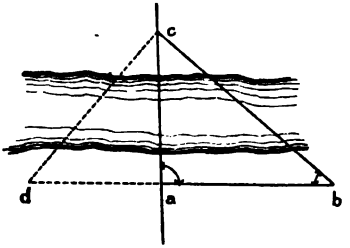

Fig. 76.
Distance across a River.

An easier method when a theodolite is at hand is to triangulate round, as shown in Fig. 75. If the angles at *a*, *b*, *c* are each made 60°, the triangle is equilateral, and $ac = ab$ or bc. If, owing to the

* See also page 202.

nature of the ground or to another obstacle in the vicinity of *b*, an equilateral triangle cannot be used, an isosceles triangle, having the sides *ab* and *bc* equal, can always be laid out, and the distance *ac* may be calculated from *ab*, *bc*, and the measured angles.

Crossing a River.—In crossing a river, as in Fig. 76, it is only necessary to run out a base as *ab* in any convenient direction along the bank, and measure the length *ab* and the angles at *a* and *b*. The distance *ac* may then be calculated from the measured side *ab* and the two angles. The length of the base *ab* should always, if possible, be greater than *ac*. The triangle *adc* may be laid out as a check, and *ac* calculated from it also.

Example of an Ordinary Small Survey with the Theodolite.—Fig. 77, Plate II., is given as an example of a survey of this kind. The main chain lines extend round the boundaries of the property, and are indicated by full lines. They are the lines AB, BC, CD, DE, EH, HK, and KA. The small projecting piece at the corner E is treated separately, so as not to complicate unnecessarily the outlines of the main polygon ABCDEHKA. It is surveyed by the lines EF, FG, and GH. Similarly the corner at B is surveyed separately by the triangle B*a*C. The subsidiary chain lines are the lines *df*, *fg*, and *ge*, run to take up the enclosure shown at *de* on the main chain line CD, and *b*H run to take up the plantation and fence shown, while *c*C takes up the fence alongside it. The angles taken are indicated in Plate II., and are first, all the interior angles of the main polygon ABCDEHKA. Confining ourselves first to this part of the survey, we notice that this polygon has seven sides, and that consequently the sum of its internal angles should be equal to $2 \times 7 \times 90^\circ - 4 \times 90^\circ = 900^\circ$. The accuracy of these angles may thus be checked before leaving the field, and with a theodolite reading to single minutes the total error should not exceed 2 or 3 minutes with ordinary care.

Starting with the longest side CD by means of the distances and angles, the sides CB, BA, AK, HK may be plotted. Taking the lengths DE and HE in the compasses, by the intersection of arcs with centres D and H the point E is plotted. The accuracy of the linear measurements may now be proved by measuring with the protractor the angles CDE, DEH, and EHK, which should agree with those observed in the field. Or otherwise we may plot by means of the lengths

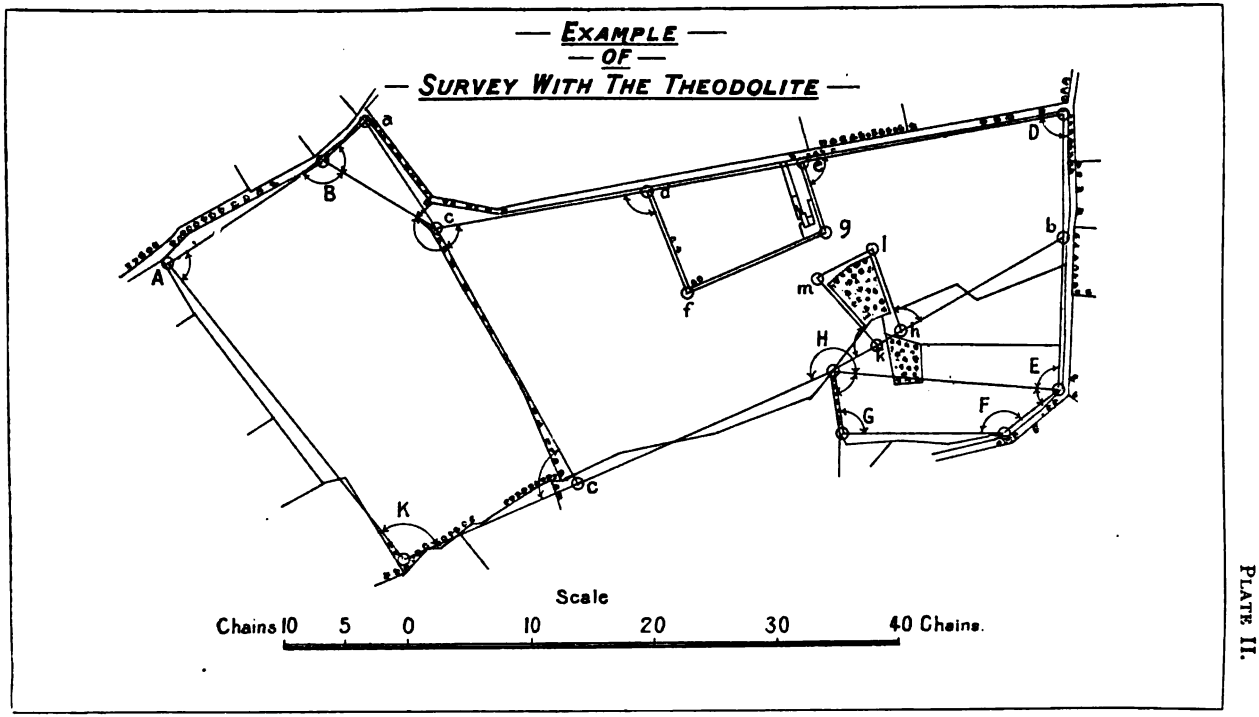

Fig. 77.

PLATE II.

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and angles all the lines DC, CB, BA, AK, KH, and HE. The distance DE should then agree with the measurement on the ground as also the angles DEH, EDC at E and D. The sides of the triangle B*a*C are measured on the ground, and the work is checked by the angles *a*BC and *a*CB, which, when measured with the protractor on the plot, should agree with their values as observed on the ground. As the theodolite is to be placed at both B and C in any case to take the angles of the main lines, two angles of the triangle *a*BC have been observed. With reference to the small polygon EFGH, all the interior angles of this polygon have been observed, and the figure being four-sided, their sum should be 360°. It is plotted by means of the lengths and angles, and checked as described for the main polygon. The line *b*H when drawn in between *b* and H should agree with its length as measured on the ground, and similarly the line C*c*. These two lines thus form valuable additional checks on the survey.

It will be noticed that at C both the angles BC*c* and *c*CD have been taken. The angle BCD only is necessary, but it was practically as easy to take both the angles when the theodolite was set at C, and by so doing, and taking also the angle K*c*C, the angles of the polygon ABC*c*KA and also the angles of the polygon CDEH*c*C may be checked separately. The former being five-sided, the sum of its angles should be equal to 540°, and the latter being also five-sided, the sum of its angles should equal the same amount. With reference to the enclosure at *de*, the lines *df*, *fg*, and *ge* are run round it, and the angle C*df* is measured. This suffices to plot this enclosure, and the work is checked by measuring the angle D*eg* at *e*. Similarly the plantation at *kh* is taken by the lines and angles shown; the other small plantation beside it is given sufficiently by the intersection of lines H*b* and HE with it and offsets from these lines. The boundaries, fences, &c., are surveyed by offsets from the chain lines and entered in the field book as already described for the chain survey (Fig. 24), and shown in the field book of that survey on pages 21 to 39.

On examination of this survey it will be seen that the measurement of several long diagonals and tie lines which would be required if the chain only were used is avoided by measuring the angles shown. The labour involved in taking these angles is much less than the labour of chaining the necessary diagonals and tie lines.

Traverse Surveying.—A traverse survey may be defined as the survey of a long, narrow, and zigzag stretch of country, chiefly undertaken for a road, railway, or other line of communication. It is possible to make a small traverse with the chain only in the manner described on page 19. For instance, suppose the lines shown in Fig. 78 are to be surveyed. The angles at B, C, D may be taken with the chain only by producing the sides and measuring the small triangles shown by the dotted lines, as described on page 19. This method is, however, very inaccurate, and a theodolite should always be used to measure the angles. A traverse survey essentially consists of a series of connected lines whose lengths are measured, and also either the angles which the lines make with each other or their magnetic or true bearings.

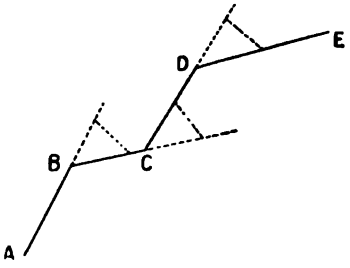

Fig. 78.—Traversing with Chain.

Example of an Unclosed Traverse Survey.—As an example of a traverse survey, let it be required to survey the portion of a road between the points A and H, as shown in Fig. 79, Plate III. The station points A, B, C, D, E, F, G, H are selected as far apart as it is possible to see between the bends in the road, and are marked by driving in pegs or spikes.

The lines AB, BC, CD, &c., are first to be chained, and the offsets taken and entered in the field book in the usual manner, as already described in Chapter I.

Ordinary Method of taking Bearings.—Next the angles are to be observed. If we simply set the instrument at B and measure the angle ABC between the lines AB and BC, and then similarly the other angles at C, D, E, &c., this would be sufficient to plot the work. There would, however, be no check on the angles. The best method, and that usually followed, is to take the magnetic bearings. By the magnetic bearing of a line is meant the angle which that line makes with the magnetic meridian as indicated by the needle of the compass.

To take the magnetic bearing of the line AB, set up the theo-

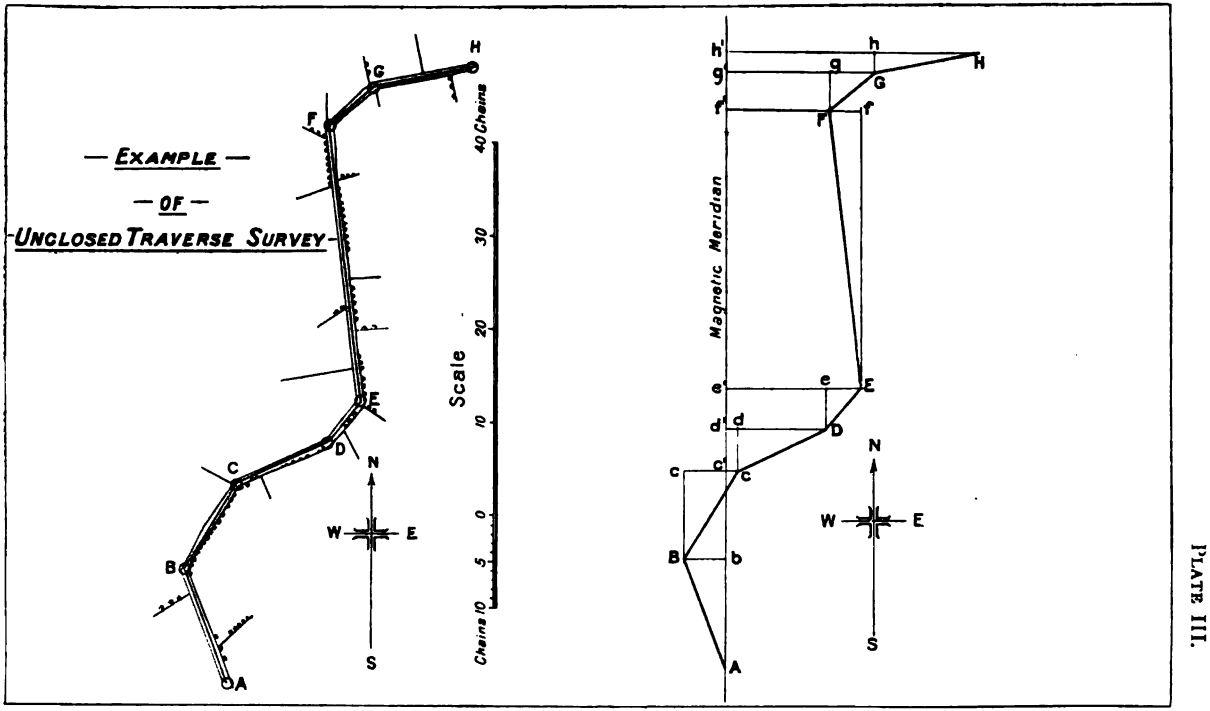

Fig. 79.

Fig. 80.

PLATE III.

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dolite at A, and set the vernier to zero of the graduated lower limb, then clamp the upper plate to the lower. Next turn the instrument bodily round until the needle of the compass on the theodolite points to the zero of its graduations, and clamp the whole instrument in that position. Loosen the upper plate, and direct the cross hairs on to B, and again clamp the upper plate. Adjust the cross hairs exactly on B by the tangent screw of the upper plate. The angle as read on the vernier is now the magnetic bearing of AB, and is to be entered in the field book. The check on the measurement of this bearing is that the needle should point to the same bearing on the graduations of the compass as the bearing already read by the vernier on the graduations of the lower plate. Now, leaving the two plates of the instrument still clamped at the bearing of AB, move the theodolite to B, and set it up there. Direct the telescope back upon the point A and revolve it vertically through 180°, so that it again points in the direction *from* A *to* B. The needle of the compass should now again indicate the previously observed bearing of AB. Again unclamp the upper or vernier plate and direct the telescope to C, and read off the bearing of BC on the vernier. Check this bearing, by observing if the compass needle indicates the same bearing, as it ought to do unless there is some local attraction, otherwise the bearing of BC has been incorrectly read on the vernier.

Having the instrument still clamped at the bearing of BC, move it now to C, and proceed precisely in the same manner, directing the telescope back on B and then taking the bearing of CD.

The same operation is repeated at each of the stations until all the bearings have been observed, care being taken to check each by seeing that it agrees with that indicated by the compass needle.

Method of avoiding Errors of Adjustment of Instrument in taking Bearings. — In revolving the telescope vertically through 180°, if the instrument is out of adjustment, and the collimation line is not exactly at right angles to the horizontal axis of the telescope, an error is introduced. This may be avoided by calculating the *back* bearing of AB, *i.e.*, the bearing BA. The *back* bearing of a line differs from its *forward* bearing by 180° plus or minus. Thus the back bearing BA is equal to the forward bearing AB minus 180°. If the addition of 180° gives a bearing greater

than 360°, the actual bearing is found by subtracting 360°. To proceed in this manner on moving the instrument from A to B the vernier is to be set to the *back bearing* BA and the telescope directed to the point A. The needle should now point to the same bearing. Keeping the object glass of the telescope still pointing in the direction BA, unclamp the upper plate and rotate it and the telescope *horizontally* until the cross hairs are directed on the point C. The vernier reading now gives the *forward bearing* of BC, which should agree with that indicated by the needle of the compass. Adding 180° to the *forward bearing* BC, we get the *back* bearing CB at which the vernier is to be set, and the instrument is placed at C and the operation repeated there, and so on. By this means the necessity for revolving the telescope *vertically* through 180° each time is obviated, and any error due to the collimation line not being at right angles to the horizontal axis is avoided. This method entails considerably more trouble, however, and need not be used unless great accuracy is required. The importance of the check afforded by the compass needle is not to be underrated, as any *large* error made in reading the vernier is at once seen on looking at the needle of the compass.

In the example on Plate III. the lengths and bearings are as follows :—

Line.	Length.	Bearing.	Line.	Length.	Bearing.
AB	1,300	339° 39′	EF	3,000	353° 42′
BC	1,110	31° 16′	FG	645	50° 12′
CD	1,075	64° 45′	GH	1,100	78° 29′
DE	590	39° 25′			

Plotting by Distances and Bearings with Protractor.—The simplest method of plotting the work is to rule a line on the paper to represent the magnetic meridian through the station A, and place a circular protractor divided into 360° with its 360° and 180° divisions on this line, the centre of the protractor being placed at A. The bearing of AB is then laid off, and the distance AB is measured by the scale along the line so protracted.

Another line representing the magnetic meridian and parallel to the original meridian line at A is then ruled through the point B and the protractor laid down on it and the bearing of BC laid

off as before and the line BC plotted, and similarly all the other lines are plotted.

Latitudes and Departures.—The most accurate method of plotting is, however, to calculate the latitudes and departures, or northings, southings, eastings, westings, as they are also called, of each of the lines. In Fig. 80, which is a diagram of the main lines of the traverse shown in Fig. 79, if a line representing the magnetic meridian is drawn through A and from B a perpendicular Bb is drawn to this line, then the distance Ab is called the "latitude" and the distance Bb is called the "departure" of the line AB. Or otherwise Ab is the "northing" or "southing," so called because it is measured in a N. or S. direction (in the case of AB, Ab is a northing because it is measured from A towards the N.), and Bb is called an "easting" or "westing," so called because it is measured in an E. and W. direction (in the case of AB, Bb is a "westing" because it is measured in a westerly direction from the meridian line through A).

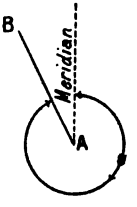


Fig. 80A. Bearings.

Example of Calculation of Latitudes and Departures.—In order to plot the survey by this method, the latitude and departure of each of the lines AB, BC, &c., *i.e.*, Ab, Bb; Bc, Cc, &c., Fig. 80, must be calculated. Referring to Fig. 80, if Ab is the magnetic meridian through A and Bb is perpendicular to Ab, then the angle $bAB = 360° -$ the bearing of $AB = 360° - 339°\ 39' = 20°\ 21'$. This is because the bearings are measured continuously round from the meridian to 360° in a right-handed direction as shown in Fig. 80A.

We therefore get $Ab = AB \cos bAB$

$$= 1300 \times \cos 20°\ 21' = 1219$$

and $Bb = AB \sin bAB$

$$= 1300 \times \sin 20°\ 21' = 452.$$

Thus for the line AB the latitude Ab is 1219, and the departure Bb is 452. Taking now the line BC, if Bc represent the meridian through B and Cc is drawn perpendicular to it, it is seen that the angle CBc = bearing of $BC = 31°\ 16'$, and latitude $Bc = BC \cos 31°\ 16' = 1110 \times \cos 31°\ 16' = 949$, and departure $Cc = BC \sin 31°\ 16' = 1110 \times \sin 31°\ 16' = 576$.

Similarly the angles DC*d*, ED*e*, GF*g*, HG*h*, are respectively equal to the bearings of CD, DE, FG, and GH, and the latitudes and departures of these lines are found by multiplying lengths of the lines by the cosines and sines of the angles. In the case of the line EF, however, the angle *f*EF is not equal to the bearing of EF, but to 360° minus that bearing similarly to the angle *b*AB of the line AB, see Fig. 80A. Similarly when the bearing is between 90° and 180° we have to subtract it from 180°, and when the bearing is between 180° and 270° we have to subtract 180° from it, in calculating the latitudes and departures. The same result is attained by reckoning the bearings from the North and South points round to the East and West points.

The whole calculation may be tabulated as follows:—

TRAVERSE SURVEY, Figs. 79 and 80, Plate III.

Line.	Length.	Bearing.	Latitude = Length × Cosine.	Departure = Length × Sine.
AB	1,300 links	339° 39′	+1219 = A*b*	− 452 = B*b*
BC	1,110 "	31° 16′	+ 949 = B*c*	+ 576 = C*c*
CD	1,075 "	64° 45′	+ 458 = C*d*	+ 972 = D*d*
DE	590 "	39° 25′	+ 456 = D*e*	+ 375 = E*e*
EF	3,000 "	353° 42′	+2982 = E*f*	− 329 = F*f*
FG	645 "	50° 12′	+ 413 = F*g*	+ 495 = G*g*
GH	1,100 "	78° 29′	+ 220 = G*h*	+1078 = H*h*
		Total	+6697 = A*h*′	+2715 = H*h*′

Traverse Tables.—To save the labour of multiplying the lines by the sines and cosines of the angles, traverse tables are used. These tables give by inspection the latitudes and departures for any given length of line and bearing within the limits of the table. Boileau's tables are calculated for every minute of bearing to five decimal places for distances from 1 to 10; Gurden's tables are calculated for every minute of bearing to four decimal places for distances from 1 to 100; Louis and Caunt's tables are calculated for every minute of bearing to four decimal places for distances from 1 to 10; Crellin's tables are calculated for every minute of bearing to four decimal places.

Example of Use of Traverse Tables.—Let the length of

the line be 1,408 and its bearing 64° 48′; the latitudes and departures are taken from the tables as under:—

Distance.	Latitude.	Departure.
1,000	425.78	904.83
400	170.31	361.93
8	3.41	7.24
1,408	599.50	1274.00

For above distance 1,000 the latitude and departure are the values given in the table for distance 10 with the decimal point removed two places to the right; similarly for 400 the values in the table for distance 4 are taken, and the decimal point removed two places to the right, for 8 the values are taken directly from the table for distance 8. The sum of the latitudes for distances 1,000, 400, and 8 is obviously equal to the latitude of the total distance 1,408, and similarly for the departures. The use of traverse tables thus reduces the calculation to simple addition.

Plotting by Latitudes and Departures. — It is convenient to denote the latitudes when northings by +, and when southings by −, and the departures when eastings by +, and when westings by −. Thus in the above table, as all the latitudes are northings, they are all marked +, while with the exception of the departures of AB and EF, which are westings, and marked −, all the departures are eastings, and are marked +. If now a line A*b*′*c*′*d*′*e*′*f*′*g*′*h*′ be drawn through A to represent the magnetic meridian, and from the points B, C, D, E, F, G, H the lines B*b*′, C*c*′, D*d*′ . . . H*h*′ be drawn perpendicular to it, it is evident that the total distance A*h*′ or the total difference of latitude of A and H is equal to the sum of the latitudes A*b*′, B*c*′, C*d*′ . . . F*g*′, G*h*′, which is +6697. Similarly the total departure of H from A or H*h*′, is equal to the sum of the departures C*c*′, D*d*′, E*e*′, G*g*′, and H*h*′ minus the departures B*b*′ and F*f*′, or in other words, if the proper signs are observed, to the algebraic sum of all the departures or +2715.

Had there been any southings in the latitudes, the total difference of latitude would of course have been found by taking the algebraic sum of the latitudes in the same way, the proper signs being observed as above explained.

Next we have—

$$Ag' = Ah' - g'h' = Ah' - Gh = 6697 - 220 = 6477$$
$$Af' = Ag' - f'g' = Ag' - Fg = 6477 - 413 = 6064$$

Similarly—

$$Ae' = 6064 - 2982 = 3082$$
$$Ad' = 3082 - 456 = 2626$$
$$Ac' = 2626 - 458 = 2168$$
$$Ab = 2168 - 949 = 1219 = \text{latitude of AB}$$

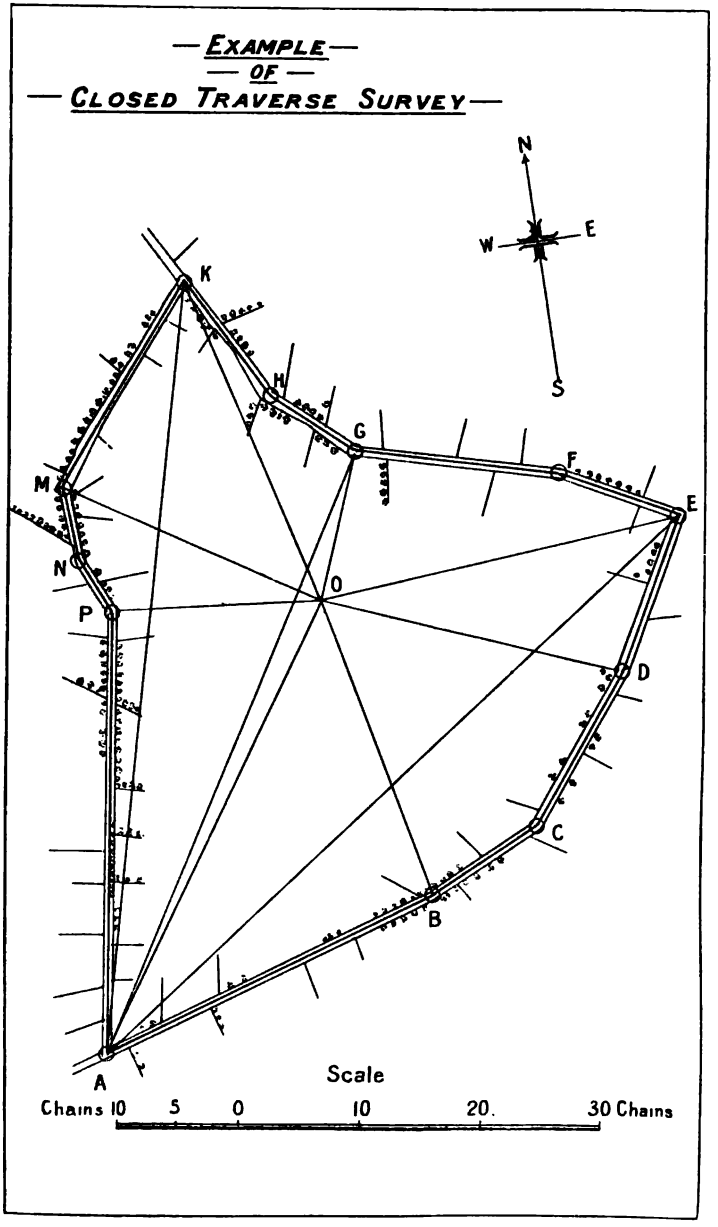
Also similarly—

$$Gg' = Hh' - Hh = 2715 - 1078 = 1637$$
$$Ff' = Gg' - Gg = 1637 - 495 = 1142$$
$$Fe' = Ff' + Ff = 1142 + 329 = 1471$$
$$Dd' = 1471 - 375 = 1096$$
$$Cc' = 1096 - 972 = 124$$
$$Bb = 124 - 576 = -452 = \text{departure of AB.}$$

Having now all the distances along the line Ah' to the points b, c', d' . . . h', and the perpendiculars Bb, Cc', Dd' . . . Hh' at each of these points, the points A, B, C, D, &c., may all be plotted from the meridian line Ah', by simply scaling the distances along Ah' and erecting the perpendiculars.

Checks on Unclosed Traverse.—As a check on the accuracy of the calculation and plotting, the lengths of the lines AB, BC, CD, &c., should when scaled agree with the measurements on the ground. In an unclosed traverse of this kind there is no check on the accuracy of the field work beyond the rough check on the bearings afforded by the compass, unless the relative positions of the beginning and end of the traverse, as A and H, Figs. 79 and 80, are known. In that case the two points A and H being first plotted, the traverse lines should fit in exactly between.

In a long traverse for an important line of communication, say 50 to 100 miles or more in length, through unmapped and undeveloped country, the bearings are checked by taking a sun observation daily, and from it calculating the true bearing of the line then being run. The error is then distributed back over the lines as far as the last bearing where a sun observation was taken. The survey, instead of being commenced with the magnetic bearing of the first line, is begun by finding from a sun observation the true bearing of this line, and the true bearings are worked to throughout (see Chapter XI.). As to checking a


— EXAMPLE —
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— CLOSED TRAVERSE SURVEY —
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Fig. 81.

traverse by bearings to conspicuous lateral objects, see also Chapter XI., page 413, Fig. 236. In a long traverse no opportunity of making this check should be lost.

Example of Closed Traverse Survey.*— Fig. 81, Plate IV., is an example of a closed traverse of the roads bounding a certain property. In this case the survey is commenced at the point A and a complete circuit is made, returning to the original starting point A. The survey is conducted in exactly the same manner as described for the unclosed traverse survey (Fig. 79). When the theodolite is first set up at A, the back bearing AP of the last line PA is taken as well as the forward bearing AB. Upon arriving at the point P the forward bearing PA of the last line should differ by 180° from the back bearing AP as observed when at the starting point A. This is a complete check upon the bearings, and proves the accuracy of the angular measurements. If, as is nearly always the case, there is a closing error, *i.e.*, if the bearing PA does not differ by 180° from the bearing AP, it must be distributed over all the various bearings from the commencement. Thus if the bearing PA is say 3 minutes in excess, then as there are 12 bearings altogether, we must deduct a twelfth part of 3 minutes or 15 seconds from each of the bearings, cumulatively.

Checks on Closed Traverse.—It will be observed that at the starting point A the bearings AE, AG, and AK have been taken. The object of taking these bearings is to see on arriving at E for instance, and taking the bearing EA back to the starting point A, if this bearing differs from the bearing AE, as taken at A, by 180°. Thus a check on the accuracy of the bearings is given on arriving at the point E. Similarly at G the bearing GA checks the bearings up to that point, and at K the bearing KA checks the work up to K. Thus any error in the bearings is discovered before finishing the whole traverse, and, if serious, is rectified before proceeding further. In fact the bearings EA, GA, and KA virtually divide the whole traverse, as far as the bearings are concerned, into the smaller closed traverses ABCDEA, EAGFE, GAKHG, and KAPNMK.† To check a closed traverse executed by another surveyor, project a line through it, see page 419, Chapter XI.

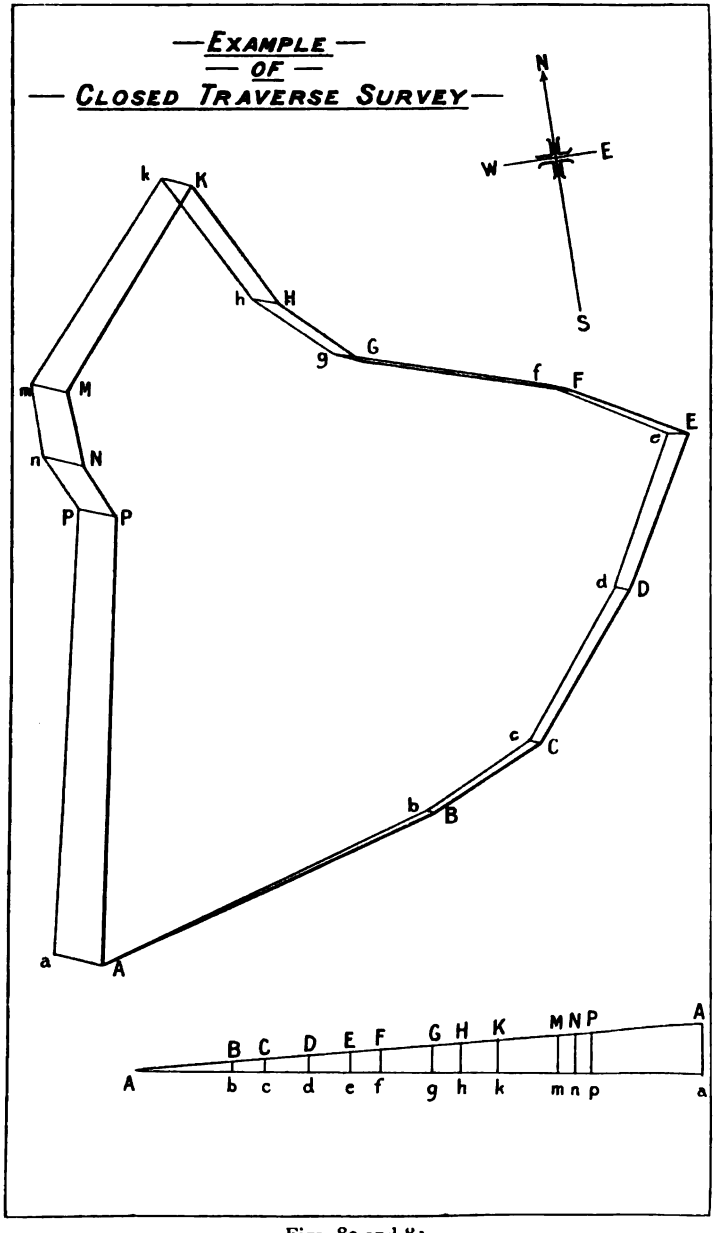
* For another example, see Chapter XI.

† In the case of stations which are not visible from the starting point of the traverse, check bearings may be taken to them from the first stations from which they are visible.

Checks on Plotting of Closed Traverse.—A station O in a commanding position near the centre of the survey, and visible from as many of the stations of the traverse as possible, should be selected, and the various bearings AO, BO, DO, EO, GO, KO, MO, PO, taken to it. The object of these bearings is to check the plotting, if done with a protractor, as it proceeds. For instance, the bearings AO and BO when plotted give by their intersection the point O. When now the work is plotted as far as D, the bearing DO should pass through the same point O, and similarly at E, G, K, M, and P the bearings EO, GO, KO, MO, and PO should all pass through the point O. In this manner any serious inaccuracy in the plotting is discovered as the work proceeds and before the whole of the work has been plotted.

Graphic Adjustment of Closing Error.—When the traverse is simply plotted with a protractor, even although the bearings have been tested and corrected as above explained, the work when plotted will seldom close. This is due to unavoidable inaccuracy in chaining the lines and also in plotting. In Fig. 82, Plate V., suppose the traverse as plotted to be A*bcde* . . . *npa*. Here the closing error is *a*A, which has to be adjusted. Owing to the smallness of the scale, for distinctness the closing error has been much exaggerated in the figure. In order to correct this error, at each of the points *b*, *c*, *d*, *e* . . . *n*, *p*, *a*, rule lines parallel to A*a*. On each of these lines set off the distances *b*B, *c*C, *d*D, *e*E . . . *n*N, *p*P. These distances are to be proportional to the distance of each of the points *b*, *c*, *d*, *e*, &c., from the beginning A of the traverse.

A convenient method of finding the distances *b*B, *c*C, *d*D, &c., is to lay down to any convenient scale, Fig. 83, Plate V., a line A*a* equal to the total length of the traverse lines A*b*, *bc*, *cd* . . . *np*, *pa*. At one end of this line erect the perpendicular *a*A, making *a*A in Fig. 83 = *a*A, the closing error, in Fig. 82. Along A*a* now measure the lengths A*b*, *bc*, *cd* . . . *pa* equal to the lengths of the lines of the traverse. Perpendiculars erected at the points *b*, *c*, *d* . . . *p* will intercept between the lines A*a* and AA the corrections *b*B, *c*C, *d*D . . . *p*P to be applied as shown in Fig. 82. The points A, B, C, D . . . N, P, A now being joined will be the adjusted traverse closing at A. It will be seen that the corrections at each end of any of the lines of the traverse being nearly equal, the length of the



Figs. 82 and 83.

adjusted traverse lines will not differ very materially from the lengths actually measured on the ground. For instance, as *c*C does not differ much from *d*D, the length CD will not differ materially from *cd*. Similarly as *d*D is nearly equal to *e*E, DE will be nearly equal to *de*.

Adjustment of Closing Error by Calculation.—When the latitudes and departures of a closed traverse are calculated, the algebraic sum of the latitudes should be zero and the algebraic sum of the departures should also be zero. This is evident, as an inspection of Figs. 81 or 82 will show that the traverse goes first a certain distance north to the point K and then returns an equal amount south to the starting point A. The sum of the northings is therefore equal to the sum of the southings, *i.e.*, the algebraic sum of the latitudes is zero. Similarly with the departures. For the reasons already stated the algebraic sums of the latitudes and of the departures will seldom equal zero and have to be adjusted.

Example of Adjustment of Closing Error by Calculation.—The following example of a closed traverse, Tables I. and II., will show the method of adjustment when the latitudes and departures are calculated. The calculations of the latitudes and departures were made as already described on pages 93 and 94.

CLOSED TRAVERSE—TABLE I.

Line.	Bearing.	Length, Feet.	Latitude.	Departure.
AB	343° 20′	1,060	+1015.5	− 304
BC	75° 32′	1,202	+ 300.3	+1163.9
CD	110° 35′	1,110	− 390.2	+1039.2
DE	221° 3′	850	− 641	− 558.2
EF	280° 20′	802	+ 143.9	− 789
FG	233° 30′	705	− 419.3	− 566.7
	Totals	5,729	+9.2 error.	−14.8 error.

The lengths and bearings being as given in Table I., upon calculating the latitudes and departures it is seen that the positive latitudes are 9.2 in excess, while the negative departures are 14.8 in excess. This is called the "closing error," and requires to be

adjusted. To do this, apportion the error proportionally to the lengths of the sides. For instance, the length of the side AB is 1,060 ft., and the total length of the sides being 5,729 ft., we have—

$$\text{Correction of latitude of AB} = \frac{1060}{5729} \times 9.2 = 1.7 \text{ ft.}$$

As the latitude of AB is positive, and the positive latitudes are in excess, this is to be deducted from the latitude of AB, and we get—

$$\text{Corrected latitude of AB} = 1015.5 - 1.7 = 1013.8.$$

Similarly the correction for BC is $\frac{1202}{5729} \times 9.2 = 1.9$, and the corrected value of the latitude of BC is $300.3 - 1.9 = 298.4$. The correction for CD is $\frac{1110}{5729} \times 9.2 = 1.8$, and as the positive latitudes are in excess, this falls to be *added* to the latitude of CD, and the corrected value is $390.2 + 1.8 = 392$. Similarly the corrections for the other latitudes are calculated and applied in the same manner, the departures are treated in exactly the same way, and we get the corrected latitudes and departures as given in Table II.

CLOSED TRAVERSE (CORRECTED)—TABLE II.

Line.	Bearing.	Length, Feet.	Latitude.	Departure.
AB	343° 20′	1,060	+1013.8	− 301.3
BC	75° 32′	1,202	+ 298.4	+1167.0
CD	110° 35′	1,110	− 392	+1042.1
DE	221° 3′	850	− 642.3	− 556
EF	280° 20′	802	+ 142.6	− 786.9
FG	233° 30′	705	− 420.4	− 564.9
	Totals	5,729	+0.1	0.0

Upon adding the latitudes and departures we see that the sum of the latitudes is 0.1 and the sum of the departures is 0. The error of 0.1 still remaining in the latitudes is owing to the corrections being calculated to one place of decimals only. The traverse is now to be plotted by ruling a meridian through the starting point A and proceeding in the same manner as described for the plotting of the traverse on Plate III., Figs. 79 and 80. The correction of the latitudes and departures evidently affects the bearings and lengths of the lines AB, BC, &c. These may be measured with the protractor and scaled after the work has been plotted from the corrected latitudes and departures, or, if more

accuracy is desired, the new lengths and bearings may be calculated from the corrected latitudes and departures.

Thus the new length of any of the lines is obviously

$$= \sqrt{(\text{corrected latitude})^2 + (\text{corrected departure})^2}$$

and the tangent of the angle made with the meridian

$$= \frac{\text{corrected departure}}{\text{corrected latitude}}$$

Adjustment of Closing Error when some of the Measurements may be considered more accurate than others.—The above method of correcting and adjusting the closing error is based on the assumption that the chaining and the measurement of the bearings are equally in error, also that the relative accuracy of the lengths and bearings of all the lines is the same. When from special circumstances the measured lengths and bearings of some of the lines may be considered to be more accurate than others, the following method may be adopted. Take one of the lines as a standard and assume that the error in this line is 1; from this basis estimate what the probable error in each of the other lines would be, taking into consideration the special circumstances of each, such as any particular obstacles to measuring, roughness or steepness of ground, number of observations made to determine bearing, if check measurement of length made, and so on; these probable errors are to be *for a distance equal to the length of the standard line.* Each of the lines being thus *weighted* with its probable error as 1.5, 2, 3, 5, &c., multiply the length of each line by its probable error and then we have—

$$\text{Correction of lat. or dep. of any line} = \frac{\text{multiplied length of given line}}{\text{sum of all multiplied lengths}} \times \text{whole error in lat. or dep.}$$

Adjustment of Closing Error when the Error is considered to be due to the Chaining only.—When it is considered that the bearings are practically correct, and that the closing error is due to the chaining alone, the correction of each line is to be computed as follows:—

$$\text{Correction of lat. or dep. of any line} = \frac{\text{given lat. or dep.}}{\text{sum of lats. or deps.}} \times \text{whole error in lat. or dep.}$$

The closing error may be assumed to be entirely due to the

chaining when the bearings have been carefully measured with the theodolite and the bearing of the first line, when redetermined at the close of the traverse, is found to agree very nearly with its true or assumed bearing at the beginning of the traverse.

Amount of Closing Error allowable in practice.*—For purposes of comparison the closing error is taken as the ratio of the length of the line joining the initial and final points of the traverse (as plotted or computed from the field notes) to the length of the whole perimeter of the traverse. In ordinary open country the closing error should not exceed 1 in 300. In town work the closing error should average about 1 in 5,000. For special purposes where greater accuracy is required, as in tunnel work, &c., the precautions in measuring the lengths and angles described in Chapters VII. and XII. must be adopted.

Compass Traverse Surveys.—A traverse survey executed with the compass only, *e.g.*, with a circumferentor or surveying compass (Fig. 60), or with Whitelaw's theodolite (Fig. 61), is executed in precisely the same manner as those already described. The essential difference is that the bearing of each line is measured from the magnetic meridian, and angular errors are therefore not cumulative, as they are in the preceding methods. A long traverse hastily executed with the circumferentor will therefore be in general more accurate than the same traverse if *hastily* executed with the theodolite, and indeed the *speed* and accuracy attainable in compass surveys is remarkable compared with theodolite surveys. At each set up of the compass the bearings of *two* lines may be taken, the line in front and the line behind, and when great speed is necessary it is usual to set up the compass over every *second* station only. To guard against local attraction, however, the compass should be set up at *each* station, and the back bearing as well as the forward bearing of each line observed. If the back bearing does not differ by 180° from the forward bearing there has either been a mistake in reading the forward bearing or else there is some local attraction. In the latter case the angles between the lines must be booked in place of the bearings, until the correspondence of the back and forward bearings indicates that the attraction has ceased. The

* See also page 420.

angle between any two lines will be correctly indicated by the *difference* of the bearings in spite of any local attraction, as both bearings will be equally affected by it. Compass surveys have been run a distance of 20 miles, coming in to 1 chain; but when any great accuracy is required the theodolite should be used.

If readings are taken sometimes from one end of the needle and sometimes from the other indiscriminately, mistakes are liable to occur. To prevent this, always point the north end of the compass box towards the object whose bearing is being observed, and always read the north end of the needle. This is also a more accurate method of procedure.

Variations of the Compass: Annual Variation.—The magnetic needle does not point to true north but to magnetic north. The magnetic north is not a fixed point, but varies with approximate regularity from year to year. The angular difference between true north and magnetic north is termed the variation. The maximum variation is about 25° east and west of true north. Near London in the year 1580 the magnetic variation was 11° 15′ E.; in 1657 the variation was 0; in 1818 the maximum westerly variation of 24° 38′ was reached. Since then the variation has steadily decreased to 18° 10′ in 1887. The average annual movement between the years 1580 and 1880 has been 8¼ minutes. At Edinburgh the variation is about 3° greater than at London. At Dublin the variation is about 3° 50′ greater.

The magnetic variation thus differs at each different place and time, and can only be correctly determined by an astronomical observation for true north by one of the methods described in Chapter X. An approximate value of the magnetic variation may be deduced from a map of the world showing the lines of equal magnetic variation with the amounts for a given year. By allowing an annual difference of 8¼ minutes since the date of the map the approximate magnetic variation may be deduced. Such a map may be found in "Hints to Travellers," published by the Royal Geographical Society. In the United Kingdom the approximate value of the magnetic variation may be deduced from the map prepared by Sir F. J. Evans, Hydrographer to the Admiralty. This map will be found in *Philosophical Transactions*, vol. 162.

Diurnal Variation.—In addition to the regular annual variation there is also a diurnal variation more or less regular. Near London there is a total difference of magnetic variation of 10 minutes in any one day. About 8 A.M. the needle is at its furthest point east, and about 1 P.M. it reaches its furthest point west. At other times of the day the position of the needle varies between these points. About 10 A.M. the needle coincides with the mean position of the magnetic meridian, and again a little before 7 P.M.

The diurnal variation may amount to $\frac{1}{4}^\circ$ at places, and is always at least several minutes. It changes continually at the rate of 1 or 2 minutes per hour, and is greater in summer than in winter. The cause has been stated to be the influence of sunlight.

The magnetic needle is liable to violent and irregular disturbances which in extreme cases may amount to 1° or 2°. These magnetic storms appear to coincide with the appearance of the aurora borealis, earthquakes, and volcanic eruptions.

Notices of magnetic storms are given by the Magnetic Department at Greenwich Observatory specially for the benefit of mine surveyors.

It may be readily imagined from the above that compass surveys are not as a rule susceptible of very great accuracy. The value of compass work lies, however, in the fact that errors are not cumulative, each error being confined to its particular line. When the lines are very short, an error of even 10 minutes in bearing is often negligible in practice; 10 minutes on a length of 1 chain is about 2 in., which on a small scale of plotting is inappreciable.

Dip of Magnetic Needle.—In the northern hemisphere the needle dips to the north, and in the southern hemisphere to the south. It is adjusted to a horizontal position by the instrument maker weighting one end.

Local Attractions.—The observer should be careful to see that there is nothing about the person which may attract the needle. Sometimes a steel band in the brim of a felt hat or steel buttons have given trouble. The glass cover of the compass may be electrified by friction and attract the needle. The electricity may be discharged by touching it with the wet finger. Magnifying glasses with guttapercha frames become highly electrified by wiping the lens, and attract the needle. They should be of brass

or German silver. Nickel has magnetic properties and attracts the needle, as does also of course iron and steel. In towns it is practically impossible to get away from local attraction, such as iron pipes in the ground, lamp posts, railings, &c., and in these situations the compass should not be used.

Deposits of iron or other magnetic ore in the ground attract the needle, so that in such districts compass work is out of the question.

Town Surveys: Example of Town Survey.—Owing to the impossibility of dividing the area to be surveyed into triangles, on account of the obstruction presented by buildings, &c., a traverse is usually the only method available for a town survey. Fig. 84, Plate VI., is an example of a survey of part of a town. As a rule it may be observed that the traverse lines should be as long as possible, and laid out as nearly in the centre of the streets as may be. The first thing to do is to fix these lines. This is done by selecting the stations A, B, C, &c., and driving in iron nails or spikes at these points, the station being accurately marked on the nail by a mark made with a steel punch. Each of these stations should be fixed by measurements taken to the corners of the buildings with a steel tape. By means of these measurements the points may be readily found, or if necessary relocated in the event of the nail or spike being knocked out. At least three measurements should be taken to each station, two to fix its position and one to check it. Thus the station A is fixed by the measurements A*k* and A*l*, and checked by the measurement A*m*. Similarly the other stations B, C, D, E, F, and G are fixed by the measurements to the corners shown in Fig. 84. Having thus selected the lines, they are next chained, and the offsets to the corners of the streets, bends, and angles of buildings, &c., taken in the usual manner. The bearings of the lines are taken as described for a traverse survey. In taking the offsets, it is to be noted that important points, such as the corner *k*, are to be surveyed by a triangle. Thus the triangle A*kn* checked by the tie line *ak* is used to fix the corner *k*. An offset at right angles is not sufficiently accurate for fixing such points, especially if the street be at all a wide one. At the points *a*, *c*, *e*, *g* the subsidiary lines *ab*, *cd*, *ef*, and *gh* are run to fix the side streets and openings shown. The main frontages of the buildings themselves only

should be surveyed, the offsets being taken to them and not to any railing, garden wall, &c., that may happen to be in front of them. This part of the work having been completed, it should now be plotted. In order to survey all the outlines of the buildings, gardens, fences, pavements, street gullies, hydrants, manholes, lamps, &c., the best method is either to trace the work already plotted or replot it on sheets about 18 in. square. These sheets are then mounted on a small light board or inserted in a sketching portfolio, which is carried outside, and the details filled in on it, each measurement being plotted to scale as taken. If preferred, the details may be sketched in by hand, and the measurements marked on the sketch, the plotting is then done in the office, but it will generally be found that owing to the closeness of the detail there is little room for figures, and that the result is confused unless the measurements are plotted outside as taken. The surveying of the detail of a town is very tedious and laborious work. It may be done through the day, but the first part of the work, *i.e.*, the surveying of the traverse lines and the main outlines of the streets, is best accomplished during the night, when the traffic is suspended. In busy thoroughfares this is in fact the only time the work can be done. It is best accomplished with the aid of flaring paraffin lamps, such as are used in public works and tunnels, the verniers being read with a candle or a small hand lamp.*

Surveying with the Box Sextant.—The chief use of the box sextant is as an aid to a chain survey. In place of checking each triangle by measuring a tie line, one or all the angles may be measured with the box sextant, and these serve as checks on the triangles, and save the measurement of tie lines. The use of the box sextant except as a check is not to be recommended, although of course it may upon occasion be used to take all the angles in place of a theodolite. For the reasons already stated on page 75 the use of the box sextant is limited to ground which is practically level.

Surveying with the Prismatic Compass.—This instrument is most convenient for making a rough preliminary traverse. The traverse is conducted in the same manner as

* See also back sight lamp, Fig. 199, Chapter VII., page 301.

already described, the bearings being of course all measured from magnetic north as described in compass traverse surveys, page 102, which see. The bearing of each line being taken independently from magnetic north, the errors are not cumulative, and a long traverse may be made with the prismatic with very satisfactory results. The prismatic compass is much used in conjunction with approximate methods and instruments for measuring distances, and with these a traveller may make a very fair traverse *en route* with but little impediment to his progress. In this connection some instruments for approximate measurement of distances may now be described.

Instruments for Approximate Measurement of Distances.—See also range finders, pages 78 and 79.

By a 2 ft. Rule.—Distances may be measured approximately with a 2 ft. rule, by opening the rule so that when held up to the eye it includes a known height at the extremity of the line whose length is required. For instance, in Fig. 85 let the distance AB be required, and let BC be any known height (BC may be a pole or ranging rod of known length, or a man on foot or on horseback). If now the foot-rule is held to the eye at *a*, Fig. 85, and opened until it includes the points B and C, we have—

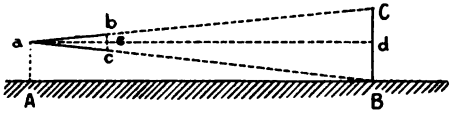


Fig. 85.—Measurement of Distance by 2 ft. Rule.

$$\frac{ad}{ae} = \frac{BC}{bc} \text{ or } ad = \frac{ae}{bc} BC$$

and as *ae* is 1 ft. we have distance $ad = AB =$ height BC divided by width *bc* between the open ends of the foot-rule; *bc* may be measured with an ordinary 10 scale of inches divided into tenths. Thus if BC is 50 ft. = 600 in., and $bc = 1.10$ in., then—

$$AB = \frac{600}{1.10} = 545 \text{ ft.}$$

Perambulator.—This is shown in Fig. 86, and consists simply of a wheel with a handle which is rolled over the line to be measured, and the number of revolutions is indicated on a

small indicator connected by gearing to the axle. The indicator expresses the number of revolutions in feet, and thus gives the total distance on inspection.

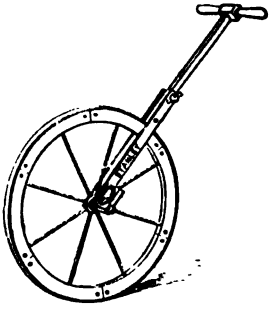


Fig. 86.—Perambulator.

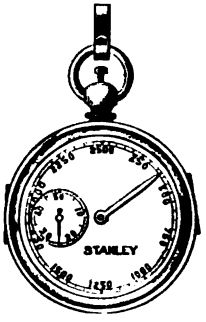


Fig. 87.—Passometer.

Pedometer and Passometer.—The pedometer records the *distance* walked over, while the passometer records the *number of paces* walked. Both instruments are similar in construction and use. Fig. 87 shows Stanley's passometer. This passometer will register the number of paces up to 25,000, or over 14 miles. The passometer should be fixed to the centre of the person. If attached to one leg, it will only count half the number of paces. The most convenient way to carry it is hanging from a waistcoat button-hole. In working with a passometer, the distance corresponding to a given number of paces should be ascertained by walking over a known distance and a "scale of paces" constructed, by which the plotting may be done directly from the registered number of paces as given by the passometer.

Trocheameter.—This is an appliance for counting the number of revolutions of a wheel. By fixing it to the axle of a carriage or other vehicle, the total distance traversed may be ascertained from the number of revolutions and the known circumference of the wheel.

Example of Prismatic Compass and Passometer Survey.—As an example of the results obtainable with prismatic compass and passometer, a closed traverse of over 2 miles in length had a closing error of 25 paces. Going uphill tends to shorten and increase the number of steps and therefore the distance as

measured by the passometer, and as the horizontal projection of the distance is less than the actual distance on the slope, steep slopes up thus lead to error when working with the passometer or pedometer. In going downhill the reverse is the case, as the tendency is to longer steps, and the distance registered by the passometer is shorter, and therefore approximates more to the horizontal projection. In the instance above quoted, the route was hilly and much twisted, some of the lines being very short, and it represents therefore about the maximum error to be expected.*

Surveying with the Pocket Compass.—A very rough traverse may be made even with the pocket compass on occasion. The bearings are judged approximately by simply holding the compass in the hand so that the needle points to 360°, and estimating the bearing of the line by the eye.

Surveying with the Plane Table.—The principle of the method of using the plane table is as follows :—

Let A, B, C, D, Fig. 88, be four points to be surveyed. First a distance must be measured as a base line. In this case AB is supposed to be measured as a base, and the distance *ab* is laid down on the paper to scale. The plane table is then set up over the point A, and the bevelled edge of the alidade or sighting ruler is laid along the line *ab*. The alidade being kept in this position, the table is turned horizontally until the sights are directed to the point B. It is then clamped in this position and a needle is fixed in the paper at the point A. The edge of the rule is now placed against the needle at the point A, and the sights are directed to C, and the ray *ac'* is drawn. Then the sights are directed to D, and a ray *ad'* is drawn. The table is now moved to the point B, and the rule being placed with its edge along the line *ab*, the table is turned horizontally until the sights are directed back along the line BA to the point A. The table is clamped in this position, and

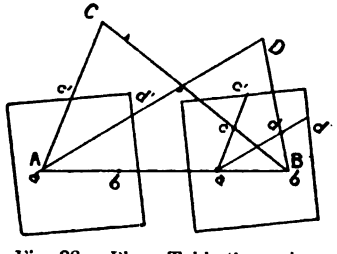


Fig. 88.—Plane Table Surveying.

* Gribble, Preliminary Survey.

the needle being fixed at the point *b* on the paper and the edge of the rule placed against it, the sights are directed successively to C and D, and the rays drawn as shown, intersecting the rays *ac* and *ad* in the points *c* and *d* respectively. The intersections *c* and *d* obviously give the positions of the points C and D.

In this manner any number of points may be taken, and everything visible from both ends of the line AB may be surveyed with the two settings of the table, the only measurement required being that of the base line AB. The survey may then be extended in any direction by intersecting rays from any pair of points now fixed on the paper.

The method is expeditious and simple, the work is cheaply executed, and the trouble often experienced in plotting from assistants' badly-kept field books is avoided.

The following problems in plane tabling will be useful :—

1. *To find the position of any point on the paper when only one ray has been drawn to it.* Set up the table over the point, and laying the edge of the ruler along that ray, sight back to the station from which the ray was drawn and clamp the table. Now stick a needle in the paper at any other station already fixed, and placing the edge of the ruler against it, sight back to that station and draw a line towards you along the edge of the ruler. The intersection of this line with the ray first drawn will give the position on the paper of the point at which the table is set up. As a check, stick in the needle at some other station already fixed on the paper, and placing the edge of the ruler against it, sight back to that station and draw another line towards you. This line should intersect the same point already found. Thus in Fig. 89, if the two points A and B have been fixed on the paper as *a*, *b*, and if one ray only, *a*, *d*, has been drawn from A to the fourth station *x*, to locate this station on the paper the table is set up at *x*, and the ruler being placed along the ray *ad*, the table is

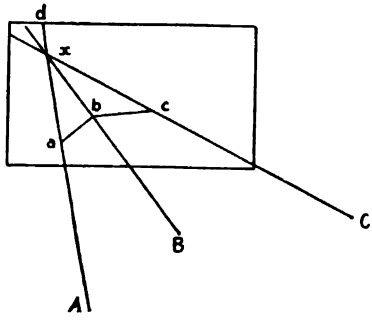


Fig. 89.—Plane Table Surveying.

turned until the sights are directed back on A and clamped in that position. A needle is then fixed in the paper at *b*, and the edge of the ruler being placed against it, the ruler is turned until the sights are directed on B. The line *bx* is then ruled, intersecting the ray *ad* in *x*, which is the position of the fourth station *x* at which the table is set up. As a check, if a third station C has been fixed as *c* on the paper the needle is now fixed at *c* and the edge of the ruler placed against it and the sights directed on C. The edge of the ruler should then intersect the same point *x* previously found.

2. *To find the position of any point when no rays have been taken to it.* When no compass bearings have been taken, this can only be done when the point is visible from three stations already fixed on the paper. The quickest method is to set up the table

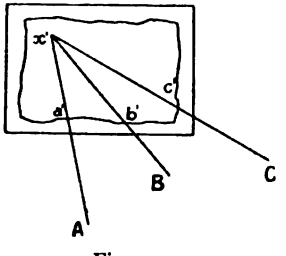


Fig. 90.

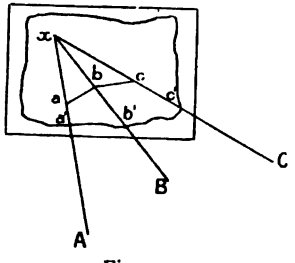


Fig. 91.

Plane Table Surveying.

at the point whose position is required and cover it with a piece of tracing paper (see Fig. 90). From any point *x'* draw the rays *x'a'*, *x'b'*, *x'c'* to the points A, B, C. Now slide the tracing paper over the table, Fig. 91, until the rays *x'a'*, *x'b'*, *x'c'* pass through the points *a*, *b*, *c* already plotted. Prick through *x'*, which is then the position *x* of the station at which the table is set up. This method will not give accurate results if the intermediate point B is more distant than A or C, as in Fig. 92. In this case it would be found that the point *x* might lie considerably to the right or left of its true position, and the lines *x'a'*, *x'b'*, *x'c'* still pass through the points *a*, *b*, *c*. Points whose relative positions are as shown in Figs. 90 and 91 should therefore be selected.

If compass bearings have been taken, the position of any point on the paper may be found from two previously determined stations

only. Thus suppose the stations A, B to be known, as *a*, *b* on the paper, Fig. 93. The table being set up at *x*, whose position on the paper is required, the edge of the trough compass is placed along the line representing magnetic north already drawn at station A, and the table turned until the compass needle points to N. The table is clamped in this position, and a needle being stuck into the paper at *a*, the edge of the ruler is placed against it, and the sights directed to A and the line *ax* drawn. Similarly the needle is stuck in the paper at *b* and the sights directed on B. The line *bx* being drawn to intersect *ax* in *x*, this point is the required position on the paper of the station at which the table is set up. As a check, if a third station C has already been fixed on the paper, as *c*, then the edge of the ruler being placed at *c*, and

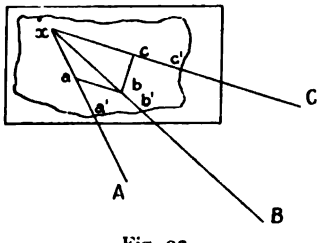


Fig. 92.

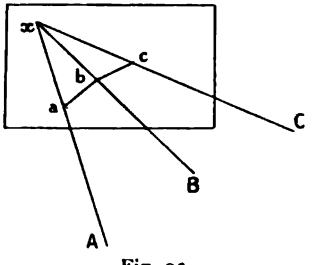


Fig. 93.

Plane Table Surveying.

the sight directed on C, the edge of the ruler should pass through the point *x* already found.

Photographic Surveying.—The theory of surveying by means of photography was originated in 1850 by Colonel Laussedat, and has since been largely developed in Germany, Austria, and Italy. The method has been much used also in the United States and Canada. The principle of the method is as follows:—If a photograph be taken from a point whose position is already known, the direction of the axis of the object glass and the focal length of the lens being also known, and the line of the horizon being marked on the picture, then the picture can be laid down on a sheet of paper on which it is desired to plot the survey, and will give the direction from the point of observation of all the points in the picture whose position is required. Two photographs of the same objects taken from different known points define completely the

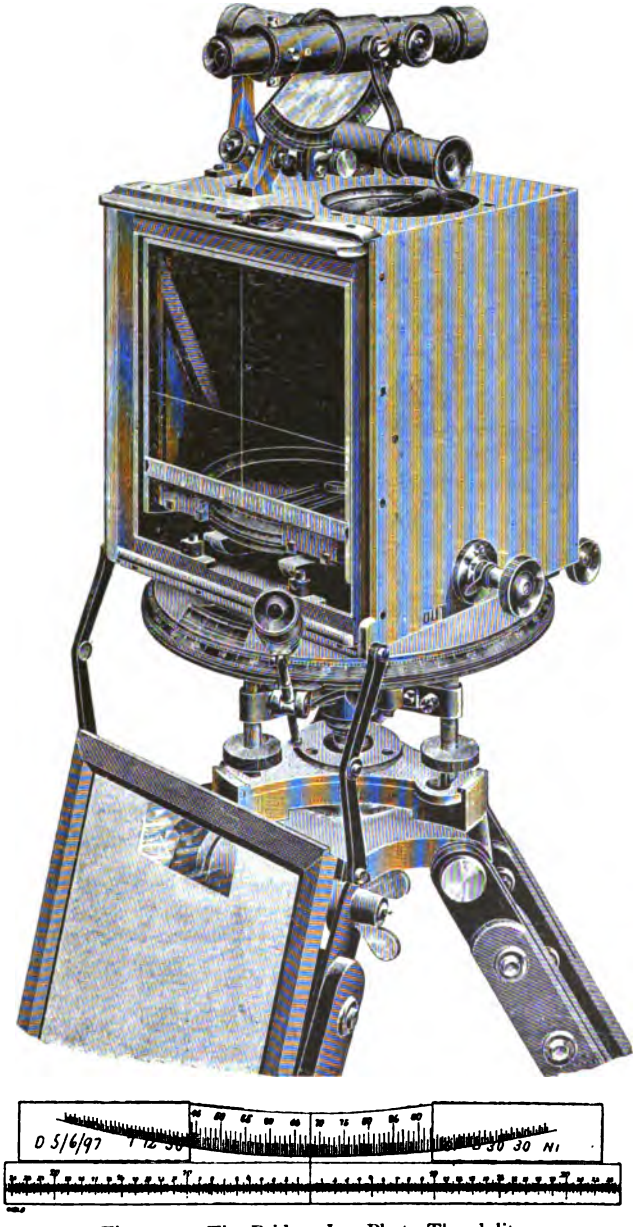


Fig. 93A.—The Bridges-Lee Photo-Theodolite.

position of each object, and also enable altitudes to be calculated or graphically determined. The method is exactly that of the plane table, the difference being that a great part of the work which with the plane table is done in the field, by the photographic method is done in the office.

In order to use a photographic camera for surveying, it must be specially fitted with the means of marking the horizon line on each picture and of observing horizontal angles. The focal length must be known accurately as well as the extent of the field of vision exempt from distortion and other optical qualities. A good lens specially suited for the work is indispensable, and the camera should be of the fixed focus type. The elements required for determining the altitudes above the horizontal plane of vision are the perpendicular distance in the picture of the image of the point from the horizon line, and the distance of the point from the observing station. The latter is taken from the plotted survey.

Fig. 93A shows a special theodolite adapted to photographic surveying, invented by Mr J. Bridges Lee. This instrument gives an automatic record of the compass bearing, which appears on the photograph, as does also a tangent scale, whose chief use is to show horizontal angles at a glance. Fig. 93B is a photograph taken by the Bridges-Lee photo-theodolite, and is one of a series taken during a survey of Trafalgar Square, London. The compass bearing and tangent scale will be seen at the top of the photograph. The horizontal and vertical lines which mark the traces of the principal horizontal and vertical planes will also be noticed in the photograph.

Advantages of the Photographic Method of Surveying.—By means of the photographic survey views of any area a more complete and reliable record of the topographical features of the ground is secured than can be obtained by any other method. Generally half an hour of clear weather at any station is sufficient to obtain a complete set of views from that station. Very often a complete set of photographic survey views may be obtained at out-of-the-way, dangerous, and inaccessible stations where it is not possible to remain for any considerable length of time, and where surveying by any other method is therefore impracticable. In very bad and unsettled weather by taking advantage of short spells of clear weather the survey may be

accomplished by the photographic method under weather conditions which render work by any other method impossible. For military purposes, in a hostile country, the rapidity and completeness of detail of the photographic method is especially valuable. The photographic method of surveying is more economical than any other method. Mr Deville found that the cost of photographic surveying in Canada was only about one-third of the cost of plane table surveying.

Fig. 93B.—Photographic Survey of Trafalgar Square.

Field Work.—Before commencing a survey any available information from existing maps and surveys should be studied and made use of as a general guide to laying out the work. If a triangulation of the country to be surveyed exists, the surveyor should be provided with a plan of this, with the triangulation points accurately shown; in cases where there is no triangulation, the exact positions of some of the principal points in the area to be surveyed must be determined by a careful triangulation. These points will serve for the basis of the photographic work,

and in many cases will be convenient for camera stations, while by their means the exact position of the other camera stations may be determined. The same general principles in selecting stations are to be observed as in other methods of surveying, with the addition that more careful account is to be taken of the illumination of the landscape and the position of the sun.

The object aimed at in all cases should be to obtain good clear views of all points which it may be desirable to plot on the map ultimately, and all such points must appear in views taken from two stations at least, and in such directions from these stations that the angle between the directions shall be neither too acute nor too obtuse.

The photographic surveyor should accustom himself to forming correct mental impressions of the kind of intersections which his photographic views are capable of yielding, and he should remember that while photographs from any particular pair of stations may give perfect intersections for points in or near middle distance, they may give only very acute intersections for distant points or very obtuse intersections for near points. Views from any given pair of stations will give good intersections for plotting purposes only within a certain definite area, and the surveyor should shade off this area on his sketch plan in the field. For all points outside this definite shaded area views from other stations will be required. The positions of camera stations within and around the area to be surveyed should be laid out in a general scheme so as to command the area most advantageously. The exact position of each station should be fixed with precision, and it is better to have views from an excessive number of stations rather than from too few; but on the whole the relative positions of the stations are more important than their number.

The surveyor must remember that the assistant in the office cannot plot any points not visible in the photographs, and that he cannot obtain good intersections from views which would not have given good intersections in the field. The assistant in the office should check his plotting from time to time by reference to subsidiary photographs presenting points from third points of view, but the really essential matter for the surveyor in the field to attend to is to make certain that his photographs shall give at least one good intersection for every important point which may possibly require to be plotted. The total number of views

required at each station will not be excessive, and will depend greatly on the angle of view of the lens employed (*i.e.*, on the ratio of the focal length of the lens to the size of the plate), but the ultimate value of the finished topographical work will depend upon the good judgment of the surveyor in the field in the selection of his stations. In taking the views the camera is levelled up so that the optical axis of the lens is truly horizontal, and the picture plane truly vertical.

Office Work.—The work consists, first, in the preparation of the photographs from the undeveloped plates exposed in the field; and second, in plotting from the photographs when ready for use. The plates should of course be handled only by a competent photographer who can be depended upon to develop them to the best advantage. The pictures must not at any stage be subjected to mechanical strains which may produce distortion of the images. The negatives or prints from them may then be handed over to the office photographer to work from. It is possible to plot from photographs of only a 6 in. focal length, and to obtain fairly correct maps if very great care is taken in the plotting; but as a rule it is not convenient to plot from such short-focus pictures, and enlargements of the photographs are generally used for the plotting.

A short-focus picture does not yield correct impressions to the eye looking at it from a greater distance than the focal distance, and may therefore tend to mislead, and if 35° or more of angle are included in a horizontal distance of some 4 or 5 in. measured along the picture, it is evident that five minute intervals will be scarcely appreciable. Also with short-focus pictures to work from, the traces of the picture planes will almost invariably fall within the area being plotted, which is generally inconvenient in practice. Suppose, however, that a 6 in. focus picture is enlarged 6 diameters; the focal length becomes 3 ft., the picture presents a much more natural appearance to the eye at a moderate distance, and one minute intervals are more easily distinguished than five minute intervals were in the 6 in. focus picture, while the traces of the picture plane will generally fall beyond the part of the map being plotted.

There is a limit to the amount of enlargement which is

practically workable and convenient, and in practice an enlargement of the photographs to a focal length of 3 ft. is found most suitable.

All pictures used for plotting should be enlarged to approximately the same extent, but although convenient, this is not strictly necessary, provided that the trace of the picture plane of each picture is drawn in its proper position on the plan, the exact focal length of each enlargement being known. The assistant in charge of the plotting should first critically examine the photographs with the object of selecting suitable pairs of pictures and a sufficient number of convenient salient points to plot. The exact positions of the station points have been already plotted on the plan, and every precaution should be taken to secure that these points are accurately plotted, as any error in the position of any station will affect nearly all results plotted from views taken at that station. It is assumed that from the photographs at hand it is possible to select a suitable pair (one from each of two stations), overlooking ground common to the two photographs.

The pair of pictures selected are placed side by side and fine red ink dots are made on corresponding points in the two pictures, and a number, also in red ink, is marked alongside each dot, the same number being used for corresponding dots in both pictures. This part of the work is best done by the person who originally traversed the ground and exposed the plates, because memory may help him in the identification of points.

Some people have complained of difficulty in the identification of corresponding points in photographs, but Deville says, "The identification of points, even under different lighting, does not offer any serious difficulties." In reality it is generally easier to identify the same point on two photographs from different stations, when the two photographs are placed side by side, than it would be to identify the point in the field after moving from one station to another.

After a number of points have been identified, marked, and numbered on the photographs, the next thing is to lay off on the plan, from the station point, the direction of the optical axis of the lens for the photographic view selected. In Mr Bridges Lee's photo-theodolite the magnetic bearing of this is recorded on the

photograph. In other instruments not fitted with this automatic bearing recorder, the bearing is read on the divided limb of the theodolite in the usual way and booked in the field. If the magnetic variation at the station is known, the true bearing may be deduced from the compass bearing, or if there is any point visible in the picture whose true bearing is known, the angle between that point and the optical axis of the lens can be ascertained by the aid of the tangent scale of horizontal angles and the direction of the optical axis of the lens laid off from it. Along the line representing the direction of the optical axis of the lens the exact focal length of the enlarged photograph is measured off from the station point, and through the point thus found a line is drawn perpendicular to the line representing the direction of the optical axis of the lens. This last line is the trace of the picture plane of the selected view at that station. The same operations are performed with regard to the other station and the other picture. The traces of the picture planes should be drawn at least as long as the width of the pictures.

Next take two narrow paralle strips of paper as long as the pictures are broad. Rule a fine line across the middle of each strip, and taking one of the strips, mark off along one edge of it the exact distance of each numbered dot on one of the pictures right and left of the middle line, noting on the strip the corresponding number alongside the distance of each dot. Having marked off on the strip of paper the respective distances of all the selected numbered and dotted points, transfer this strip of paper to the plan, so that the edge carrying the distances of the dots coincides exactly with the trace of the picture plane, while the transverse line across the strip is exactly in line with the line through the station representing the direction of the optical axis of the lens. This strip is then fixed in place, and the other strip is treated in the same way with the other picture and similarly fixed at the other station.

Both strips being now accurately in position with the distances of the numbered dots along the picture traces, the next step is to fix two small pins at the station points. Then take two long fine threads of silk, make a loop at one end of each thread, and slip one loop over each of the pins at the station points. To the other end of each thread fasten a piece of elastic, and to the end of the elastic fix a paper-weight. Now

shift the weights on the drawing-table (keeping the threads always sufficiently tight to mark straight lines) until the threads pass through dotted points bearing corresponding numbers on the two strips, taking care that the thread of each station covers the point on the picture line appropriate to that station. The intersection of the two threads will then be the position of the point on the plan. The threads represent rays from the stations corresponding to the rays which would have to be drawn on the plan of a plane table survey. They do not require to be drawn on the plan of the photographic survey, because the plotter is working from both stations at the same time, and he can plot the intersections directly without incurring any risk of confusion among a number of rays. All the selected points common to the two pictures can in this way be plotted in an incredibly short space of time; and as each point is plotted, it may be seen whether the intersection is a good one or not, and all points originally plotted from very acute or very obtuse intersections may be pencil marked and checked and if necessary altered if better intersections are found for them from other stations. This process may be repeated indefinitely with other photographs from the same or other stations, and outlines between the plotted points may be filled in as the work proceeds.

The above is the method employed in Canada and elsewhere for plotting plans by the graphic method, and experience has shown that with reasonable care and good judgment it gives thoroughly satisfactory results. It will be seen that there is nothing difficult or complicated about plotting by this method, and no special knowledge of perspective or descriptive geometry is necessary to apply it.

Levels.—The altitude of any point may be determined after the survey is plotted, and for this purpose the vertical angle subtended by the point at the camera station may be measured. Let α = vertical angle; d = distance of point from camera station, which is found by scaling off plotted survey; f = focal length of photograph or enlargement; h = vertical height of image of point on photograph above or below horizontal line drawn across centre of photograph; then for the required altitude H we have—

$$\text{H} = d\frac{h}{f} \text{ or } \text{H} = d \tan \alpha$$

In reducing the levels the height of the axis of the instrument above ground or peg at the station is to be measured and added or deducted as the case may be.

As on a scale of 200 ft. to 1 in. the distances cannot be scaled nearer than about 2 ft., the levels as above found will be within from 1 to 2 ft. of the truth, the error being proportional to the vertical angle, and to the error in the scaled distance.

Contours by Photographic Surveying.—With regard to contours, it may be noted that every horizontal line on every picture intersects all points at the same level as the station points, and no others. It is evident, therefore, that if the positions of points along the horizon line are plotted, a contour line can be drawn at once by joining these points. It is evident that, in ordinary course, station points in hilly country are likely to be at various altitudes, so that without any special selection of stations for contouring purposes, a number of contour lines can be obtained from photographs taken at different stations. If special contours are required, the surveyor in the field should select some stations at the particular levels at which contours are required.

These methods are the simplest and the best for obtaining plans and contours from photographs. Subsidiary methods and instruments may also be employed on occasions, such, for example, as the perspectograph for rapidly tracing a rough outline of the ground plan from a single photograph overlooking the ground; or a perspectometer, for quickly sketching in the outlines of lakes, bays, or other flat surfaces; or a transparent protractor, for the direct reading of angles vertical or horizontal. Also it is possible on occasion to plot portions of the ground plan from single photographs overlooking the ground from a comparatively high altitude, by using vertical instead of horizontal intersections for fixing points; but all such methods are to be regarded as subsidiary only, and as a rule, less accurate, although sometimes more rapid, than the ordinary methods described. Heights are best obtained by computation from the observed angular altitudes referred to horizon lines whose altitudes are known, after the positions of the points on a plan have been definitely fixed, as described on preceding page.

The author is indebted to Mr J. Bridges Lee, M.A., for information on photographic surveying.

Cost of Photographic Surveying.—In 1888 photographic surveying was adopted in the main range of the Rocky Mountains near the Canadian Pacific Railway.* The total area surveyed by this method up to the end of 1892 was 2,025 sq. miles. The results are plotted to a scale of 1 to 20,000, and reduced for publication to half that scale. As an example of practice, one of the published sheets represents an area of 63 sq. miles. It was surveyed from six stations inside the sheet and eleven stations outside, and 1,075 points were fixed by intersections from thirty-five photographs. Thus on this sheet 17 points per square mile were fixed. The cost of surveying by this method was about £1. 10s. per square mile, or ½d. per acre.†

Plotting the Survey: Protractors.—A protractor is essentially a circle or a semicircle divided into degrees and more or less

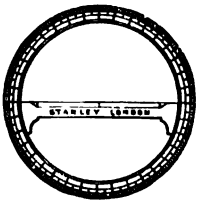


Fig. 94.—Protractor.

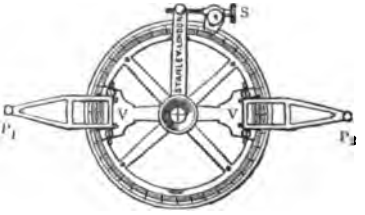


Fig. 95.—Protractor with Verniers.

minute subdivisions, depending on the size of the protractor. Fig. 94 shows an ordinary 12 in. brass protractor as made by Stanley. These may also be had of vulcanite. Fig. 95 shows a more accurate protractor having a tangent or slow motion screw at S and verniers at V, V. The points are pricked off with the pricking needles at P_1 and P_2. With this protractor an angle may be protracted to 1 minute.

In plotting an angle as BAC, Fig. 96, the angle should never be laid off with a small protractor and the line produced to the required length. For instance, if the dotted circle represents the protractor, the zero or 360° being placed at *b* on the line AB and

* Minutes Proceedings Institution of Civil Engineers, vol. cxxviii.

† A complete treatise on Photographic Surveying, containing 232 pages and many illustrations, by E. Deville, Surveyor-General of Public Lands for the Dominion of Canada, is published by the Government Printing Bureau, Ottawa, Canada.

the centre of the protractor at A, the angle BAC might be laid off by marking the point *c* on the protractor at the required angle, and then producing A*c* to C. This would be a most inaccurate way of proceeding, and should never be done. The position of C as found by producing A*c* might be very much out. When a sufficiently large protractor is not at hand, the length of the perpendicular B*d* and the hypotenuse A*d* should be calculated. For instance, B*d* = AB tan BAC, and A*d* = AB sec BAC. By the intersection of arcs described with the beam compasses from A and B as centres the point *d* is found, and the line A*d* drawn at the required angle with AB. The length of the line AC may now be scaled along A*d* and the point C thus fixed. Or otherwise, take A*d* = AB, and calculate the chord subtended by the required angle. As a rule it is to be observed that *producing* a line with a straight-edge leads to inaccuracy.

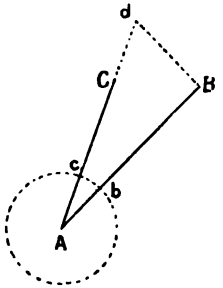


Fig. 96.
Plotting an Angle.

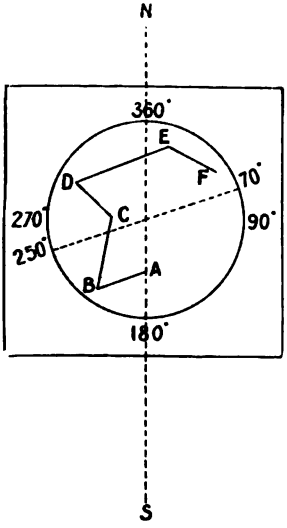


Fig. 97.—Large Protractor.

A very good form of protractor is that shown in Fig. 97. It consists of a large square sheet of cardboard with a circle cut out of it. The circle may be of large diameter, 18 in. or 2 ft. To use this protractor it is laid down with its 360° and 180° divisions coinciding with the meridian line drawn through A. Suppose the bearing of AB is 250°. Take a parallel ruler and set its edge to 250° and 70° on the protractor. Now roll the parallel ruler until its edge passes through A and draw the line AB. As a check roll the ruler back again and see if its edge still coincides with 250° and 70° on the protractor. In this way all the bearings AB, BC, CD, DE, EF may be plotted without shifting the protractor. The protractor

should of course be kept down with weights. Even in the case of a large survey it will not be necessary to shift the protractor more than a few times. To do this, rule a line to represent the meridian through the last station plotted, and again place the protractor with its 360° and 180° divisions on this line and proceed as before. With a large protractor of this kind most of the lines of a survey may be plotted without producing lines or calculating B*d* and A*d*, Fig. 96, as would be necessary with a small protractor. It is divided into degrees, half and quarter degrees.

Section Paper.—This is paper ruled into squares of 1 in. and $\frac{1}{10}$ in. by faint lines. It may also be had divided to various other divisions or specially to order. For work abroad on the metric system it is usually divided to centimetres and millimetres. For plotting longitudinal and cross sections during survey it is very useful, as no scaling is necessary.

Computation of Areas: Areas of Triangles.—The usual method of computing the area of a survey is to divide it into triangles and scale their bases and perpendiculars as described in Chapter I., page 52. The area of each triangle may, however, be calculated by the formulæ given on page 81, from the given sides and angles, and the area is thus obtained from the figures in the field book only. For areas adjacent to irregular boundaries see pages 52, 53, Chapter I. See also planimeters and computing scale, pages 53 to 56, Chapter I.

Calculation of Area of Closed Polygon from Lengths and Bearings.—The area of a closed polygon or closed traverse survey, the lengths and bearings of whose sides are known, may be calculated as follows:—

In Fig. 98, draw a line representing the meridian through A, one of the corners of the polygon. Calculate the perpendicular distances *aa*, *bb*, *cc*, &c., from the middle of each of the lines AB, BC, &c., to the meridian line through A. Let these distances *aa*, *bb*, *cc*, &c., be called the middle distances. Now multiply the latitude of each line by its middle distance. Add together all the products of the lines whose latitudes are northings, and similarly add together all the products of the lines whose latitudes are southings. Deduct the less sum from the greater, and the result will be the area of the polygon. The middle distance of AB is equal to half the departure of the line AB. The middle distance of BC is equal

to half the sum of the total departures of the lines AB and BC from NS, and so on for the other middle distances.

In the case of Fig. 98 we have area = lat. of AB × *aa*, − lat. of BC × *bb*, − lat. of CD × *cc*, + lat. of DE × *dd*, − lat. of EF × *ee*, + lat. of FA × *ff*. The plus signs are applied to those lines whose latitudes are northings, and the minus signs to those lines whose latitudes are southings. For an example see Traverse, Chapter XI., page 419.

As the boundaries of a survey are usually irregular or curved lines, this method is as a rule not of much use, as the areas of the parts adjacent to the irregular boundaries have to be taken

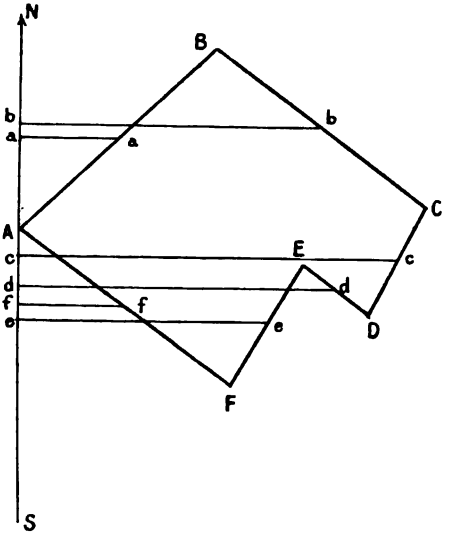


Fig. 98.—Calculation of Area of Closed Traverse Survey.

out separately by planimeter or by methods given on pages 52, 53, Chapter I. When the boundaries of a property are straight lines, and the survey lines can be run along the boundaries themselves so as to include the whole area, the method is, however, useful, especially if the latitudes and departures have been calculated for adjusting the survey and plotting it. This will apply to surveys of townships, mining areas and estates, &c., in countries such as Australia, South Africa, United States, Canada, &c., where the boundaries are often straight-lined polygons of more or less regular shape.

See also Chapter I. for measures of length and area, scales, computation of areas, plotting, copying, enlarging and reducing plans, &c., pages 49 to 59.

To Cut Off a Given Area by a Straight Line Starting from a Given Point in the Boundary of a Survey.—Having plotted the survey, join the given point with one of the corners of the survey so that the nearest approximation to the desired area is cut off. Compute the length and bearing of this line and the area cut off by it. Take the difference between this area and the area required to be cut off, and the remainder is the area to be added or subtracted in the form of a triangle of which we are given the length and bearing of one of the sides, the bearing of another side, and the area. That is, in a triangle ABC we are given one side c, one angle A (the difference of the given bearings), and the area of the triangle Q. The area of the triangle being $\frac{1}{2}bc \sin A$, which is equal to Q, we have $b = \frac{2Q}{c \sin A}$. Having then the two sides b and c and the included angle A, the length of the other side a and the other angles B and C may be calculated. The bearing of the side a is then deduced and the line BC run; the length b of the side AC may be measured as a check, as also the length a of the line BC itself.

To Cut Off a Given Area from a Survey by a Straight Line with a Given Bearing.—Draw a line with the given bearing through whichever corner will give the nearest approximation to the area required to be cut off. Compute the length of this line and the length of the part of the other side of the survey cut off by it, and also the area cut off. Take the difference between this area and the area required to be cut off, which difference is the area to be added or subtracted by a line parallel to the trial line. This area will be a trapezoid whose area, length, and bearing of one of the parallel sides and the bearings of the other sides are known. From these the lengths of the other sides of the trapezoid may be computed. Then measure one of the end lengths and run the line, measuring the length of the line and the other end length as a check.

CHAPTER III.

LEVELLING.

Instruments: Water Level.—The simplest form of level is the water level (Fig. 99). It consists of a tube of tin, or lead, copper, &c., with its ends bent up at right angles. In each of these ends is inserted a piece of glass tube or a glass phial. The whole is mounted on a steady flexible joint to a tripod stand. The tube being filled with water preferably coloured red or blue, and the instrument set up in any position, the surface of the water will always be at the same level in each of the glass tubes or phials. By sighting along the surface of the water in each of the tubes, therefore, we get a horizontal line.

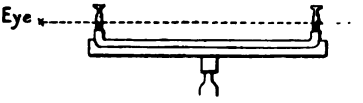


Fig. 99.—Water Level.

Spirit Level.—The spirit level (Fig. 100) consists essentially of a curved glass tube nearly filled with ether, but with a bubble of ether vapour left within. This bubble will always fill the highest part of the tube, and therefore by its movement indicate any change in the position of the tube. If the bubble, by raising or lowering one end of the tube, is brought between the two marks *a*, *b*, which are equidistant from the middle or highest part of the tube, or in case the bubble may have expanded under the influence of change of temperature, so that it extends for an equal distance on each side of the marks *a*, *b*, then if the tube be set in a block the under side of which AB is parallel to a tangent to the centre of the tube, AB will be horizontal. Or if sights CD are fixed parallel to this tangent, then the line of these sights, CD, will be horizontal. The tube is graduated from the centre towards the

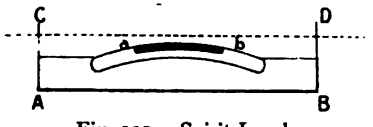


Fig. 100.—Spirit Level.

ends by divisions etched on the glass. When instead of the sights C and D the tube is attached to a telescope so as to be parallel to its axis, we have the engineer's spirit level. The upper surface of the tube is usually an arc of a circle, and the horizontal line is parallel to a tangent to the tube at its middle or highest point. In Fig. 100 the curvature of the tube is much exaggerated.

Radius of Curvature of Bubble Tube.—The radius of curvature of the tube may be found as follows:—In Fig. 101, a staff AB being held at B, and the bubble adjusted to a horizontal position *af*, the reading on the staff at *f* is noted. Next the bubble

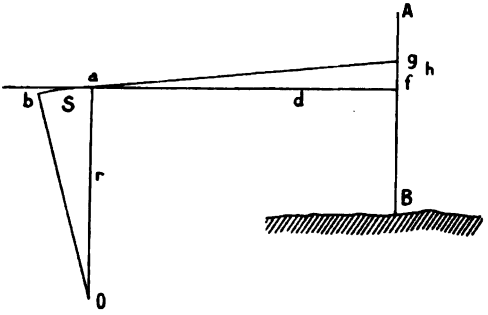


Fig. 101.
To find Radius of Curvature of Bubble Tube.

Fig. 102.
Sensibility of Level.

is moved through the arc *ab* by raising or depressing one end, and the distance $ab = s$ is noted from the divisions on the bubble tube. The reading on the staff at *g* is now noted, and the difference *h* of the readings at *g* and *f* found. The distance $af = d$ being also measured, we have, as the triangles *fag* and *oab* are approximately similar—

$$\frac{r}{s} = \frac{d}{h}$$

$$\text{whence } r = \frac{sd}{h}$$

o being the centre of the circle of which the bubble tube is an arc.

For example, at 50 ft. distance the difference of staff readings was 0.04 ft. and the bubble was moved through 0.03 ft. Then—

$$r = \frac{0.03 \times 50}{0.04} = 37\tfrac{1}{2} \text{ ft.}$$

Sensibility.—The sensibility of a spirit level is estimated by the distance through which the bubble moves for a given change of inclination. The more sensitive the level the greater is the distance through which the bubble moves for any given change of inclination.

In Fig. 102, let s = length of arc through which the bubble moves for a change of inclination of 1″—

r = radius of curvature of bubble tube.

Then—

$$\frac{s}{\text{whole circumference}} = \frac{1''}{360^\circ}$$

$$\text{or } \frac{s}{2\pi r} = \frac{1''}{360^\circ}$$

$$\text{whence } s = \frac{2\pi r \times 1''}{360^\circ} = \frac{r}{206265}$$

The sensibility is therefore directly proportional to the radius. It is also equal to that of a plumb-line level whose plumb line is equal in length to the radius of curvature of the spirit-level tube. A bubble is also sensitive in proportion to its length, a long bubble being more accurate than a short one. This refers to the length of the bubble itself apart from the length of the tube.

Accurate Measurement of Small Vertical Angles by means of Bubble Tube of Spirit Level.—Small angles may be measured by means of bubble tube readings with greater accuracy than by any other method known.

*Angular Value of one Division of Bubble Tube.**—To find the angular value of one division of the bubble tube, proceed as follows:—Measure a line on level ground 300 to 500 ft. long. Set up at one end of this line and hold the staff on the other end. Bring the bubble near one end of its run by moving the instrument vertically and read both ends. Also read the staff. Bring the bubble near the other end of its run and again read bubble ends and staff. Repeat this a great many times, and take the half difference of the two end readings each time, thus getting the movement of the centre of the bubble from the centre of tube for each reading. Take the average of these results for each end of tube. These average results being for opposite ends of the tube, their sum will give the average movement of bubble. Similarly take the average

* See also pages 191, 194.

of the upper readings and the average of the lower readings on the staff, and the difference gives the average movement of the collimation line.

Let A = bubble movement in divisions of scale on bubble tube.
„ S = movement on staff in feet.
„ L = length of base-line in feet.

Then—

$$\text{Angular value of 1 division of bubble tube in seconds} = \frac{S}{AL \sin 1''}$$

$$\text{Log sin } 1'' = 4.6855749$$

Measurement of Small Angles with Bubble Tube.—To measure a small angle, note reading of two ends of bubble to divisions and tenths, and take half the difference. Move the bubble the given amount and note both ends again, and take half the difference. The difference of these two amounts in divisions of the scale of the bubble tube multiplied by the angular value of one division of the bubble tube is the vertical angle through which the tube was moved. The bubble tube is supposed to be graduated from the centre towards the ends, as is usually the case. Thus the half difference of end readings is the movement of the centre of the bubble from the middle graduation.

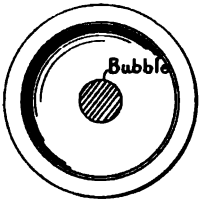


Fig. 103.
Circular Spirit Level.

Circular Spirit Level (Fig. 103).—The upper surface of this level is spherical. It therefore indicates a level in *every* direction in place of only in one direction as the preceding. It is adjusted in the same way, but in *two* directions at right angles to each other.

Dumpy Level.—The most common form of engineer's spirit level is the dumpy level, invented by W. Gravatt. The instrument as made by Stanley is shown in Fig. 104. It consists of the *telescope ab* similar to the telescope of the theodolite (see page 60), having the *diaphragm* at *c* carrying the cross hairs or a glass with fine lines engraved on it. The *spirit level de* is attached to the top of the telescope, being hinged at one end *d* and fixed by the capstan adjusting screws at *e*. By means of the latter the end of the spirit level may be raised or depressed until it is parallel to the axis and collimation line of the telescope. There is also a

small *cross level* at *f* similarly fixed by adjusting screws, by means of which the levelling at right angles to the axis of the telescope may be effected. Underneath the telescope is a compass for taking magnetic bearings, read by the prism or microscope shown at *g*.

The vertical axis of the telescope is attached by arms to the three *levelling screws s,s*, which rest in bearings on the plate *p* which is fixed to the top of the *legs l,l,l*.

Fig. 105 shows Cooke's patent reversible level, as made by Messrs Cooke, York. The peculiarity of this instrument is that by having enlarged collars on the telescope at *a* and *b*

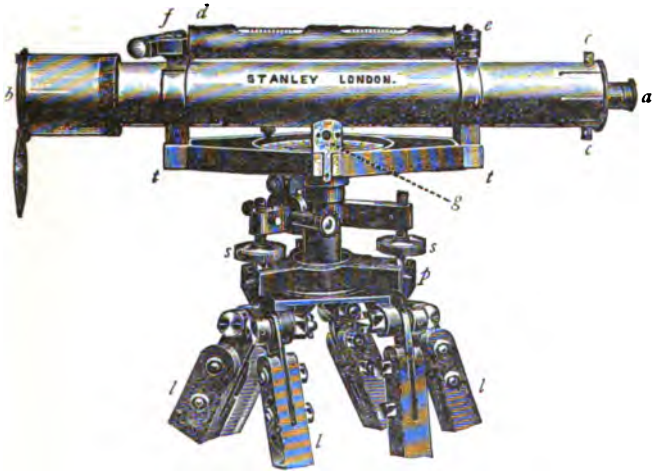


Fig. 104.—Dumpy Level.

the whole telescope may be withdrawn lengthwise and replaced end for end, or it may be completely rotated by twisting round the telescope tube with the hand. This enables the adjustment to be made more easily than the adjustment of the ordinary level, and somewhat after the manner of adjusting the Y level. It may be done indoors, but the instrument may also be adjusted on a long base outside, in the usual manner, as described in Chapter IV.

Fig. 106 shows Cushing's patent reversible level, as made by Messrs Cooke, York. This instrument is the invention of Mr Cushing, Inspector of Instruments to the India Office. The eye-piece end of the telescope at *a* carrying with it the *diaphragm* is

made interchangeable with the object glass end *b*. These pieces being interchanged, the adjustment of the instrument is facilitated.

The Y Level.—Fig. 107 shows the Y level, as made by Troughton & Simms, London. In this instrument the telescope rests in clips at *a* and *b* which may be opened by taking out the pins shown. The telescope may then be lifted out and replaced end

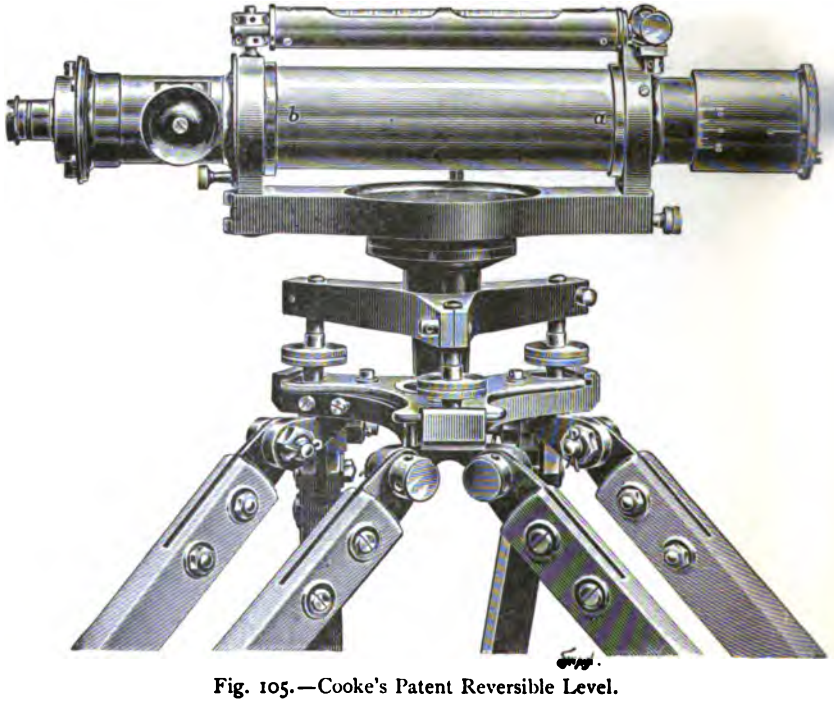


Fig. 105.—Cooke's Patent Reversible Level.

for end. It may also be completely rotated on its axis by twisting it round with the hand. The object of this is to test the adjustment or the readings on the staff. For instance, suppose the bearing of the telescope in the clips at *a* is higher than the bearing at *b*, or that the spirit level is not parallel to the collimation line. If the clips are opened and the telescope is reversed end for end and again directed to the staff, as the bearing *b* is now at the eye end of the telescope and the bearing *a* at the object end, the error

will be reversed, and the mean of the two staff readings will be the correct reading. Similarly if the telescope is twisted half round on its axis, the error if any due to the cross hairs not being in the axis of the telescope will be reversed and the mean of the two readings will be correct.

Levels are made with either three or four levelling screws.

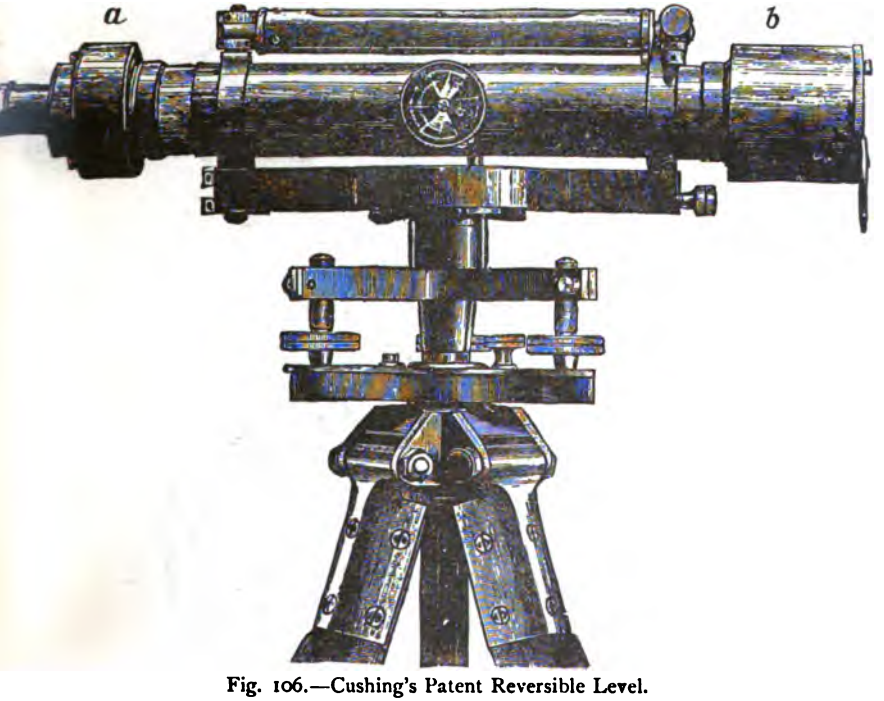


Fig. 106.—Cushing's Patent Reversible Level.

Three screws are better than four, as four screws will jam if screwed up very far. Four screws also require the use of both hands.

Levelling Staff.—Fig. 108 shows Cooke's ordinary Sop- with telescopic levelling staff divided into feet and $\frac{1}{100}$ parts of a foot. Fig. 109 shows the same staff graduated to feet and $\frac{1}{50}$ parts of a foot only. Fig. 110 is another staff of the same kind

divided by figures and dots only. Fig. 111 is a metric staff divided to metres and half centimetres.

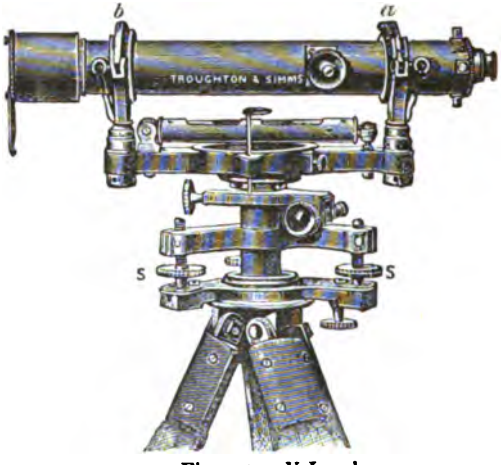


Fig. 107.—Y Level.

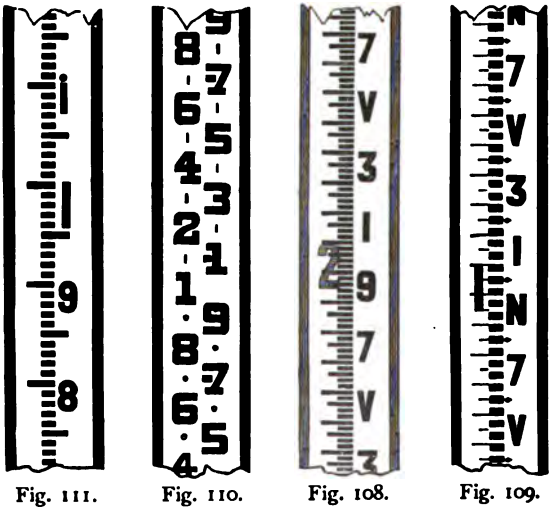


Fig. 111. Fig. 110. Fig. 108. Fig. 109.

Levelling Staves.

The staff is usually 14 or 16 ft. long, and is made in three pieces, the two upper pieces fitting telescopically into the lower

piece so that it may be conveniently carried when not in use. The figures indicating the feet or metres are usually painted red and the others black.

Levelling with the Spirit Level: Detached Levels.—The simplest case of levelling is finding the difference of level between two points so situated that the level may be set up somewhere between them and both points sighted on. Suppose the difference of level between two points A and B, Fig. 112, is required. The method of procedure is as follows:—Set up the level at C anywhere between A and B. Now turn the telescope so that it is parallel to one pair of screws, and by means of them adjust the telescope so that the bubble is in the centre of its run. Now turn the telescope through 90° so that it lies over the third screw, or in the four-screw instrument, over the other pair of screws,

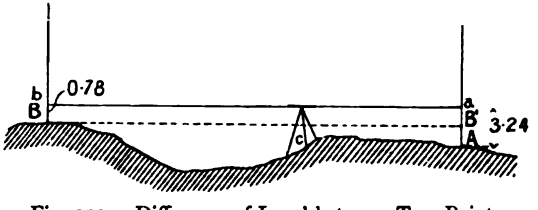


Fig. 112.—Difference of Level between Two Points.

and again level the bubble. Repeat these operations until the bubble remains in the centre of its run in every position of the telescope. The instrument is now said to be "levelled up." Now having directed the chainman to hold the staff at A, the telescope is directed on to the staff, focussed, and the reading booked, care being taken to see that the bubble remained in the centre of its run while the reading was being taken. The chainman is then directed to hold the staff at B, and the telescope is directed on to the staff there and the reading booked. Suppose the reading *Aa* at A is 3.24 ft., and the reading *Bb* at B is 0.78 ft. Then the collimation line *ab* being horizontal and the horizontal line BB' being drawn through the point B, it follows that the difference of level AB', Fig. 112, is equal to the staff reading *Aa* less the staff reading *Bb*, or $3.24 - 0.78 = 2.46$ ft., or the point B is 2.46 ft. *higher* than the point A.

Continued Series of Levels.—To take the next case, suppose the difference of level between two points A and B, Fig. 113, is required, and that owing either to the configuration of the ground or to the distance between A and B it is impossible to see the staff at both these points from any one intermediate position. Set up the level at C and level it up. Suppose the staff reading at A is 4.15. Book this, and then direct the chainman to hold the staff at E, which point should be as high up as it is possible to see. A little practice will soon enable the observer to select the point C where to set up the instrument so that it will be about *halfway* between A and the highest visible point E. In holding the staff at E, a well-defined point, such as a stone firmly imbedded in the ground, should be selected by the chainman to hold the staff on, otherwise the foot of the staff may be shifted during the observation and while moving the instrument from C to D. Having directed the telescope to E, and booked the reading, which is say 0.23, the staff is simply turned round so as to face the point D, and the level is carried to that point and set up there. The reading on the staff still held on the same point E is now taken, say it is 3.57. The staff is then moved to F and the reading there taken, say 2.98. In this case again a firm point, such as a stone, to hold the staff on, is selected by the chainman at F. The level is now moved to G and set up there, and the staff still held at F is read, say the reading is 0.02. Lastly, the chainman holds the staff on the point B and the reading there is taken, say 4.06, which concludes the operation. Now the reading at A being 4.15 and that at E 0.23, it follows, as already explained for Fig. 112, that the point E is $4.15 - 0.23 = 3.92$ ft. *higher* than A. Similarly the reading at E being 3.57 and that at F 2.98, it follows that the point F is $3.57 - 2.98 = 0.59$ ft. *higher* than E; and again the readings at F and B being 0.02 and 4.06, the point B is therefore $4.06 - 0.02$

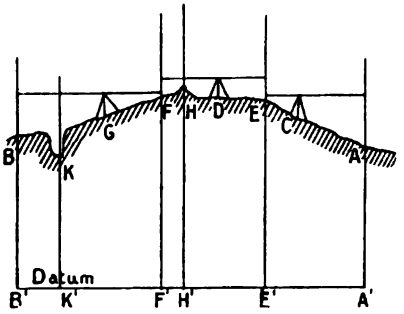


Fig. 113.—Continued Series of Levels.

= 4.04 ft. *lower* than F. We have now a rise of 3.92 from A to E, and a rise of 0.59 from E to F, and therefore a total rise of 3.92 + 0.59 = 4.51 ft., from A to F. Deducting from this the fall from F to B, which is 4.04, we find that the point B is 4.51 − 4.04 = 0.47 ft. higher than A.

When levelling in this way between any two points the sights CA, DE, and GF are called *back sights*, and the sights CE, DF, and GB are called *fore sights*.

The points E and F on which the staff is held while the level is moved forward, and to which both a fore sight and a back sight are taken, are called "*change points*."

Field Book: "Rise and Fall" System.—The usual form of level book for booking the levels is as follows:—

1 Back Sight.	2 Intermediate Sight.	3 Fore Sight.	4 Rise.	5 Fall.	6 Reduced Level.	7 Distance.	Remarks.
4.15					143.52		A
3.57		0.23	3.92		147.44		E
0.02		2.98	0.59		148.03		F
		4.06		4.04	143.99		B
7.74		7.27	4.51	4.04			
7.27			4.04				
0.47 rise			0.47 rise				

The first column contains the back sights, the second the intermediate sights, the third the fore sights, the fourth the rises, the fifth the falls, the sixth the reduced levels, and the other page of the book is reserved for the distances (column 7) and remarks.

The use of the second column for intermediates and the sixth column for reduced levels will be explained after.

Booking the Levels.—The method of booking the levels shown in Fig. 113 is as follows:—The first sight of 4.15 on A is a back sight, and is entered in the back sight column as shown, and is described as A on the same line in the space for remarks. The next sight on E is a fore sight, and as this is to a *new* point, the reading on E, viz., 0.23, is entered on the *next* line below and is described as E in the space for remarks. The next sight being a

back sight to the *same* point E, is entered on the *same* line in the back sight column, viz., 3.57. Similarly the next sight being a fore sight to a *new* point F, it is entered in the fore sight column on the *next* line, viz., 2.98, and is described as F in the remarks space. The back sight 0.02 to the *same* point is entered on the *same* line. The last sight being to the *new* point B, and being a fore sight, it is entered on the *next* line in the fore sight column, viz., 4.06, and is described in the remarks as B. It may be noted that the last sight of a series of levels is *always a fore sight.*

Reducing the Levels.—It is now to be observed that whenever the fore sight is less than the back sight, the difference of level between the two points is a *rise*, and *vice versâ*. Thus we see that the fore sight on E, viz., 0.23, is less than the back sight on A, and the difference of level between A and E is therefore $4.15 - 0.2\dot{3}$, or 3.92 rise, which is entered in the rise column as shown. Similarly the fore sight to F being less than the back sight to E, the result is $3.57 - 2.98 = 0.59$ rise, or 0.59 rise from E to F, which is also entered in the rise column as shown. Again, the fore sight on B being *greater* than the back sight on F, we get $4.06 - 0.02 = 4.04$ *fall*, or 4.04 fall from F to B, which is entered in the fall column as shown.

The total difference of level between A and B is now the difference of the sums of the rises and falls in columns 4 and 5, or $4.51 - 4.04 = 0.47$, and as the sum of the rises is the greater, the result is a rise of 0.47, *i.e.*, there is a rise of 0.47 from A to B.

Check on Reducing the Levels.—Obviously the same result will be arrived at by adding up all the back sights and all the fore sights in columns 1 and 3. When the sum of the fore sights is less than the sum of the back sights, the result is a rise, and *vice versâ*. In this case we have sum of fore sights = 7.27, and sum of back sights = 7.74. The difference of level is therefore $7.74 - 7.27 = 0.47$ *rise* as before. This is a valuable check on reducing levels, and should never be neglected.

Datum.—In all levelling operations of any importance it is necessary to work to a fixed "datum," *i.e.*, to find the heights of the various points above a certain fixed plane which is called the "datum." The datum adopted for the levels of the Ordnance Survey of the United Kingdom is mean sea-level at Liverpool.

Bench Marks.—A bench mark is a mark on a building or wall or other prominent object, the level of which mark above a certain datum is known. The Ordnance bench marks are distributed at numerous points all over the country, and are marked by an arrow (Fig. 114) cut into the stone of the various buildings, walls, &c. The positions of these bench marks and their heights above Ordnance datum are marked on the Ordnance maps, and they may be readily found by consulting the maps. In some instances, when bench marks were marked on a stone in a fence-wall, or other building, cases have occurred where part of this wall or building has been taken down and rebuilt and the stone with the bench mark cut on it has been built in the wall again at a considerably different level from its original position. Even a bench mark on a milestone has been known to be considerably displaced from its original position. The surveyor should always, therefore, where possible, avoid selecting a bench mark on a building of a temporary nature. In countries where there is no reliable survey or levels the surveyor must fix his own datum and bench marks, either by starting from approximate mean sea-level or by assuming a datum and working to it throughout.

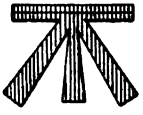


Fig. 114. Ordnance Bench Mark.

Reduced Levels.—Having now explained the meaning of a datum, the use of column 6, page 137, may be shown. For instance, suppose the height of the point A, Fig. 113, above Ordnance datum, or any other selected datum, to be known, we use column 6 to find the heights of the other points E, F, and B above this datum. Suppose that the point A is known to be 143.52 ft. above a certain datum. This is called the *reduced level* of the point A, and is entered opposite the back sight of A in column 6, as shown on page 137. On referring to the rise and fall columns we see that E is 3.92 ft. above A. To get the reduced level of E, therefore, we add 3.92 to the reduced level of A, viz., $143.52 + 3.92 = 147.44$, which is entered in column 6 as the reduced level of E. Similarly the reduced level of F is $147.44 + 0.59 = 148.03$, while B being 4.04 ft. lower than F, its reduced level is $148.03 - 4.04 = 143.99$. Obviously, when all the levels are reduced, the difference of level between any two points is equal to the difference of their reduced levels. For instance, the

difference of level between A and B is 143.99 − 143.52 = 0.47, as previously found.

Plotting the Levels: Longitudinal Section.—If the distances of the points from each other, A′E′, E′F′, and F′B′, Fig. 113, have been measured and entered in the distance column, in order to plot the levels between A and B all we have to do is to draw a line to represent the datum, and on it lay off the distances A′E′, E′F′, F′B′. By erecting perpendiculars A′A, E′E, &c., at these points, and making them equal to the reduced levels 143.52, 147.44, 148.03, and 143.99, the points A, E, F, B are found; and on joining these points, we get the "longitudinal section" AEFB of the ground between A and B, as shown in Fig. 113. In cases where the perpendiculars A′A, E′E, &c., are inconveniently long, the datum line A′B′ is taken as a certain amount *above* the datum of the levels. Suppose the datum A′B′ is assumed to be 100 ft. *above* the datum of the levels, the perpendiculars A′A, E′E, &c., will then be plotted equal to 43.52, 47.44, &c., instead of equal to 143.52, 147.44, &c. The assumed level of the datum line A′B′ is to be noted along the line A′B′ thus: Datum = 100.00, or as the case may be.

Readings near Top of Staff.—When the line of sight or collimation line cuts the staff near the top, so that the staff reading is large, care should be taken to have the staff held perfectly vertical, otherwise the reading may be considerably out. Thus in Fig. 115, if AB is the collimation line, and the staff be held vertically, the reading is BD. If, however, the staff be held off the plumb as DC, the reading is DC, and as DC is greater than DB in the ratio of the hypotenuse of a right-angled triangle to its longest side, the error may amount to several hundredths of a foot, *i.e.*, two or three figures in the second place of decimals. This error may be avoided by causing the chainman to slowly "wave the staff" from the position DC to the position DE on

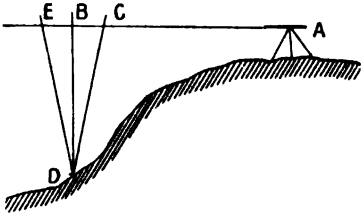


Fig. 115.—Readings near Top of Staff.

each side of the vertical until the *smallest reading* DB has been observed.

Field Book: "Collimation" or "Height of Instrument" Method.—The system of booking and reducing the levels already described is called the rise and fall system, and is somewhat more laborious than the collimation or height of instrument method which will now be described. Taking the same example as shown in Fig. 113 and already described, the level book under the collimation method is kept as under:—

Back Sight.	Int. Sight.	Fore Sight.	Height of Inst.	Red. Level.	Distance.	Remarks.
4.15			147.67	143.52		A
3.57		0.23	151.01	147.44		E
0.02		2.98	148.05	148.03		F
		4.06		143.99		B

Here it will be noticed that in place of the rise and fall columns we have one column marked "Height of Instrument." Referring to Fig. 113, we notice that if the reduced level of the point A is 143.52 ft. and the back sight on A is 4.15, the reduced level of the collimation line of the instrument at C is $143.52 + 4.15 = 147.67$, or in other words, the height of the axis of the instrument or the height of the collimation line, usually called height of instrument, is 147.67. This is entered in the height of instrument column. To get the reduced level of the point E, the staff reading on E being 0.23, it is obvious from Fig. 113 that E is 0.23 below the axis of the instrument, and we get $147.67 - 0.23 = 147.44$ for the reduced level of E. Again, the height of instrument when placed at D = reduced level of E + back sight on E $= 147.44 + 3.57 = 151.01$. Again, deducting the fore sight on F from this height of instrument we get $151.01 - 2.98 = 148.03$ as the reduced level of F. Similarly for the new height of instrument at G we have $148.03 + 0.02 = 148.05$, and for the reduced level of B we get $148.05 - 4.06 = 143.99$.

On examining this process it will be seen that it simply consists in *adding* all the back sights to the reduced levels of the points on which they are taken to get the heights of instrument, and *subtracting* each fore sight from the previous height of instrument to get the reduced levels of the points to which the fore

sights are taken. We have therefore the simple rule, "Add the back sights and subtract the fore sights."

Intermediate Sights.—As regards intermediate sights, suppose now there is an eminence at H and a depression at K, the levels of which are also required (Fig. 113). The intermediate sights are to be booked on their proper lines in the intermediate column, and are to be treated exactly like fore sights. By the rise and fall method, supposing the staff reading at H to be 1.16, and the reading at K to be 6.23, the level book will be as under:—

Back Sight.	Intermediate Sight.	Fore Sight.	Rise.	Fall.	Reduced Level.	Distance.	Remarks.
4.15					143.52		A
3.57		0.23	3.92		147.44		E
	1.16		2.41		149.85		H
0.02		2.98		1.82	148.03		F
	6.23			6.21	141.82		K
		4.06	2.17		143.99		B

Having reduced the levels as far as the point E, the back sight on E being 3.57 and the intermediate sight on the intermediate point H being 1.16, we see that H is higher than E by $3.57 - 1.16 = 2.41$ rise, which is entered in the rise column and added to 147.44 to get the reduced level of H, *i.e.*, $147.44 + 2.41 = 149.85$. Again, the reading on the intermediate point H being 1.16, and the fore sight on F being 2.98, we see that F is lower than H by $2.98 - 1.16 = 1.82$ fall, which is entered in the fall column and deducted from 149.85, giving 148.03 as the reduced level of F. Proceeding in this way, we get the reduced level of the intermediate point K = 141.82, and the reduced level of B = 143.99 as before.

By the collimation or height of instrument method the level book would be as under:—

Back Sight.	Int. Sight.	Fore Sight.	Height of Instrument.	Reduced Level.	Distance.	Remarks.
4.15			147.67	143.52		A
3.57		0.23	151.01	147.44		E
	1.16			149.85		H
0.02		2.98	148.05	148.03		F
	6.23			141.82		K
		4.06		143.99		B

— EXAMPLE —
— OF —
— LONGITUDINAL SECTION —

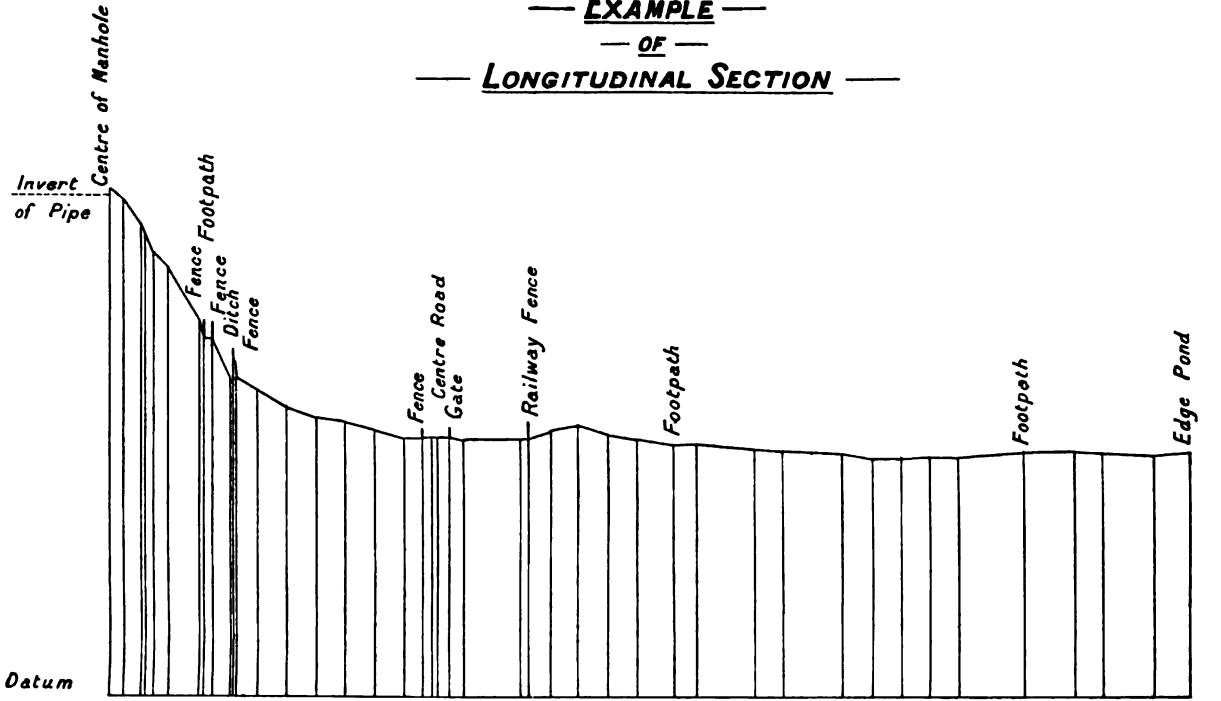


Fig. 115A.

PLATE VII.

In this case we simply have to deduct each intermediate from the previous height of instrument to get the reduced level, in the same way as we deduct each fore sight. It will be noticed that there is no new height of instrument opposite the intermediate sights, the intermediate and the next following fore sight being both deducted from the same height of instrument. The reason for this will be obvious from an inspection of Fig. 113.

Comparison of "Rise and Fall" and "Height of Instrument" Systems.—On comparing the two methods it will be seen that by the collimation or height of instrument method in the case of the intermediate sights there is a saving of either a subtraction or an addition in the reducing, which when there are a great many intermediate sights is a not inconsiderable saving of labour. By the rise and fall method there is, however, a check on the reducing of the intermediate sights, as any error in the reducing of an intermediate sight affects the reducing of the next following fore sight, and therefore of all the levels after it. On the other hand, by the height of instrument method any intermediate sight may be wrongly reduced without affecting any of the other levels. For instance, by the height of instrument method in the above example the reduced levels of the intermediate sights at H and K might have any value in the reduced level column without affecting any of the other reduced levels. In this way we might check in our levels correctly on to a bench mark and still have the intermediate sights wrongly reduced. By either method, however, there is no check on the actual staff readings of the intermediate points in the field. Important points should therefore always be made "change points" and not intermediates. This may always be effected by moving the instrument, although it might not be necessary to do so as far as being able to see points beyond is concerned.

Example of Longitudinal Section.*—Fig. 115A, Plate VII., is another example of a "longitudinal section." The level book of this section is given on pages 144, 145, and it will be instructive for the student to copy out the back sights, intermediate sights, and fore sights, together with the reduced level of the starting point at the pond, and work out the reduced levels for himself, and compare them with those given on pages 144, 145. He might

* For another example see page 230, Plate X.

then plot the section to a scale of say 3 chains to 1 in. horizontal and 30 ft. to 1 in. vertical.

Example of Level Book.

LEVEL BOOK OF FIG. 115A.

Back Sight.	Intermediate Sight.	Fore Sight.	Height of Instrument.	Reduced Level.	Remarks.
3.84			88.75	84.91	Edge pond.
	4.60			84.15	1 ch. 14 ft.
4.71		3.98	89.48	84.77	3 ch.
	4.15			85.33	4 ch.
	4.50			84.98	5 ch. 47 ft., footpath.
5.66		6.70	88.44	82.78	8 ch.
	5.80			82.64	9 ch.
	5.50			82.94	10 ch.
	5.10			83.34	11 ch.
	4.00			84.44	12 ch.
7.24		2.90	92.78	85.54	14 ch.
	6.50			86.28	15 ch.
	5.70			87.08	17 ch.
	4.40			88.38	17 ch. 48 ft., footpath
7.44		3.24	96.98	89.54	19 ch.
	6.50			90.48	20 ch.
	3.20			93.78	21 ch.
0.25		3.11	94.12	93.87	
	2.20			91.92	22 ch.
	3.75			90.37	22 ch. 43 ft., railway fence.
	4.10			90.02	23 ch.
	5.10			89.02	25 ch.
	4.50			89.62	25 ch. 30 ft., gate.
8.18		4.37	97.93	89.75	25 ch. 47 ft., centre road.
	7.70			90.23	26 ch.
	8.20			89.73	26 ch. 26 ft., fence.
	7.90			90.03	27 ch.
	5.00			92.93	28 ch.
	2.40			95.53	29 ch.
10.47		0.98	107.42	96.95	30 ch.
	7.00			100.42	31 ch.
12.98		0.70	119.70	106.72	32 ch.
	8.50			111.20	32 ch. 48 ft.
	4.00			115.70	32 ch. 53 ft., fence.
	9.20			110.50	32 ch. 53 ft., ditch.
	9.20			110.50	32 ch. 60 ft., ditch.
10.37		0.29	129.78	119.41	
	5.50			124.28	33 ch. 32 ft., fence.
	4.20			125.58	33 ch. 49 ft., fence.
12.97		2.07	140.68	127.71	
	9.50			131.18	34 ch.

Example of Level Book—*continued.*

Back Sight.	Intermediate Sight.	Fore Sight.	Height of Instrument.	Reduced Level.	Remarks.
13.68		0.76	153.60	139.92	
	5.20			148.40	35 ch.
12.96		0.50	166.06	153.10	
	11.00			155.06	35 ch. 33 ft.
	5.30			160.76	35 ch. 51 ft.
	2.05			164.01	36 ch.
12.88		0.90	178.04	165.16	
	5.40			172.64	36 ch. 40 ft.
	2.06			175.98	37 ch., centre of manhole.
		4.42		173.62	Invert of pipe.
123.63		34.92			
34.92					
88.71 rise.					
84.91 edge pond.					
173.62 invert of pipe.					

Cross Sections.—The object of cross sections is to get an idea of the ground on each side of the line of the longitudinal section. The cross sections should be taken only at those points where there is a change in the surface contour of the ground. The surface between two adjacent cross sections will thus approximately coincide with planes joining these cross sections. Cross sections should always be taken at right angles to the line of the longitudinal section. They may usually be squared off by the eye with sufficient accuracy, or if more accuracy is required, by the optical square, box sextant, or theodolite.

A level is taken at each point in each cross section where the slope of the ground changes, and the distance of each of these points from the centre line or line of longitudinal section is measured.

Thus in Fig. 116, at the points C, D, E, F, G cross sections are taken, the line of each cross section being at right angles to AB. The distances AC, CD, DE, EF, FG, GB are to be measured. Then in the first cross section the slope of the ground changes at *a*, *b*, *c* on the right, and at *d*, *e* on the left. The distances C*a*, C*b*,

c*c*, c*d*, and c*e* are therefore measured, and levels are taken at these points. These levels being plotted, will be as represented in Fig. 117.

In taking cross sections, a convenient method is to first of all insert at each of the points *a*, *b*, *c*, *d*, *e*, &c., a small stick, at the same time measuring the distances c*a*, c*b*, c*c*, &c., from the centre line AB. Insert in a cleft in each stick a paper marked C.S. No. 1, 15 ft. R, C.S. No. 1, 20 ft. R, and so on, meaning that the point at which the paper is, is on C.S. No. 1 and 15 ft. to right of the centre line, or on Cross Section No. 1 and 20 ft. to right of centre line AB, and so on. The staff-holder, when he comes to hold the staff at each point, pulls out the paper and calls out to the leveller what is written on it, afterwards destroying the paper. By adopting this method, all the highest points of all the cross sections may be taken at one set up of the level, and the next lower set of points at the next set up of the level. This is a saving of labour, as all the levels may then be taken with two or three settings of the level. On the other hand, if each cross section is taken separately, each might require three or four settings of the level on steep ground. The most usual method of working is, however, to have an assistant who remains at the centre line and calls out the taped distances to where the staff is held, the staff-holder taking the ring of the tape with him.

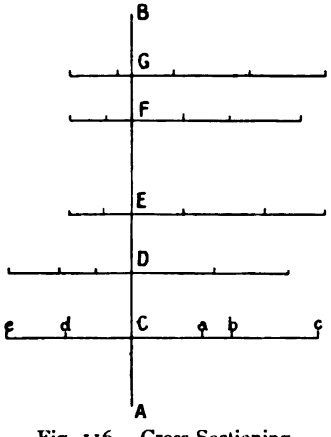


Fig. 116.—Cross Sectioning.

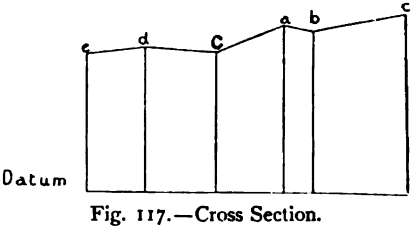


Fig. 117.—Cross Section.

The length of each cross section, *i.e.*, c*c* or c*e*, Fig. 116, will depend on the object of the survey. For a road or a railway

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the cross sections must extend as far as the top or bottom of the slope of the cutting or embankment and for a few feet beyond. When the longitudinal section has been taken and the gradients fixed, the depths of cutting or height of bank at each point on the centre line are known, and the distance to which the slope will run out may be easily calculated mentally. Otherwise the distance to which the cross sections must extend must be estimated as nearly as the circumstances of the case permit, a certain amount of margin being allowed by taking them rather farther than necessary. This also provides for alterations, such as flattening the slopes, &c.

The levels for cross sections are booked in the same way as already described for the levels of a longitudinal section, each level being described in the remarks column as C.S. No. 3, 35 ft. R, or L, and so on, as the case may be.

Checking on to Bench Marks or "Checking Back."—All levels of any importance should start from a bench mark whose level is accurately known, and should be carried on to another similar bench mark at the conclusion, or failing a convenient bench mark at the end of the line of levels, it is necessary to "check back" to the original starting point. This affords a complete check on the accuracy of the levels, with the exception of the intermediate sights. Cases may occur in which a serious mistake has been made in the levels and again rectified by another mistake of equal amount but contrary direction. The levels will then check in on the bench marks and still be wrong. Such cases, however, very rarely occur.

In checking on to a bench mark after taking all the levels required for the work in hand, of course no intermediate sights are taken, only back and fore sights along the shortest and most convenient route to the nearest bench mark or to the starting point. The levels should check in to .01, .02, or .03 ft.

Hand Level.—This instrument, as made by Stanley, is shown in Fig. 118. It is simply a tube with a mirror inside which reflects the bubble of the spirit level. Upon looking through the tube the bubble is seen reflected in the mirror, and by moving the end of the tube up or down, the reflection of the centre of the bubble may be made to coincide with the line of sight *ab*, indicated by a scratch on the mirror and a cross hair in the tube, which line is

then horizontal if the spirit level is parallel to the line of sight, as it should be if the instrument is in adjustment. The hand level is very useful for taking short cross sections, and is much used by American engineers on railway work when the height of bank or

Fig. 118.—Hand Level.

depth of cutting does not exceed 5 or 10 ft., and the cross sections are therefore short. For the method of cross sectioning with the hand level, see pages 414, 415, 416, Chapter XI.

Clinometer. — One of Stanley's clinometers is shown in Fig. 119, Fig. 120 shows one of Cooke's, and Fig. 121 one of Troughton & Simms'. These instruments are in principle the same as the hand level, the difference being that the clinometer may be used on an inclined line, and the spirit level then made level by turning a milled-headed screw actuating a pinion. This moves a vernier, and the angle of inclination is then read off the graduated arc. To use the clinometer we must sight on to a point at the same height above the ground as the eye. Thus in Fig. 122, the clinometer being held to the eye at *a*, to get the angle of

Fig. 119.—Clinometer.

the slope *cd* we must sight on to a point *b*, such that $bd = ac$. The line of collimation of the clinometer is then parallel to *cd*, the slope of the ground, and the angle as read on the graduated arc is the angle of the slope *cd*. A very good way to use the clinometer is

to cut a notch in a ranging rod or pole at a convenient height for the eye. Hold the clinometer in this notch and sight on to another rod having a mark at the same height or cut off to that height.

The use of the clinometer is to get the slope of the ground, and thus save the labour of taking levels for cross sections.

Cross Sectioning with the Clinometer.—For many purposes, such as railway cross sections, the clinometer is most use-

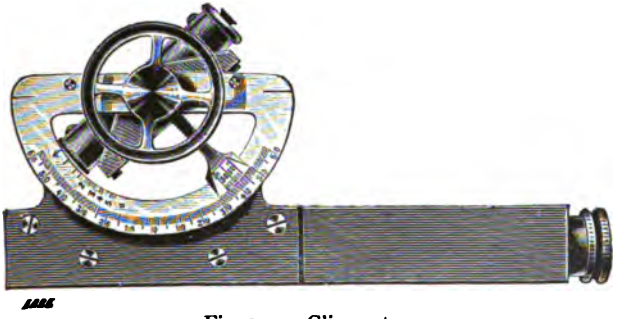

Fig. 120.—Clinometer.

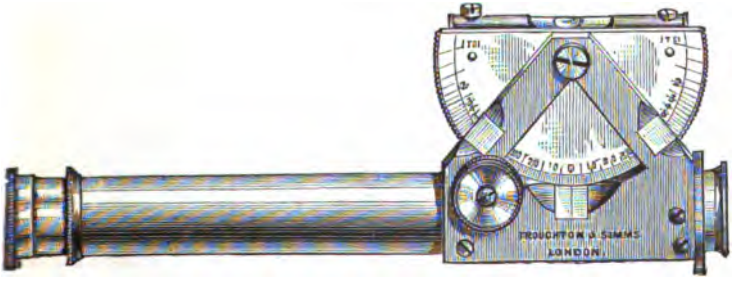

Fig. 121.—Clinometer.

ful. Thus in Fig. 122 the angle of slope *fcd* being known, the slope *cd* may be plotted, and if the distance *cd* = *ab* along the slope is also known the point *d* may be plotted. Similarly the angle *kde* being also measured, and the distance *de* or *bg*, the point *e* is fixed. In this manner all the points of a cross section may be fixed by the clinometer, and of course much more rapidly than by the level. The results of the clinometer are not so accurate as levels, but for

ordinary purposes, such as the cross sections for locating a road or railway, the clinometer is quite near enough.

Cross sections for contours for locating purposes in railway surveys are almost invariably taken with the clinometer. For final earthwork quantities, however, the level should be used.

Cross Sectioning with the Theodolite.—The most rapid method of taking long cross sections on steep hillsides is with the theodolite. The instrument is set up over the centre line peg and the level of the axis found by measuring from the peg, whose level is known from the longitudinal section levels. When taking the longitudinal section, levels should be taken *on* the pegs for the purpose at those places where it is proposed to

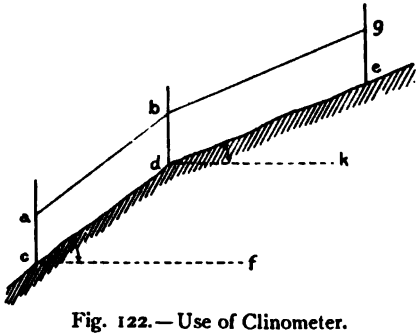

Fig. 122.—Use of Clinometer.

use the theodolite for cross sectioning. The theodolite being set up and levelled, the collimation line is set approximately parallel to the general slope of the ground, and the vertical arc clamped at that angle. First the height of the axis of the theodolite above the peg and the vertical angle are booked. The staff-holder then holds the staff at the points where the slope of the ground changes, at the same time holding the ring end of a tape at the point where the collimation line cuts the staff. There ought to be an assistant at the instrument to take the tape distances to the axis. The staff readings and distances of each point are thus read and booked. To plot the cross section all that is necessary is to plot the height of the axis of the instrument, then the collimation line passing through this point at the proper angle; the distances are then measured along the collimation line, verticals at each

point measured down from the collimation line and equal to the various staff readings, then give the points on the surface where the staff was held. Upon joining these points the desired cross section is obtained. With stadia hairs in the telescope the distances along the collimation line may be readily found on the principles described in Chapter VI., without measuring with the tape. For very long cross sections two tapes joined together are required, and on a windy day this is awkward. With the stadia hairs the process is rapid and quite accurate. By having sliding sights at right angles to the staff it may be held at right angles to the collimation line, and in that case the distances along the collimation line may be calculated mentally. In plotting the staff readings are of course then plotted *perpendicular to the collimation line.* The calculation is troublesome if the staff is held vertical. This method is also useful for cross sections of cuttings on steep hillsides with some rock in the bottom and long earth slopes up the hill. For these it may be necessary to set up the theodolite nearer to the side of the cutting than the centre line, or further out, in order to get the collimation line approximately parallel to the general slope and not too high on the staff. In this case the distance of the instrument from the centre peg is also to be measured and booked and plotted accordingly. The height of the axis of the instrument may be found by levelling with a spirit level a common straight-edge, placed on the centre peg, and measuring from the straight-edge to the axis of the theodolite. The index error of the vertical arc should be ascertained by testing the collimation line on two level pegs 5 or 6 chains apart, and the error is to be added to or deducted from each vertical angle as the case may be.

Contours.—In order to fix the position of any point in space we must have its three co-ordinates—that is, its two horizontal co-ordinates and one vertical co-ordinate. The horizontal co-ordinates fix the position of the point in plan and are determined by surveying, while the vertical co-ordinate fixes the position of the point in elevation and is determined by levelling. The plan ought to show the position of each point both horizontally and vertically, and this is effected by drawing *contours* on the plan.

Suppose a hill to be cut by a number of equidistant horizontal planes, and suppose we draw the intersections of these

planes with the surface of the hill, as in Fig. 123. Then the lines representing the intersections of the planes with the surface of the hill are "contour lines." They are the lines which would be formed by water surrounding the hill and rising a certain height at a time until it reached the top. The edge of the water at each successive rise would form a contour line. It is evident that the steeper the slope the nearer the contour lines will be on the map, while the flatter the slope the further apart the contour lines will be. A right cone would thus be shown by a

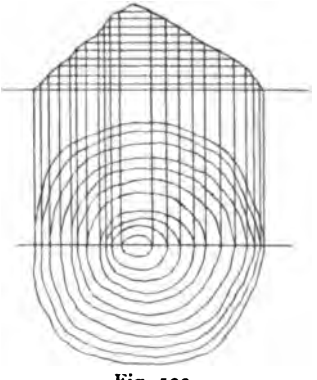

Fig. 123.
Contours of a Hill.

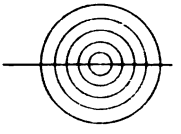

Fig. 124.
Right Cone Contours.

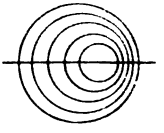

Fig. 125.
Oblique Cone Contours.

series of concentric circles, as in Fig. 124; an oblique cone, as in Fig. 125.

Vertical Distance between Contour Lines.—This is fixed by the object of the survey, the population and importance of the country, the degree of irregularity of the surface, and the scale of the map. In mountainous districts the contours may be 100 ft. apart vertically. For engineering purposes, such as the general plan of a railway, the contours may be about 5 ft. apart vertically. A good rule is to make the vertical distance between the contours equal to the denominator of the ratio of the scale of the map divided by 600. Thus on the 25 in. Ordnance scale or $\frac{1}{2500}$, the vertical distance between the contours would be $2500 \div 600$, or about 4 ft., which would be about right for locating

a road or railway. On very steep slopes the contours must be further apart vertically, otherwise the contour lines will come too close to each other.

Determination of Contour Lines.—There are two general methods. The first method is to determine the contours on the ground at once; the second method is to determine the highest and lowest contours only on the ground, and interpolate for the intermediate ones. In the first method, locate each contour on the ground with the level, driving in pegs where the line of the contour bends, *i.e.*, at the salient points. The contour lines thus laid out on the ground are then surveyed by any of the ordinary methods, traversing being perhaps best adapted for this purpose. On a long narrow piece of ground, such as that required for locating a road or a railway, take a section across it at every $\frac{1}{2}$ or $\frac{1}{4}$ mile, as at *ab*, *cd*, Fig. 126. These sections should be about in the line of steepest slope. Set pegs on these sections at the levels of the desired contour lines. Then the contours are located and marked by pegs by levelling between the corresponding points on *ab* and *cd*. Thus contour 10 is located by levelling from 10 on *ab* to 10 on *cd*, the levels being checked in between these points. Having thus levelled in pegs at all the points marked *x* in Fig. 126, these points are then surveyed and plotted on the plan.

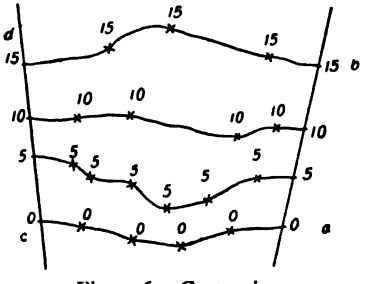

Fig. 126.—Contouring.

Second Method.—This method consists in determining the levels and positions of the principal points where the surface changes its slope either in amount or direction. Intermediate contours are then obtained by proportion or interpolation between these points.

The most usual way of effecting this is from cross sections, as described in Chapter V. (see page 226).

Interpolation of Contours.—The levels and positions of

the principal points being fixed, points of any intermediate level corresponding to any required contour are found by proportion. Thus in Fig. 127 the position of the points *a*, *b*, *c*, *d*, *e*, *f*, *g* being as shown, suppose the position of the 45 contour between *e* and *a* is required, the distance *ae* being 100 ft. The total fall between *a* and *e* being 15 ft., the distance in which the fall is 5 ft. will be $\frac{5}{15} \times 100 = 33\frac{1}{3}$ ft., or the 45 contour will be $33\frac{1}{3}$ ft. from *a* towards *e*. Similarly the positions of any number of contours may be found by interpolation when a certain number of points of known level have been plotted on the plan.

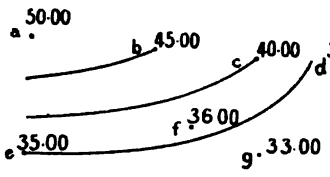

Fig. 127.—Interpolating Contours.

Graphic Method of Interpolating in Contours.—The following method, given by Mr Neil Kennedy, M. Inst. C.E.,* is the simplest and quickest for interpolating in contours. Prepare a diagram similar to that shown in Fig. 128, and trace it on tracing paper or linen. Now suppose that *x* and *y* are two points whose levels have been determined, the level of *x* being 81.00 and the level of *y* 107.00, and it is required to interpolate in contours every 5 ft. between *x* and *y*. Consider the lowest dotted line on the diagram to represent the 80.00 contour, then the tracing paper is moved over the plan until the point *x* lies on the line of the diagram representing 81.00, while at the same time the point *y* coincides with the line on the diagram representing 107.00 as shown in Fig. 128; at the same time the diagram must also be so adjusted that the line joining *x* and *y* is parallel to the lines *a*, *a*, *a*, which are ruled across the diagram as a guide. If now we prick through the dotted lines of the diagram between *x* and *y* we get at once the positions of the 85.00, 90.00, 95.00, 100.00, 105.00 contours between *x* and *y*.

When the two points *x* and *y* are very near each other, the consecutive radial lines of the diagram may be made to represent contours 5 ft. apart in place of every fifth line of the diagram as in above example.

* Kennedy, Surveying with the Tacheometer.

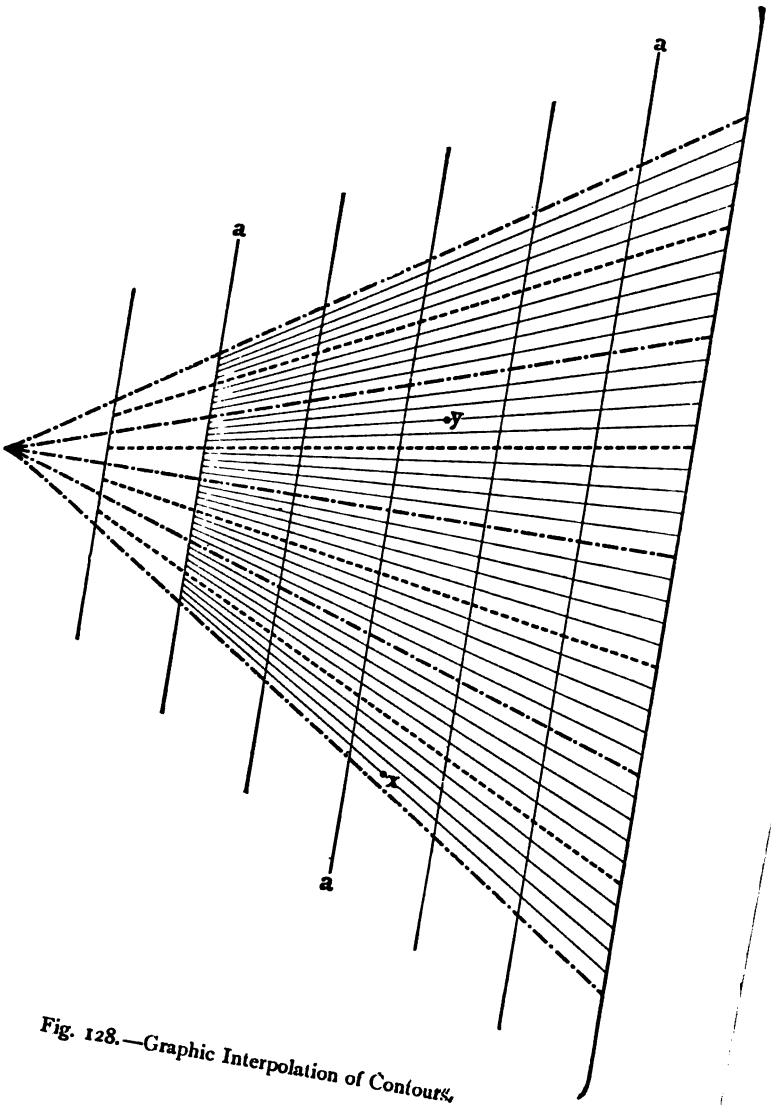

Fig. 128.—Graphic Interpolation of Contours.

Ridge and Valley Lines.—Ridge lines are lines which part water falling upon them, and from which it flows on opposite sides. A valley line is the opposite of a ridge line, and is indicated by the watercourse which runs in it.

Sketching and Inking in Contours.—A good guide is to observe that the contour lines are perpendicular to the ridge or valley lines. On a long slope or hill sketch first the top and bottom contours, and the middle one; then interpolate the others. Always remember that two contours can never run into one another except on a vertical face, and that if a contour runs entirely round a hill or hollow, it will come back to its commencing point.

In a contoured map, if the levels of the contours are not marked against them, there may be some doubt as to which are the highest and lowest and which are the ridges and which the valleys.

Watercourses indicate the slopes. Hatchings on the under sides of the contour lines as if water were flowing off will also remove the ambiguity. A good way to ink in contours is to use burnt sienna, making every fifth contour line thicker and darker than the others. The contours are, however, often inked in with Indian ink the same as the rest of the map.

Calculation of Contents from Contour Lines.—The cubic contents of a hill to be excavated may be conveniently calculated from contours, the contents of each part between successive contours being approximately equal to the average of the areas of the upper and lower surfaces multiplied by the vertical distance apart of the contours. This method is also applicable to finding the contents of a hollow to be filled in, a reservoir, &c.

Delineation of Ground by Hatchings.—This system is quick and effective but not accurate. The hatchings may be guided by contour lines lightly drawn in. The hatchings should be drawn exactly perpendicular to the contour lines. When the contours are very far from each other, draw in intermediate contours. Hatchings in adjacent lines should "break joint," so

as to show the position of the contour lines, which are lightly drawn in to guide the hatchings and then erased.

Delineation of Ground by Shades from Light Falling Vertically.—Consider the two slopes *ac* and *ad*, Fig. 129, then we have—

$$\frac{ad}{ac} = \frac{\cos a_1}{\cos a_2}$$

Now as the longer the slope is the less lighted it will be, since from Fig. 129 it is seen that the same amount of vertical light falls on each slope, and therefore the amount of light is inversely proportional to the length of the slope, or which is the same thing, directly proportional to the cosine of the angle of the slope. Tables may therefore be prepared giving the relative amounts of light for different slopes. In practice the difference of shade is usually exaggerated. The different shades may be put on with Indian ink or sepia, the tints being made light for flat slopes and heavy for steep slopes, a slope of 60° being quite dark, and one of 30° a shade about half-way between that and white, and so on.

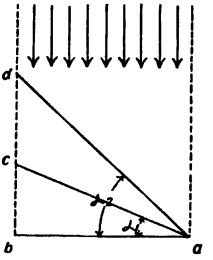

Fig. 129.
Shading Slopes.

The shading may also be done by contour lines—that is, by interpolating in contour lines, and making them more numerous on steep slopes and less numerous on flat slopes. This, however, is apt to confuse the map.

The most usual method of shading is to make the thickness and distance between the hatchings proportional to the angle of the slope. The required degree of shade may be made either by varying the thickness of the hatching lines or their distance apart, or both.

French Method.—In this method the degree of slope is shown by varying the spaces between the centres of the hatchings. The rule is—"The distance between the centres of the hatchings equals $\frac{2}{100}$ in. plus $\frac{1}{4}$ of the denominator of the fraction denoting the slope (*i.e.*, the tangent of the angle of the

slope) in $\frac{1}{100}$ in." The lines are made fine for flat slopes and thicker for steeper slopes. Only slopes from $\frac{1}{1}$ to $\frac{1}{64}$ inclusive are shown by this method.

German Method or Lehmann's Method.—In this method there are nine shades for slopes from 0° to 45°, the first is white and the last black. For intermediate slopes the ratio of white to black is as follows:—

$$\frac{\text{white}}{\text{black}} = \frac{45° - \text{angle of slope}}{\text{angle of slope}}$$

For example, for a slope of 30°—

$$\frac{\text{white}}{\text{black}} = \frac{45° - 30°}{30°} = \frac{15°}{30°} = \frac{1}{2}$$

Slopes steeper than 45° are shown by short thick lines parallel to the contour lines. The angle of slope may always be found from a contoured map, since the tangent of its angle is equal to the vertical height between contours divided by the horizontal distance between them.

Delineation of Ground by Shades from Light Falling Obliquely.—The light is supposed to come from the upper left hand corner of the map, and those slopes facing the light have a light shade, and those on the opposite side a dark shade. This method is effective but not accurate.

By drawing a map with contour lines, however, and shading it from oblique light, it may be made both effective and accurate.

Correction of Levels for Curvature.—The line given by the level is tangent to the surface of the earth, as AC, Fig. 130, where AE represents the earth's surface and O its centre.

Thus we have—

$$AC^2 = CE\ (CE + 2EO).$$

As EC is very small compared with the earth's diameter 2EO, we may put $AC^2 = EC \times 2EO$, and we get—

$$\frac{AC^2}{2EO} = EC$$

AE being the true level line, EC is the correction for curvature, which is therefore equal to the square of the distance divided by the earth's diameter. Taking the earth's diameter as 7,916 miles, the correction for 1 mile would be—

$$\frac{1}{7916} \text{ mile or } \frac{1 \times 5280 \times 12}{7916} = 8 \text{ in.}$$

The correction for 10 chains is $\frac{1}{8}$ in. The correction is proportional to the square of the distance and the effect of curvature is to make distant points appear *lower* than they really are. The effect of curvature and also of refraction is eliminated by placing the instrument midway between the two points the difference of whose levels is required.

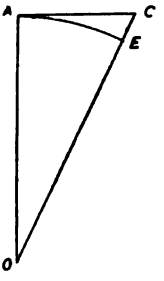

Fig. 130. Correction for Curvature.

Refraction.—Rays of light coming through the atmosphere are *refracted* or curved downwards. The effect is to make objects appear *higher* than they really are. The error due to refraction is thus in a contrary direction to that due to curvature, and its amount is on an average about $\frac{1}{7}$ of the error due to curvature. It, however, varies with the state of the atmosphere, and is somewhat uncertain. See also pages 185, 370.

Other Instruments: Plumb Line Levels.—The A level is shown in Fig. 131. It is so made that when the plumb line

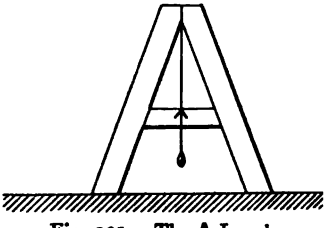
Fig. 131.—The A Level.

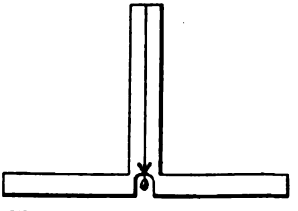
Fig. 132.—Plumb Line Level.

is adjusted to the mark on the cross-piece, the feet of the level are at the same height, and a line joining them is horizontal.

Another form of the plumb-line level is shown in Fig. 132. In this the cross-piece is at right angles to the plumb line, and is therefore horizontal when the plumb line coincides with its mark. These forms are not convenient for producing a line. For this purpose the last form is inverted. By sighting along the cross-piece we get a level line when the vertical piece is plumbed with the plumb line.

Reflecting Levels.—These instruments are made on the principle that a ray of light which strikes a reflecting plane at right angles is reflected back in the same direction. When the eye is reflected in a plain mirror the line joining the eye and its

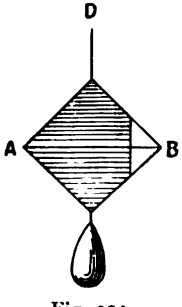

Fig. 133.
Reflecting Level.

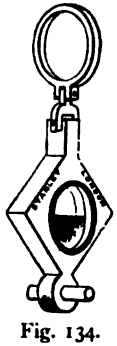

Fig. 134.
Reflecting Level.

image is perpendicular to the mirror, and if the mirror is vertical this line is horizontal, and may therefore be used for finding points at the same level as the eye. The first form (Fig. 133) is a rhomb of lead about 2 in. in the side and 1 in. thick. On one side, the shaded part of Fig. 133, is a mirror. The right hand part of the rhomb is cut off as shown in the figure, and a wire AB is stretched across it. To use the instrument, hold it up by the string D, with the mirror opposite the eye, so that the eye is seen bisected in the mirror by the wire AB. Then look through the opening at B and any point in line with the eye and wire will be on the same level with them. The instrument is made to hang vertical by means of the weight shown. Fig. 134 shows this instrument as made by Stanley. The second form is a hollow

brass cylinder with an opening at the top as in Fig. 135. Inside is a small mirror, and this mirror is made to hang vertical by means of a weight within the cylinder. The instrument is used similarly to the first already described, the lower edge of the opening serving the same purpose as the wire in the first instrument. The third form is a small steel cylinder (Fig. 136), about 5 in. long and ½ in. diameter, highly polished, and hung from the centre of one end by a fine thread. To use it, it is

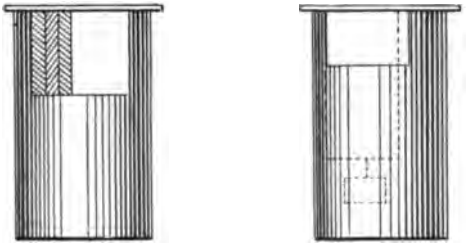

Fig. 135.—Reflecting Level.

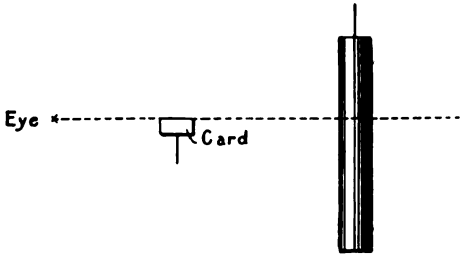

Fig. 136.—Reflecting Level.

held by the thread with one hand, and with the other hand a card is held between the eye and the instrument, the upper edge of the card as seen reflected in the cylinder being used in the same way as the wire in the first form of the instrument.

Boning Rods.—These are used by foremen for working to levels given to them by the engineers on pegs. For instance, in Fig. 137, suppose the pegs at A and B are driven in to a certain

level. Then by placing a boning rod on each of these pegs and sighting forward to a third boning rod held at C, when the boning rods are truly in line the foot of the third boning rod will be at the

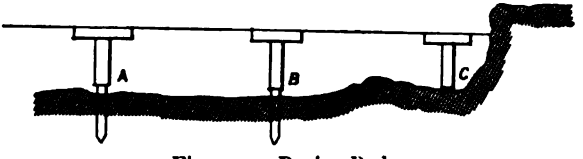

Fig. 137.—Boning Rods.

same level as the pegs at A and B, or if the pegs at A and B are driven in to a certain gradient, the foot of the boning rod at C will be on this gradient. The boning rods must of course be all exactly the same height.

Practical Hints, Obstacles and Difficulties.—Always if possible set up the level midway between the back-sight and fore-sight points. Any error of adjustment of the instrument is thereby eliminated, as also errors due to curvature and refraction. For instance, if the instrument is out of adjustment, and the reading on the back sight is 0.10 too small, the reading on the fore sight will also be 0.10 too small, if the level be midway. The *difference* of the readings will, however, be correct, as they are both in error the same amount; the correct difference of level is therefore determined.

Change Points.—In soft ground, where there are no solid stones to hold the staff on for the change points, as for instance in pasture land, &c., it is a good plan to make the staff-holder carry a chaining arrow which he drives into the ground up to the head. The staff is then held on the chaining arrow as in Fig. 138.

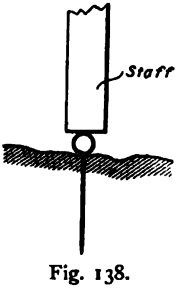

Fig. 138.
Change Point.

Figs. 139 and 140 show foot plates specially made for the same purpose. These are driven into the ground and the staff is held on the knob.

Steep Slopes.—In ascending or descending a hill the instrument will be set up so that the collimation line strikes as near the top of the staff as possible in the back sights and near the bottom in the fore sights. The

sights will thus be of unequal length, and if the instrument is out of adjustment errors will be introduced. To avoid this, set up *on one side of the line* along which the levels are being taken. When the slope is very steep, the greatest distance up the hill at which the collimation line will strike the staff may be so little that the staff cannot be read. In that case also the level must be set up to one side of the line along which the levels are being taken.

Levelling Across a Hill.—It will save time to set up the level on one side and sight over as shown in Fig. 141, in place of

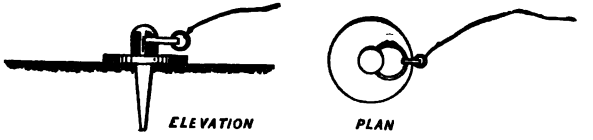

Fig. 139.—Foot Plate.

Fig. 140.
Foot Plate.

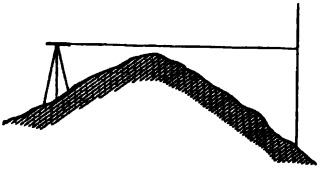

Fig. 141.—Levelling over a Hill.

setting up on the top of the hill; this also applies to levelling across a hollow.

When the Staff is too low or too high.—In levelling over a line of pegs already driven in, when the staff is too low, direct the chainman to raise it until it can be read, and then measure from the bottom of the staff to the top of the peg and add the measurement to the staff reading. If the staff is a little too high, measure from the top of the peg to where the line of sight cuts it, and book the measurement with a contrary sign, *i.e.*, if a back sight minus and if a fore sight plus.

When the Staff is too near to read the Divisions.—When no figure is visible, direct the chainman to raise the staff slowly until a figure comes in sight, and then lower it again. If

the staff is too near to be focussed, have a field book placed against it, and raise or lower the field book until its edge is in the collimation line. The staff may then be read where the edge of the field book cuts it.

The methods in the two last paragraphs are generally not sufficiently accurate for back and fore sights, but they may usually be used for intermediate sights.

Board Fence.—Run a knife through one of the joints, and keeping the knife horizontal, hold the staff on it, and take a fore sight; then, still keeping the knife in the same position, move the level to the other side of the fence, and again holding the staff on the knife, take a back sight to it.

Wall.—Take a fore sight to the wall, and mark where the collimation line cuts it. Measure from this point up to the top of the wall, and book this as a fore sight with a *plus* sign. Shift the level to the other side of the wall, and take a back sight to it, and mark where the collimation line cuts it. Measure from this point up to the top of the wall, and book this as a back sight with a *minus* sign. The same object will be effected by holding the staff upside down, its foot being held flush with the top of the wall, and the back and fore sights being booked with opposite signs as above described.

House.—First try to find some place to set up the instrument from which you can see through the house by opening doors and windows. Failing this, try to find some place inside the house where the staff may be held so that it may be seen from both sides of the house by opening doors and windows.

Sun.—The sun often causes trouble by shining on the object glass. If the instrument has a shade for the object glass, pull it out. If not, shade the object glass with your hat or hand, or place the instrument on one side of the line so that the sun will not shine on the object glass while sighting.

Personal Error.—Different persons do not see things exactly alike. Each person may have an inaccuracy peculiar to himself. One may take an observation different from another equal in skill. Also one's right and left eye may differ. This difference in persons is called their "personal equation." To try the accuracy of the eye, turn the head sideways, so that the

eyes are in the same vertical line, and sight to the staff held horizontally. Read where the vertical hair strikes the staff. Then turn the head the other way so as to reverse the position of the eyes, and sight again with the other eye. The average of the two readings is correct.

To Locate a Given Level.—See page 253, Chapter V.

Reciprocal Levelling.—When it is impossible to set up midway between two points, this is useful. In Fig. 142, set up the instrument at A, and sight on the staff held on B. The difference between the height of the instrument above the peg at A and the staff reading is *a* difference of level. Then set up the instrument

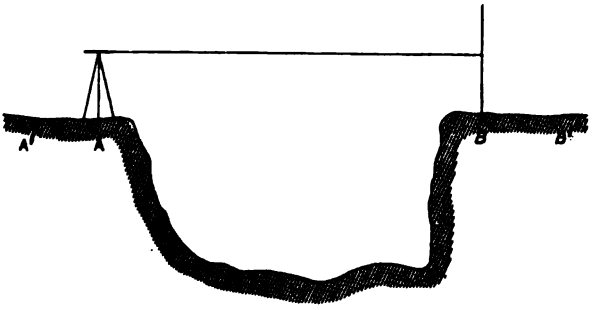

Fig. 142.—Reciprocal Levelling.

at B and sight on the staff held on A. This gives *another* difference of level. The mean of these two differences of level is the correct difference. Or otherwise set up at A′ and read the staff held on A and B. The difference of these readings gives *a* difference of level. Then set up at a point B′ so that the distance BB′ is equal to AA′, and again sight on B and A. The difference of these staff readings gives *another* difference of level. The mean of the two results is the correct difference of level. See also instructions for precise spirit levelling, levelling across a river, page 199.

Hypsometry or Levelling with the Barometer.—The difference in height of any two places may be determined by the barometer. As the height of the column of mercury balanced by the atmosphere decreases as we ascend above sea level, the reading

of the barometer is therefore an indication of the altitude, and by making the necessary corrections for temperature, gravity, &c., we may arrive at a fairly good approximation of relative differences of level.

The column of mercury in the barometer is supported by the weight of the column of air at the point of observation, and the mercury falls when the barometer is carried higher, and *vice versâ*. The weight of the air diminishes from the surface of the earth to the supposed surface of the atmosphere. It has been found by experiment that as the heights to which the barometer is taken increase in arithmetical progression, the weights of the column of air at the barometer, and therefore the readings, decrease in geometrical progression. Therefore the difference of the levels of any two not very distant points on the earth's surface is equal to the difference of the logarithms of the barometer readings at these two points multiplied by some constant coefficient.

By experiment this coefficient is found to be 60384.3 at freezing point, or 32° F., the readings of the mercury being in inches, and the difference in height being in feet.

Or approximate difference of height in feet

$$= 60384.3 \ (\log H - \log h)$$

where H = height of mercury in inches at lower station

h = " " " upper "

Correction for Temperature of Mercury.—If the temperature of the mercury in the barometer is different at the two stations, it will be expanded at the one station and contracted at the other. This will therefore affect the height of the column of mercury and the barometer readings. To reduce the readings to the same temperature multiply the difference of the temperatures of the mercury in degrees F. by $\frac{1}{10000}$. The reading of the barometer at the upper station is then to be multiplied by this product, and the result *added* to it if the mercury at the upper station is the colder, and *subtracted* if the mercury at the upper station is the warmer. This corrects *both* barometer readings for temperature of mercury.

If a brass scale is used to read the height of the mercury in the barometer, to allow for the expansion or contraction of the brass scale as well as the mercury multiply by $\frac{9}{100000}$ in place of $\frac{1}{10000}$ as above.

Correction for the Temperature of the Air.—A column of air of any height will weigh less when it is warm than when it is cold. The readings of the barometer are therefore affected by the temperature of the air. To correct for this, multiply the approximate height as already determined by $\frac{t+t'-64^\circ}{900}$, where t and t' are the temperatures of the air at the two stations. Add the result algebraically to the already determined approximate height. This correction is positive when the sum of the temperatures exceeds 64°, and *vice versâ*.

Correction for Latitude.—The latitude of the place affects the result to some extent. To correct for this, multiply the approximate height already determined by 0.00265 cos 2L where L = latitude, and algebraically add this result to the last found approximate height. This correction is negative from the pole to 45° where it is zero, and positive from 45° to the equator.

Correction for Height of Lower Station.

Let x = last corrected height.

s = height of lower station above sea level.

Then the correction is $\frac{x+52251}{20888629}+\frac{s}{10444315}$

This correction is always to be added to the last found approximate height.

Correction for Decrease of Gravity on a Vertical.—For this correction, see Table I., Part IV.

The last three corrections are only required for very great accuracy.

For ordinary engineering purposes it is usually sufficient to correct the barometer readings for the temperature of the mercury, and then make the correction for the temperature of the air.

The formula will then be—

(1) Difference of height of two stations in feet

$$= 60384.3\ (\log \text{H} - \log h)\left(1+\frac{t+t'-64^\circ}{900}\right)$$

where

H = corrected barometer reading at lower station in inches

h = „ „ „ upper „ „

t = temperature of air at lower station in degrees F.

t' = „ „ upper „ „

Mercurial Barometric Tables.—To facilitate the calculation when all the corrections are made, Table I., compiled by Mr Francis Galton, F.R.S., is given.* Part II. of Table III. may be used to correct barometer readings for temperature of intermediate air, as it applies to barometer readings as well as to boiling point thermometer readings. Thus all the calculations may be dispensed with by the use of these two tables.

Example of Use of Tables—

	Upper Station.	Lower Station by Sea.
Thermometer in open air	70.3°	77.5°
Thermometer in barometer	70.3°	77.5°
Barometer readings	23.66 in.	30.046 in.

The latitude being 21°.

Part I., Table I., gives for 30.046 in.	27649.7
" " " 23.66 "	21406.9
Difference	6242.8
Part II., Table I., gives for $77.5° - 70.3° = 7.2°$	− 16.9
Approximate difference of heights	6225.9
Part II., Table III., gives for $\frac{77.5° + 70.3°}{2} = 74°$ nearly	× 1.0933
Nearly correct difference of heights	6806.8
Part III., Table I., gives for above attitude and latitude 21°	+ 13.3
Part IV., Table I., gives for above altitude	+ 19.3
Part V., Table I., is not used in this case	0.0
Correct difference of heights	6839.4 ft.

French Barometers.—French barometers are divided in millimetres, and the thermometer in degrees Centigrade, the freezing point being zero and boiling point 100°.

Then difference of height in metres

$$= 18409 (\log H - \log h) \left(1 + \frac{2 (t + t')}{1000}\right)$$

* By permission of the Royal Geographical Society, from "Hints to Travellers."

Babinet's Simplified Formula.—Difference of height in metres

$$= 16064 \frac{H - h}{H + h} \left(1 + \frac{2\,(t + t')}{1000}\right)$$

where the barometer readings are in millimetres and the temperatures in Centigrade.

In English measure Babinet's formula becomes—

Difference of height in feet

$$= 52703 \left(\frac{H - h}{H + h}\right) \left(1 + \frac{t + t' - 64^\circ}{900}\right)$$

where the barometer readings are in inches and the temperatures in Fahrenheit.

Belville's short approximate rule will be useful for rough work. It is as follows:—

Altitude in feet

$$= 55000 \frac{\text{difference of barometer readings in inches}}{\text{sum of barometer readings in inches}}$$

$$\text{Or altitude in feet} = 55000 \frac{H - h}{H + h}$$

For engineering purposes, however, barometer observations are practically worthless without temperature corrections.

The best barometric tables are Professor Guyot's. The most concise are those in *Annuaire du Bureau des Longitudes.*

A difference of $\frac{1}{10}$ in. in the barometer reading corresponds to about 90 ft. difference of height near 32° F. and near sea level. At greater temperatures and greater elevations the difference of elevation corresponding to a given difference of barometer reading increases. Thus at 70° F. $\frac{1}{10}$ in. in barometer corresponds to a difference of height of 95 ft.

Heights by Aneroid Barometer.—When an aneroid barometer is used, as it is mechanically compensated for temperature, no correction of the barometer readings themselves is required.

The formula for heights by aneroid barometer is as follows:—

Let x = difference of height of two stations in feet.
„ H = reading in inches of barometer at lower station.
„ h = „ „ „ upper „
„ t and t' = temperatures (Fahr.) of the air at the two stations.

Then—

$$x = (\log H - \log h) \times 60384.3 \times \left(1 + \frac{t + t' - 64^\circ}{900}\right)$$

For greater accuracy corrections in Parts III., IV., V., Table I., must also be made.

Aneroid Barometric Tables.—To facilitate the use of the formula, Table II., taken from the Professional Papers of the Corps of Engineers, U.S.A., is given.

For example, let the barometer reading at lower station $H = 29.58$ in.

Let the barometer reading at upper station $h = 27.32$ in.

Temperature of air at lower station $t = 67^\circ$

" " upper " $t' = 62^\circ$

Then Table II., Part I., gives for 29.58 in. 28440.9

and for 27.32 in. 26356.6

Difference = 2084.3

Also $t + t' = 129^\circ$, and Table II., Part II., gives for 129°, .0722.

Therefore difference of height of two stations = 2084.3×1.0722

= 2234 ft.

Accuracy of Barometric Observations.—This depends on the number of observations and the skill of the observer. With great care they may be correct to a very few feet, as shown by the following results by Professor Guyot :—

Heights by Barometer.	Heights by Spirit Level.
6,707 ft.	6,711 ft.
2,752 "	2,752 "
6,291 "	6,285 and 6,293 ft.

To get good results *simultaneous* observations at the two stations should be made. If this is not possible the observations should differ in time as little as possible. Repeated observations should be made when possible.

Determination of Heights by Boiling Point Thermometer.—The temperature at which water boils differs with the pressure of the atmosphere, and decreases as we ascend. Thus the thermometer may be used in place of the barometer in measuring heights.

Boiling Point Thermometer Tables.—Table III., compiled by Mr Francis Galton, F.R.S., will facilitate the computation of heights by boiling point thermometer.*

For example from Table III.—

At station A, boiling point = 195°.1, tabular No. = 9,040
„ B, „ „ = 210°.3, „ = 887

Approximate difference of height = 8,153 ft.

To correct for temperature of air—

At station A, temperature = 65° F.
„ B, „ = 73° F.
Mean temperature ½ (65° + 73°) = 69° F.

From Table III., Part II., the multiplier for 69° is 1.082, and corrected difference of height = 8153 × 1.082 = 8,821 ft.

Instruments: Aneroid Barometer.—The most useful barometer for surveying purposes is the aneroid. A 5 in. aneroid, with a range of about 4,000 or 5,000 ft., is about the best size. Fig. 143 shows a 5 in. aneroid by Stanley. When reading the instrument, hold it horizontally, and tap it several times. As the aneroid is mechanically compensated for temperature, the correction for temperature of the instrument itself (correction for temperature of mercury) is not required. The correction for temperature of the air must, however, always be made, as also the other corrections when any great degree of accuracy is aimed at. As a rule, when several trips are made between two stations, it will be found that the mean of the "uphill" journeys is more correct than the mean of the "downhill" journeys. The reason of this is that the pressure of the atmosphere puts a strain on the spring inside the vacuum box of the aneroid, and in going "uphill" the pressure of the atmosphere decreases, and this strain

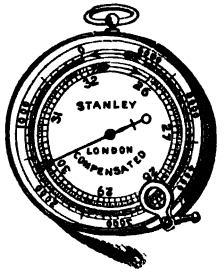

Fig. 143.
Aneroid Barometer.

* Given by permission of the Royal Geographical Society, from "Hints to Travellers."

is relieved. In going "downhill" the reverse is the case, and the instrument is apt to be slower in responding to the change of atmospheric pressure. The instrument should always be held in the *same position* when taking the readings, *i.e.*, either always horizontally or always vertically, as a mere change of position usually affects the readings.

An aneroid should be compared with a standard mercurial barometer, and have its index error ascertained. Most aneroids will not agree with a standard mercurial barometer at low pressures, even if correctly graduated in the first instance. The index error is fairly constant in good instruments, but will always be considerably increased when low pressures have been the rule for any length of time.

Aneroids should be treated with almost as much care as a chronometer, and should not be dangled about or shaken in the pockets. As the index error is apt to change, for absolute heights the results should be compared with the portable boiling point thermometer (Fig. 147).

Above 8,000 ft. elevation the aneroid is not reliable.

Mountain Barometers.—These may be either *cistern* or *syphon* barometers. The best cistern barometer is Fortin's, as improved by Guyot. This is shown in Fig. 144, and consists of a column of mercury in a glass tube, its lower end being in a cistern of mercury. The tube is covered by a brass case terminated at the top in a ring C for suspending the instrument. At the bottom is a flange G to which the cistern is attached. At D is a vernier from which the height of the mercury is read. The zero point is a small ivory point at H, the mercury in the cistern being raised or lowered by the screw K until its surface is in contact with this ivory point. At F is the attached thermometer which gives the temperature of the mercury.

The syphon barometer is shown in Fig. 145. In this form the cistern is dispensed with, and the tube is turned up at its lower end as shown. A small hole at S admits the air. The height of the mercurial column is equal to the difference of the heights of the mercury in the two branches of the tube. The whole is protected by a brass case, and is fitted with verniers, thermometers, &c., as in Fortin's barometer. The best syphon barometer is Gay-Lussac's, as improved by Bunten.

Fig. 144.
Cistern
Barometer.

Fig. 145.
Syphon
Barometer.

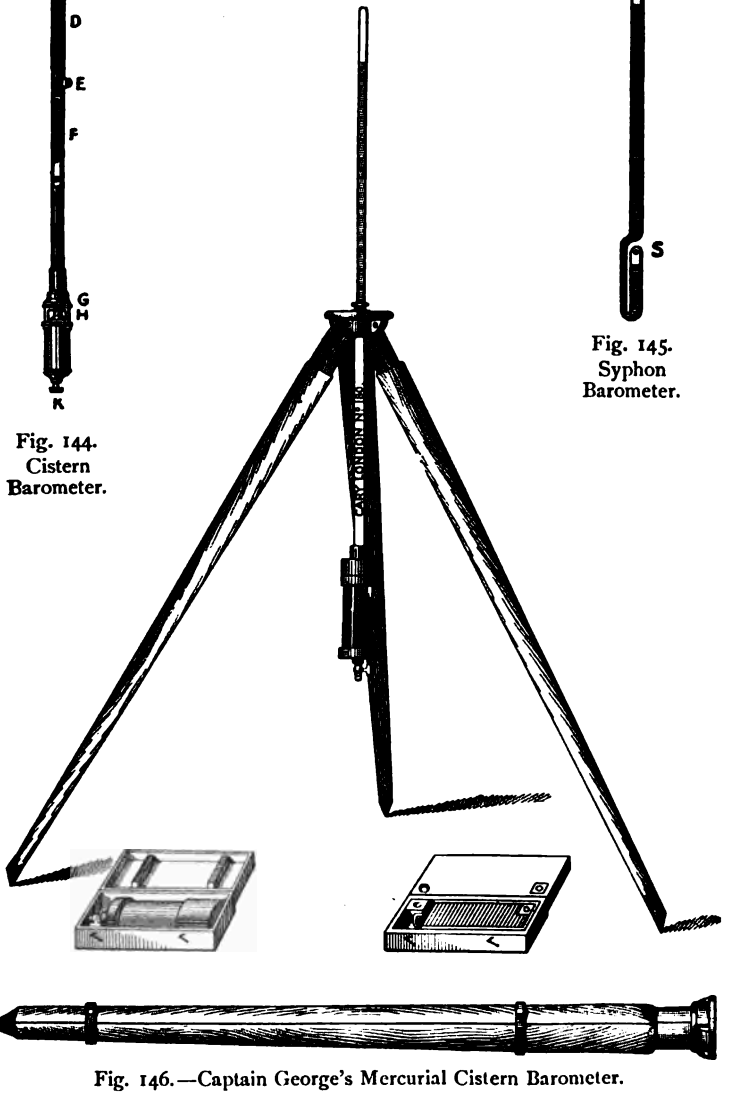

Fig. 146.—Captain George's Mercurial Cistern Barometer.

Fig. 146 shows Captain George's mercurial barometer (cistern barometer), as made by Cary, London.

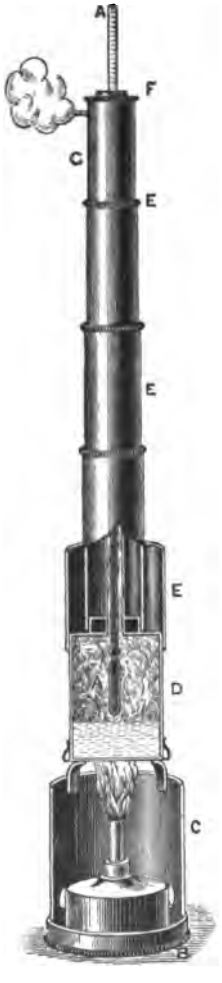

Fig. 147.—Boiling Point Thermometer.

Boiling Point Thermometer.—This instrument is shown in Fig. 147, and consists of a thermometer AA graduated from 180° to 215°, a spirit lamp B which fits into the bottom of a brass tube C that supports the boiler D, and a telescopic tube EEE, which fits tightly on to the top of the boiler. The thermometer is passed down the tube E from the top until within a short distance of the water, *which it should never touch*, and is supported in that position by an indiarubber washer F. The steam passes from the boiler up the tube E and escapes by the hole G. The whole of the apparatus fits into a cylindrical tin case 6 in. long and 2 in. diameter, with the exception of the thermometer, which is packed in a brass tube lined with indiarubber, and having a pad of cotton wool at each end.

In using the boiling point thermometer, wait until the mercury becomes stationary before reading, and at the same time take the temperature of the air in the shade with an ordinary thermometer. As the lamp is very often too small, in purchasing see that the lamp will hold a proper supply of spirit. A screen of some sort is usually required on the windward side of the thermometer, and at low temperatures is indispensable. The heat of the lamp will otherwise be carried off too rapidly for the water to boil properly.

Levelling by Angular Measurements or Trigonometrical Levelling.—The simplest case for short distances in which curvature and refraction are neglected is as

shown in Fig. 148. In this case, if the distance PC and the angle BPC are measured, we have—

$$BC = PC \tan BPC \quad \ldots\ldots\ldots\ldots (1)$$

As the best conditioned triangle has the angles at P and B each equal to 45°, the distance PC should be selected as nearly as possible equal to the height BC. In most cases it will happen that the distance PC cannot conveniently be directly measured as in Fig. 149. In this case measure the distance PD to any convenient point D in the same vertical plane as P and B. Also measure the angles of elevation CPB, CPD, and the angle PDB at D.

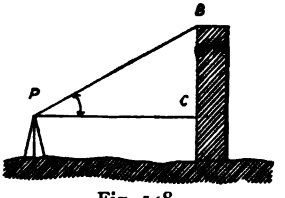

Fig. 148.
Trigonometrical Levelling.

Then in the triangle PBD, having given the side PD and the angles DPB, PDB, calculate the side PB, and we get—

$$PB = PD \frac{\sin BDP}{\sin PBD}$$

PBD is of course equal to 180° − (BPD + BDP). Then from PB and the angle BPC calculate BC, and we get—

$$BC = PB \sin BPC$$

and substituting the value of PB—

$$BC = PD \frac{\sin BDP \sin BPC}{\sin PBD} \quad \ldots\ldots\ldots\ldots (2)$$

If the height of B above D is required, we have—

$$DE = PD \sin CPD.$$

Then DE = FC and BF = BC − FC. Otherwise measure the angle of elevation FDB at D, and in the triangle PBD calculate BD, then we have—

$$BF = BD \sin BDF.$$

When a distance PD in the same vertical plane as P and B cannot be measured, run out a line PD in any convenient direction as shown in Fig. 150, and measure PD. Measure also the angle of elevation BPC and the horizontal angles CPD, CDP. Then in the triangle CPD we have—

$$PC = PD \frac{\sin CDP}{\sin PCD}$$

Also BC = PC tan BPC, and substituting for PC, we get—

$$BC = PD \frac{\sin CDP \tan BPC}{\sin PCD} \quad \ldots\ldots\ldots (3)$$

In the converse way, inaccessible distances may be found from known heights. Thus in Fig. 148, if the height BC is known, we have—

$$PC = BC \cot BPC.$$

Similarly if the inaccessible distance DP, Fig. 151, is required, at B measure the angles of depression of D and P, which are equal to the angles of elevation CDB, CPB. Also measure at B the horizontal angle DCP. From the known height BC and the angles of elevation at D and P calculate CD and CP.

Having then the two sides CD and CP, and the included angle DCP, the required distance DP may be found.

D and P are supposed to be in the same plane, and the height BC above this plane known.

In Figs. 149, 150, 151, the distances and vertical angles are shown as measured from one point on the ground to another. In practice the angles and distances are actually measured from the axis of the instrument as shown in Fig. 148. The height of the axis of the instrument above the ground is therefore to be allowed for.

Calculation when Curvature is allowed for. — For greater distances the effect of curvature is appreciable, and must be taken into account.

Thus in Fig. 152 the horizontal line given by the instrument is AC, the tangent to the earth's surface at A. The observed angle of elevation of the object B is BAC, and the calculated height will be BC. The true level line being really the arc AC', the true height of B is BC'.

For moderately great distances ACB may be taken as equal to 90°, and we get for CC'—

$$CC' \frac{d^2}{2R}$$

where d = distance AC', R = earth's radius AO.

If the earth's mean radius be taken as 20,888,629 ft., then—

$$CC' = 0.000000023936d^2$$

Then—

$$BC' = d \tan BAC + 0.000000023936d^2 \quad \ldots\ldots\ldots (4)$$

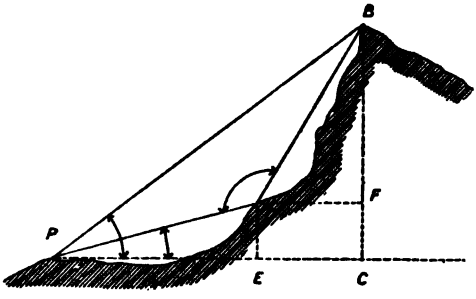

Fig. 149.—Trigonometrical Levelling.

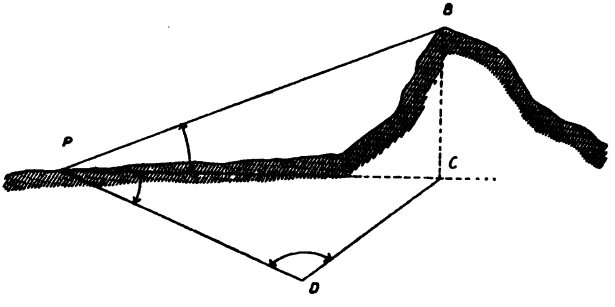

Fig. 150.—Trigonometrical Levelling.

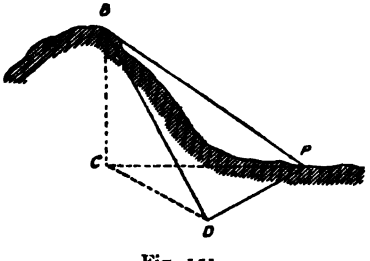

Fig. 151.
Trigonometrical Levelling.

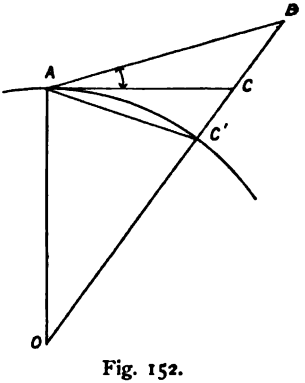

Fig. 152.
Trigonometrical Levelling.

The arc AC′ and the lines AC′ and AC are taken as approximately equal.

Another method is to correct the observed angle BAC by adding the angle CAC′, which is equal to half the central angle AOC′.

With the above value of the earth's radius we have then—

$$\text{Angle CAC}' \text{ in minutes} = 0.0000823d$$

Adding this to the observed angle BAC, and taking AC′B as 90°, we have—

$$\text{BC}' = d \tan (\text{BAC} + 0.0000823d) \quad \ldots\ldots\ldots \quad (5)$$

For very great distances the angle ACB cannot be considered equal to 90°, and in this case in Fig. 152 we have in the triangle ABC—

$$\text{BC} = d \frac{\sin \text{BAC}}{\sin \text{B}}$$

Also $\text{B} = 180° - (\text{O} + \text{BAO})$, where O is the angle AOB,

but $\text{BAO} = 90° + \text{BAC}$

therefore $\text{B} = 180° - (\text{O} + 90° + \text{BAC})$

or $\text{B} = 90° - (\text{O} + \text{BAC})$

therefore $\sin \text{B} = \cos (\text{O} + \text{BAC})$

Substituting this value of sin B in the last equation for BC, we get—

$$\text{BC} = d \frac{\sin \text{BAC}}{\cos (\text{O} + \text{BAC})}$$

Taking CC′ as before equal to $0.00000002\,3936d^2$, we get—

$$\text{BC}' = \text{BC} + \text{CC}' = d \frac{\sin \text{BAC}}{\cos (\text{O} + \text{BAC})} + 0.00000002\,3936d^2$$

Substituting for O its value in minutes $0.0001646d$, we get—

$$\text{BC}' = d \frac{\sin \text{BAC}}{\cos (0.0001646d + \text{BAC})} + 0.00000002\,3936d^2 \quad \ldots\ldots \quad (6)$$

For the other method, by correcting the angle BAC to BAC′, in the triangle BAC′ we have—

$$\text{BC}' = d \frac{\sin (\text{BAC} + \frac{1}{2}\text{O})}{\cos \text{BAC} + \text{O}}$$

Substituting the values of O, we have—

$$\text{BC}' = d \frac{\sin (\text{BAC} + 0.0000823d)}{\cos (\text{BAC} + 0.0001646d)} \quad \ldots\ldots\ldots \quad (7)$$

Calculation when both Curvature and Refraction are allowed for.—The effect of refraction is to increase the observed angle BAC. The amount will vary with the state of the

atmosphere, temperature, wind, &c., but on an average is usually taken as 0.16 of the curvature cc′, Fig. 152. We have therefore to deduct 0.16 cc′ from the calculated height, or we may deduct 0.16 of the angle CAC′ from the observed angle BAC.

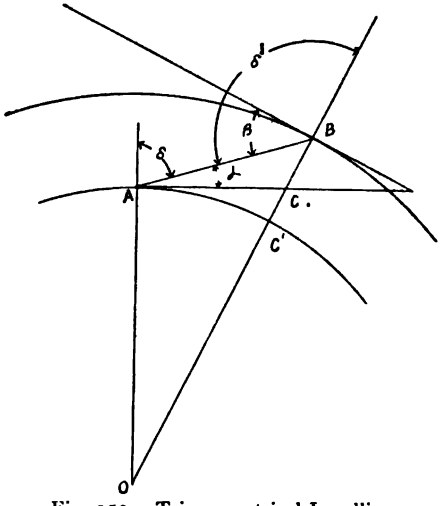

Fig. 153.—Trigonometrical Levelling.

In this case formula (4) becomes—

$$BC' = d \tan BAC + 0.00000002d^2 \quad \ldots\ldots\ldots \quad (8)$$

Formula (5) becomes—

$$BC' = d \tan (BAC + 0.000069d') \quad \ldots\ldots\ldots \quad (9)$$

Formula (6) becomes—

$$BC = d \frac{\sin (BAC - 0.00001316d')}{\cos (BAC + 0.00015143d')} + 0.000000023936d^2 \quad \ldots \quad (10)$$

Formula (7) becomes—

$$BC' = d \frac{\sin (BAC + 0.00006913d')}{\cos (BAC + 0.00015143d')} \quad \ldots\ldots\ldots \quad (11)$$

Reciprocal Observations for Cancelling Refraction.—The effects of refraction may be eliminated by taking reciprocal observations at each station as shown at A and B, Fig. 153, where the angle of elevation α is measured at A and the angle of depression β is measured at B.

In this case we have—

$$BC' = d\,\frac{\sin \frac{1}{2}(\alpha+\beta)}{\cos \frac{1}{2}(\alpha+\beta+o)} \quad \ldots\ldots\ldots\ldots (12)$$

where o is the angle AOB as before.

If the zenith distances δ and δ' are measured, we have—

$$BC' = d\,\frac{\sin \frac{1}{2}(\delta'-\delta)}{\cos \frac{1}{2}(\delta'-\delta+o)} \quad \ldots\ldots\ldots\ldots (13)$$

When o is very small compared with α and β or δ and δ' it may be neglected, and (12) and (13) become respectively—

$$BC' = d \tan \tfrac{1}{2}(\alpha+\beta) \quad \ldots\ldots\ldots\ldots (14)$$

$$BC' = d \tan \tfrac{1}{2}(\delta'-\delta) \quad \ldots\ldots\ldots\ldots (15)$$

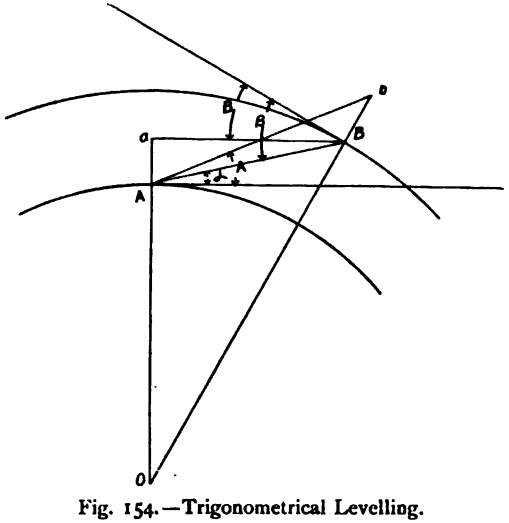

Fig. 154.—Trigonometrical Levelling.

Reduction of the observed Angles from the Summits of Signals.—When the stations cannot be seen from each other, signals are erected, as A*a* and B*b*, Fig. 154. In this case the observed angles are A and B, and these have to be reduced to α and β. Let the heights of the signals above the instrument be h and h', then—

$$\alpha = A - \left(\frac{h \cos A}{d \sin 1''}\right) \quad \ldots\ldots\ldots\ldots (16)$$

$$\beta = B + \left(\frac{h' \cos B}{d \sin 1''}\right) \quad \ldots\ldots\ldots\ldots (17)$$

In (16) and (17) the expressions in brackets are in seconds of arc.

If zenith distances are observed as A′ and B′, the corrected angles will be—

$$\alpha' = A' + \left(\frac{h \sin A'}{d \sin 1''}\right) \quad \ldots\ldots\ldots\ldots \quad (18)$$

$$\beta' = B' + \left(\frac{h' \sin B'}{d \sin 1''}\right) \quad \ldots\ldots\ldots\ldots \quad (19)$$

If the height of a conical spire, as AB, Fig. 155, cannot be measured, take the circumference at *a* and *b*, and deduce the radii

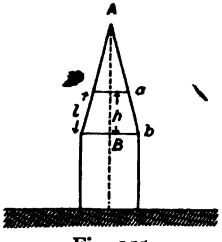

Fig. 155.
Height of Spire.

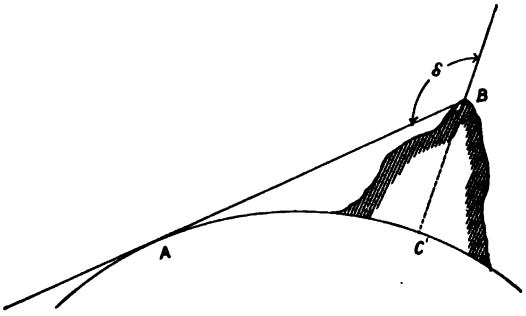

Fig. 156.—Heights by Observation to Sea Horizon.

r and R. If the vertical height *h* between *a* and *b* be measured, we have—

$$AB = \frac{Rh}{R - r}$$

If the slant height *l* be measured—

$$AB = \frac{R}{R - r}\sqrt{\{l + (R - r)\}\{l - (R - r)\}}$$

Measurement of Heights by Observation to the Sea Horizon.—If in Fig. 156 the zenith distance δ be measured, then expressing $\delta - 90°$ in seconds, and taking *m* as the coefficient of refraction (0.08 on an average), *r* as the earth's mean radius, we get—

$$BC' = \tfrac{1}{2}r\left(\frac{\sin 1''}{1 - m}\right)^2(\delta - 90°)^2\{1 + \tfrac{1}{4}\left(\frac{\sin 1''}{1 - m}\right)^2(\delta - 90°)^2\}$$

All observed angles in trigonometrical levelling are first to be

reduced to allow for height of axis of instrument above ground or station; or the height of instrument must otherwise be allowed for.

Precise Spirit Levelling.—Precise spirit levelling may be defined as the carrying of a very long line of levels, usually from the sea coast to inland points, for the accurate determination of the elevation of the principal bench marks in a large general survey of a country such as the Ordnance Survey of the United Kingdom, the United States Coast and Geodetic Survey, &c. Lines of precise levels are carried across the country, upwards of 1,000 miles in length, and from the bench marks established on these lines the levelling of the whole country is executed.* Lines of precise levels are also required for the execution of special engineering works, the determination of water slopes, canals, &c. In order to carry such a long line of levels with accuracy, special precautions and methods of working must be observed.

Instruments: Level.—The great accuracy effected in precise spirit levelling is as much owing to the methods employed and the precautions observed in making the observations as to the special instruments used.

When a *special level* is used, it is usually of a magnifying power of telescope of about 40 or 50 diameters, and with three levelling screws. A mirror is provided fixed above the bubble tube in which the bubble is seen by the observer, and its position is observed without the eye being moved from the eyepiece. There is a screw called the "elevating screw" under the support of the telescope, next to the eyepiece, by means of which the bubble is brought accurately to the centre of its run while taking the observation, so that its position as seen in the mirror is correct at the instant of reading the staff. This screw of course moves *both* the telescope and the bubble tube without altering their relative positions. The spirit level should be very sensitive, and one division

* It is not possible to carry elevations from one triangulation station to another with very great accuracy by means of vertical angles or "trigonometrical levelling" owing to the great variations and uncertainty of refraction. The elevations of the very high principal trigonometrical stations are, however, usually found by trigonometrical levelling. In any triangulation the elevations of at least the base lines above mean sea level must be found by a line of precise spirit levels from the coast in order that the triangulation distances may be reduced to mean sea level.

of the bubble tube should correspond to a vertical angle of about 3 seconds. The bubble tube should be chambered so that the length of the bubble may be adjusted to a nearly constant length under different temperatures. The diaphragm must have three horizontal cross hairs, one axial and two stadia hairs, as the distance of the staff from the instrument must be ascertained for each sight. For stadia hairs and stadia measurement of distances, see Chapter VI. The tripod legs should be covered with white cloth to shield them from the sun and render them less liable to variations of temperature.

Kern Level. — The levelling instrument adopted by the International Geodetic Commission is made by Kern & Co., of Aarau, Switzerland, who make a specialty of levels for "precise" work. It is of the wye type, similar to that shown in Fig. 107, and otherwise as just described. Usually a tangent or slow motion screw is provided for adjusting the cross hairs on to the staff. The motion given by this screw is merely horizontal or in azimuth. Even the ordinary spirit level, as made in the United States, generally has a tangent or slow motion screw. The bubble tube rests on the upper surface of the rings on the telescope which are supported in the wyes, and is carried in the hand between instrument stations.

A Kern level with a focal length of $14\frac{1}{2}$ in. has an object glass $1\frac{7}{16}$ in. diameter, and a magnifying power of telescope of 50 diameters.

Levelling Staff.—The *levelling staff* is made in one piece, and is about 10 ft. long. It is divided to feet and 100th parts of a foot, the 1000ths of a foot or third place of decimals being read by estimation. If working on the metrical system, the staff is graduated to metres and centimetres, the millimetres or third place of decimals being read by estimation. The staff should be made of good well-seasoned dry pine, and should be about 4 in. wide on the face and have a vertical stiffening piece up the back of it, thus making its cross section T-shaped. A small spirit level is attached to the staff, to enable the staff-holder to hold the staff truly vertical. Two wooden handles are provided for holding the staff, one on each side of the graduated face. The foot of the staff should have an iron cylindrical piece about 1 in. diameter fixed on it, and the graduations of the staff are reckoned from the extremity of this. The zero of the graduations is, however, some-

times at some distance above the foot of the staff. A foot-plate is always used, and it should have a cylindrical hollow in it of slightly larger diameter than the cylindrical piece on the foot of the staff. The bottom of this hollow is spherical, and on this convex spherical surface the piece on the foot of the staff rests.

A tent must be carried to protect the level from strong winds, also a large sun umbrella to shield it from the sun when the tent is not used, or to shield it from light winds.

Daily Adjustments.—Perfect accuracy of adjustment is not to be aimed at, but the instrumental errors are to be reduced to a reasonably small amount, this amount accurately ascertained, and each day's work corrected for the instrumental error.

The instrument must be tested for parallelism of spirit level and collimation line at the beginning and end of each day's work, adjustment 2, Chapter IV. A true horizontal line is determined as described in Chapter IV., Fig. 165. The instrument being then set up as in Fig. 166, the tangent of the angle which the actual collimation line makes with a truly horizontal line is equal to the error of reading on staff at B, Fig. 166, divided by the distance apart of the points A and B. Or calling this error of staff reading e, the distance d, the vertical angle made by the collimation line with a truly horizontal line v, we have—

$$\text{Tan } v = \frac{e}{d}$$

The value of v is to be determined at the beginning and at the end of each day's work, the average value of v for the day being used to correct that day's work.

Adjustment 1, Chapter IV., must also be tested as often as necessary, and the collimation line kept in the axis of the telescope. The centring of the object glass should also be tested (see Chapter IV.). The staff level is to be adjusted daily by means of a plumb line attached to the staff.

Any error in the absolute length of the levelling staff affects only the final result of the work, and may be corrected for at the conclusion of the levelling. For moderate heights this correction may usually be neglected.

Field Work: Errors from Unstable Supports.—The principal errors to be guarded against in the field work are errors

from *unstable supports* of instrument and levelling staff and *atmospheric errors.*

Any movement of the change point on which the staff is held is usually a downward movement, and if this takes place in the interval between a fore sight and a back sight (as it usually does) the final resulting elevation obtained will be too high. Again, if the instrument itself settles down during the interval between a back sight and a fore sight, the final resulting elevation from this cause also is too high. Errors due to settling of supports, therefore, result in too high a final elevation, and are obviously *cumulative.* The only way in which these errors can be eliminated is by doing all the levelling over again, going over the *same* ground in the *opposite* direction. Very often, however, this is not practicable, and the levelling party usually consists of a *leveller* with a *check leveller* following up behind. Sometimes the leveller runs his own line of check levels by taking two change points and four staff readings at each set up of the level. As the same setting of the instrument is used for each line of levels, by this method the two lines of levels are not entirely independent. The instrument may, however, be shifted, and a new set up made for the duplicate line back and fore sights. By this means one leveller can carry an independent line of check levels along with the first line of levels. A line of check levels run in the *opposite* direction to the first line is, however, always preferable, as the mean of it and the original line will be nearer the truth than the mean of two lines run in the same direction.

Atmospheric Errors.—*Atmospheric errors* are those due to wind, a heated atmosphere which causes the image of the levelling staff to tremble, and variable refraction. During a moderate wind the level may be shielded in a tent or by a screen, but when the velocity of the wind exceeds 8 or 10 miles an hour work must be stopped. During a heated atmosphere short sights must be taken, but if the length of sights has to be reduced to less than about 150 ft. to enable the image to be read clearly, it is better to stop, as the errors arising from the large number of stations occupied by the instrument and levelling staff become excessive. Refraction is greatest early in the morning, diminishes till 10 A.M., is nearly constant till 4 P.M., then begins to increase.*

* Great variation in refraction sometimes occurs over ground which is passing from light to shade, or *vice versâ.*

Available Working Hours.—In clear weather, as a rule, not more than three or four hours a day are available for good work, and at times when the days are very hot and the nights very cold it is not possible to get in an hour's good work in the day.

Observing Staff Readings.—All three horizontal cross hairs are read, the mean being taken as the staff reading, and the two differences between the axial and extreme hair readings are taken out. The stadia hairs being spaced at as nearly as possible equal distances from the axial hair, these two amounts should agree very nearly, otherwise one or more of the three readings is in error. This is a most valuable check on any serious error in the readings, and should be made in the field for each reading before the level or staff is shifted to the next station. Two levelling staves and two staff-holders are employed to save time between back sight and fore sight which would be lost with only one staff.

Length of Sights.—Under the most favourable conditions, for the best work the length of the sights should not exceed about 300 ft., or say 5 chains (66 ft. chains). With sights of about this length, in favourable weather an experienced man can level about a mile an hour, or 4 miles a day, of precise levels. The instrument must always be shaded from the sun, both while set up on a station and while being moved between stations. It is essential to do this in order to keep the adjustments anything like constant and the length of the bubble from continually changing.

Corrections.—If the lengths of the back sights and fore sights are made exactly equal for each set up of the level, all instrumental errors are eliminated. On the other hand, any instrumental error obviously merely applies to the excess of the sum of the lengths of the back sights over the sum of the lengths of the fore sights, or *vice versâ*. The length of each of the sights must therefore be computed from the stadia hair readings, as described in Chapter VI. This is easily done with the aid of a short table, such as that described in Chapter VI. The lengths of the back sights are then added up, as also the lengths of the fore sights,

and the difference ascertained. Calling this difference δ, and the correction for instrumental error c, we have—

$$c = \delta \sin v^*$$

where v is the average value of the vertical angle for the day, described on page 184.

This correction is to be added to or subtracted from the elevation of the finishing point of each day's work, and the levels of any intermediate bench marks are to be corrected in a similar manner. For instance, if the sum of the lengths of the fore sights is in excess, and the vertical angle v is minus or an angle of depression, the sum of the fore sight staff readings is too small, and the resulting elevation of the finishing point of that day's work is too high, and the correction c is therefore to be subtracted. The similar application of the correction when the back sight lengths are in excess and also when the vertical angle v is plus or an angle of elevation will be sufficiently obvious.

The difference between the sum of the lengths of the back sights and the sum of the lengths of the fore sights should also be corrected for curvature and refraction, but since the errors due to curvature and refraction tend to neutralise each other, and since δ is usually so small a quantity, it will generally be found that this correction is unnecessary, if reasonable care has been taken to have the back and fore sights of approximately equal length. The correction for curvature and refraction may, however, be applied when required. For curvature and refraction see page 159, Chapter III. Also, owing to the uncertainty of the amount of the refraction, it is much better to avoid the necessity of applying a correction for it by keeping δ within small limits.

Accuracy and Cost of Precise Spirit Levelling.—On the United States Coast and Geodetic Survey the limit of difference between check lines is 5 millimetres $\times \sqrt{2K}$, where K is the distance in kilometres. On the United States Lake Survey it was 10 millimetres $\times \sqrt{K}$, and on the Mississippi River Survey it was 5 millimetres $\sqrt{K}$. These limits are respectively equal to .029, .041, and .021 ft. multiplied by the square root of the distance in miles.† Where greater discrepancies occurred, the line of levels

* Sin v is used because the distances as found from the stadia hair readings are the distances along the inclined line of sight or collimation line.

† Johnson, Theory and Practice of Surveying.

was run again. The "probable error" of the mean of several observations may be found as follows:—

Let d_1, d_2, d_3, &c., be the differences of the various results from the mean, and let Σd^2 be the sum of the squares of these differences, n the number of observations, then the probable error of the mean is—

$$\pm .6745 \sqrt{\frac{\Sigma d^2}{n(n-1)}}$$

When there are only two observations, the formula becomes $\pm \frac{1}{3}d$ where d is the difference between the two results.

The European International Geodetic Association decided upon the following probable errors in the mean or adopted result:—

± 5	millimetres	per kilometre	= large.
± 3	,,	,,	= medium.
± 2	,,	,,	= fair.
± 1	,,	,,	= very precise.

On the United States Coast and Geodetic line of precise levels from Sandy Hook to St Louis, 1,109 miles, the probable error per kilometre of the adopted result was ± 1.2 millimetres. Professor J. B. Johnson ran 670 miles of precise levels on the Mississippi River Survey with a probable error in the mean result of 23.5 millimetres for the whole distance, which is less than 1 in., while the probable error per kilometre was ± .7 millimetre.*

In the precise spirit levelling undertaken for the general levelling of France, of 28,700 kilometres the probable error for systematic error was 0.12 to 0.18 millimetre per kilometre, and for accidental error 0.79 millimetre per kilometre. The average closing error of polygons averaging 550 kilometres in length was ± 60 millimetres. Taking into account the systematic error, the probable error in the difference of level found between Marseilles and Dunkirk does not exceed 60 millimetres. The cost of the levelling of the most precise order was 35 francs per kilometre.†

On the United States Geological Survey, using a 20 in. level

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* Johnson, Theory and Practice of Surveying.

† The radius of curvature of bubble tube used for the most precise work was 50 metres; for less degrees of precision the radii of bubble tubes were 30 metres and 20 metres. A system of total reflecting prisms was used to enable the observer to read the staff and note position of bubble simultaneously.

with bubbles graduated to 10 seconds of arc, there were 10,968 miles levelled and 1,924 permanent bench marks fixed, at an average cost per lineal mile of 19s. 1½d., exclusive of cost of instruments. In 111 closed polygons of an average length of 51 miles each, the maximum discrepancy between duplicate determinations in feet was $0.05\sqrt{\text{distance in miles}}$, minimum $0.02\sqrt{\text{distance in miles}}$, and mean $0.03\sqrt{\text{distance in miles}}$.

Adjustment of Errors in Closed Circuits of Precise Levels.—When a line of levels returns to its starting point the actual error is of course known, as the total difference of elevation should be zero. There are two general methods of applying corrections. One is to distribute the error among the lines composing the circuit proportionally to the length of the lines or to the square root of the length of the lines. The other method is to make the corrections proportional to the computed "probable error" of each line as calculated from the differences between each separate result and the mean result. The corrections are computed by the method of least squares, so that they are the most probable ones, *i.e.*, so that the sum of the squares of the corrections is a minimum.

The latter method is very laborious; and if the closing error is simply distributed so that the error on each line is taken as proportional to the square root of the length of the line, the result is practically identical with that arrived at by the more laborious methods. Experience shows that errors in levelling are more nearly proportional to the square root of the distance than to the distance only.

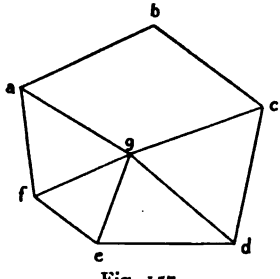

Fig. 157.
Adjustment of Closed Circuits of Precise Levels.

When the levelling includes a number of closed circuits, as in Fig. 157, the closing error of polygon *abcg* is first distributed among the sides *ab*, *bc*, *cg*, *ga*. In adjusting the polygon *gcd*, the error is to be distributed between the sides *cd* and *dg* only, the error in side *gc* having already been adjusted. Similarly in correcting *gde* the error is distributed between *de* and *eg* only, and for the polygon *gfa* its closing error is adjusted in the side *fa* only.

It is necessary to correct first the polygon whose closing error is the greatest, then that with the next greatest error, and so on.

American Practice in Precise Spirit Levelling.—On pages 194 to 201 are given the instructions of the Mississippi River Commission for Precise Levelling.* These are the result of many years' experience, and will be a good guide in this class of work. In the United States the levelling staff is called the "rod," and the cross hairs are called "wires" or "threads." The levelling instrument used is of the wye type, similar to Fig. 107, or the Kern level already described. The bubble tube is adjusted to be parallel to the upper surface of the rings on the telescope tube which rest in the wyes, the bubble tube resting on these rings; the collimation line is adjusted to coincide with the axis of the wyes, and any difference in the size of the rings themselves is to be determined. Thus three corrections take the place of the single correction given on page 187. The single correction given on page 187 may also be used for the wye level instead of the above three corrections, as it is obvious that if the bubble tube, rings, and collimation line are all corrected with reference to the same line (the axis of the wyes), the result is the same as correcting for their relative positions to each other, *i.e.*, one single correction for the angle of inclination between bubble tube and collimation line gives the same result; the bubble tube being made level by levelling up with the levelling screws and elevation screw for each sight, this angle is the angle made by the collimation line with a truly horizontal line when the bubble is at the centre of its run, as given on page 184. The methods of determining and applying these separate corrections are, however, given below.

It is sometimes the practice to read the staff when the bubble is not exactly in the centre of its run, and a correction is then made to the staff reading for the amount of the deviation of the bubble from the centre of its run. For this purpose the value of one division of the bubble tube, *i.e.*, the vertical angle corresponding to a movement of the bubble of one division, is carefully determined. As a general rule the practice of reading the staff with the bubble deviating from the centre of its run is to be avoided, and the instrument should be carefully levelled up in the

* Johnson, Theory and Practice of Surveying.

usual way, so that the bubble is at the centre of its run at the instant of reading the staff. When bisecting a bench mark, however, it is convenient to use the elevating screw and note the deviation of the bubble.

Value of one Division of the Bubble Tube.—This is found by sighting on to the levelling staff, whose distance from the instrument is carefully measured. The divisions of the bubble tube are numbered from the centre towards the ends, and the bubble being moved one division at a time, staff readings are taken for each position of the bubble. Readings need only be noted for extreme positions of the bubble, as central and intermediate positions are of little use in finding the mean value of one division.

Let E_1 = mean of all the north end readings of the bubble when run to the north end of tube.
E_2 = ditto, for bubble at south end of tube.
F_1 = mean of all the south end readings of the bubble when run to the north end of tube.
F_2 = ditto, for bubble at south end of tube.
S_1 = mean reading of staff for bubble at north end of tube.
S_2 = ditto, for bubble at south end.
d = distance of staff from instrument.
v = value of one division of bubble tube (sine of the angle of inclination) at unit distance.

Then—

$$v = \frac{S_2 - S_1}{d\left(\frac{E_1 - F_1}{2} - \frac{E_2 - F_2}{2}\right)} \quad \ldots\ldots\ldots 1$$

$$v \text{ in seconds} = \frac{S_2 - S_1}{d \sin 1''\left(\frac{E_1 - F_1}{2} - \frac{E_2 - F_2}{2}\right)} \quad \ldots\ldots\ldots 2$$

The value of one division of a bubble tube should be constant, but is often altered by changes of temperature of the fastenings of the tube in its case.*

* By experiments on the level of Ramsden's 3 ft. theodolite it was found that though at the ordinary temperature of 66° the value of one division was about 1 second, yet at 32° it was 5 seconds.

Correction for Inclination of Bubble to Upper Surfaces of Rings.—This correction is determined in terms of divisions of the bubble tube. It is to be found by reversing the bubble tube on the telescope and taking readings in both positions. An odd number of observations should be made.

Let N_1 = mean of north end readings for bubble tube direct.
N_2 = „ „ „ „ reversed.
S_1 = „ south „ „ direct.
S_2 = „ „ „ „ reversed.

Then—

$$i = \tfrac{1}{2}\left(\frac{N_2 - S_2}{2} - \frac{N_1 - S_1}{2}\right)$$

where i is the inclination of the bubble tube to the upper surface of the rings in terms of divisions of the bubble tube.

Correction for Collimation Line.—This is found by reading the staff with the telescope normal, and then with the telescope inverted, *i.e.*, rotated 180°, about its own axis.

Let S_1 = staff reading for telescope normal.
S_2 = „ „ inverted.
d = distance of staff from instrument.

Then —

$$c = \frac{S_2 - S_1}{2d}$$

where c is the correction for unit distance. The correction for any distance is therefore $c \times$ distance.

Correction for Inequality in the Size of the Rings.—This is determined by reversing the bubble on the rings, and also reversing the telescope in the wyes. The following is an example of the method of ascertaining this correction :—

	Reading of Bubble.	
	North.	South.
Telescope normal, level normal	4.2	5.6
„ „ „ reversed	4.5	5.4
Average . . .	4.35	5.5
Half difference . . .	−0.57	

	Reading of Bubble. North.	South.
Telescope reversed, level normal	6.4	3.1
„ „ „ reversed	6.5	3.0
Average . . .	6.45	3.05
Half difference . . .	+ 1.70	
Telescope normal, level normal	4.1	5.7
„ „ „ reversed	4.6	5.5
Average . . .	4.35	5.6
Half difference . . .	− 0.62	
Mean reading telescope normal	− 0.59	
„ „ reversed	+ 1.70	
Movement of bubble	− 2.29	

Therefore the bubble moves 2.29 divisions *towards the object glass* when the telescope is reversed in the wyes. This is evidently *twice* the difference in the rings, and the angle between the axis of a cone and its slant side being half the apex angle, therefore the line of sight makes an angle with the tops of the rings of one quarter of 2.29 or 0.57 division of the bubble tube. In this case the eye end ring is the smaller, and therefore when the upper surfaces of the rings are level the line of sight is depressed. This correction is termed the "pivot correction," and alters only with unequal wear of the rings.

The angular value of one division of the bubble tube and the inequality of the size of the pivot rings need only be determined once each season. The constant for the stadia hairs which is used for the distances as well as the absolute length of the levelling staff are usually also determined once each season. The inclination of the bubble tube to the upper surfaces of the rings and the collimation line correction are, however, determined daily, at the beginning and at the end of each day's work.

Final Correction.—If D is the difference between the sum of the back sights and the sum of the fore sights, or *vice versâ*, then the final correction is—

$$D\{c + v\,(i + p)\}$$

where c is the collimation line correction, $v = \sin$ of angle of inclination corresponding to one division of bubble tube, equation 1, page 191, i the correction for inclination of bubble to upper surfaces of rings, p the pivot correction.

Instructions for Precise Spirit Levelling under the Mississippi River Commission.*

1. Before commencing operations the constants of the instruments will be determined. The most important of these is the *value of one division of the level tube.* This can best be determined by means of a level trier. It can also be determined in the field as follows:—

Set up the instrument firmly, if possible mounting it on a wooden post, or better still, on a stone pier. Set up a rod in its tripod at such a distance that it can be distinctly read through the telescope. The distance should be at least 50 metres, or if the air is very still, 100 metres, and should be carefully measured. Adjust the instrument carefully, taking such length of bubble in the level tube that its ends will be about the middle or tenth graduated line on each side. Direct the telescope to the rod, and by means of the elevation screw cause the bubble to run to near one end of the level.

Carefully note the position of the three wires on the rod and the reading of the level. Now by means of the elevation screw cause the bubble to run to near the other end of the tube, and note the reading of the wire and bubble as before. One result for value of one division of level can then be obtained. This operation should be repeated ten times.

The elevation of the rod should be changed occasionally between sets, in order to avoid estimating the same part of the same centimetre on the rod. It will be sufficient to run the bubble five divisions each side of its central position.

If k = distance from instrument to rod

d, d' = distance through which eye and object ends of bubble move when run from near eye end to near object end

$\frac{d+d'}{2}$ = amount of displacement of bubble between two readings

r, r' = corresponding means of three thread readings on rod, and

v = value of one division of level in seconds of arc.

$$\text{Then } v = \frac{2\,(r'-r)}{k \sin 1'' \,(d+d')}$$

2. With the value of one division of the level, *tables will be constructed* showing the correction to be applied to a rod reading for an observed inclination of the level and for a distance determined by interval between extreme threads. If the level bubble is well ground, equal displacements of the bubble, say of two divisions, will correspond to equal displacements on the rod.

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* Johnson, Theory and Practice of Surveying.

3. Before using the level or determining its value, *the fastening of the tube in its case* should be examined. One end should be clamped down just tight enough to prevent the tube from moving easily, but not tight enough to strain the glass. The other should be lightly clamped so that the tube may be free to expand and contract with temperature changes. The cotton packing at the ends should not exert a lateral strain on the tube. All level tubes will be numbered, and have their numbers marked on them.

4. *In order to determine the inequality in the telescope rings*, the instruments should be mounted on a stone pier or other firm support and carefully levelled. The level should be carefully adjusted, and the instrument clamped to prevent its moving in azimuth. Now with the eyepiece of the telescope over the elevating screw, note the reading of the bubble when level is set on telescope, both in direct and reversed position. Now reverse the telescope in the wyes and read the level as before. Several sets of observations should be made.

Let b, b' = inclination of telescope as denoted by means of level readings with telescope direct and reversed, then the inequality of rings—

$$p = \frac{b - b'}{4}$$

Sixteen determinations of the value of p of two instruments in use on the Lake Survey gave probable errors of $\pm 0.046''$ and $\pm 0.041''$. The inequality may be expressed in seconds of arc if desired, but for purposes of computation is best expressed in terms of level divisions, as it can then be combined directly with the error of adjustment of level.

5. *The centring of the object glass* will be examined. This may be done as follows:—Draw out the eyepiece until the threads are no longer visible. Direct the telescope on some well-defined object, and when looking at it rotate the telescope on its wyes.

If the object remains steady, the object glass is sufficiently well centred. Should the object appear unsteady, the fault can only be remedied by a maker. The objective should be firmly screwed into the telescope.

6. *The values of the wire intervals* will be determined as follows:—Set up a rod at carefully measured distances of 10, 20, 30 to 100 metres from the instrument. Read the rod ten times at each distance. The rod may be altered in elevation, the level may be caused to change and the telescope may be rotated 180° (reversed) in order to change the position of the threads on the rod.

Taking the mean of the ten observed differences of readings of the extreme threads at each station occupied by the rod, a table will be constructed giving in metres the distance of the rod from the instrument for any observed difference of reading between extreme wires.

7. *Unless the rods used have been previously compared with some known standard, they will be compared with each other and their relative lengths determined.* This may be done by establishing two fixed points or two foot plates at equal distances from the instrument and differing in elevation about 2.7

metres. The distance should be about 10 metres. Determine the difference of elevation of the points by reading each rod on each point. A comparison of the resulting differences of elevation will give relative lengths of metres on rods.

Ten measurements with each rod will be determined. The elevation of the instrument will be slightly changed between each set in order to eliminate errors in estimating the millimetres. Each rod will be numbered and have its number marked on it. The rods should also be kept dry and provided with canvas covers to protect them when being carried to and from work.

The distance of the zero graduation above steel spur on which the rod stands will be well determined. This may be done with a right angle triangle and rule. It may also be determined by means of another levelling rod, the graduations of which commence at the foot of the rod, by determining the height of the instrument above some fixed point and subtracting it from the reading of the rod to be determined. The relative lengths of the rods must be known.

Whenever a bench mark is connected with in such a way that the rod is not placed directly on the bench mark, this quantity enters into the computation of difference of elevation.

8. Before commencing work at any time all adjustments will be carefully made.

The telescope will be collimated by having a rod set up at a distance of 50 metres and noting the position of the wires on the rod when the telescope is normal and when inverted or rotated 180° about its axis. The collimation error of the mean of the horizontal thread must not exceed 1.25 millimetres at a distance of 50 metres.

The horizontality of the horizontal wires will be examined by moving the telescope in azimuth so that the rod shall appear to move through the field of the telescope. If the threads are horizontal, the reading on the rod will be the same, the position of the level, which should be closely watched, remaining the same. If the threads are found to be not horizontal, they will be made so by turning the telescope a small amount in the wyes. When the thread wires have once been made horizontal, small screws which abut against projection of wye above elevating screw should be so adjusted that when they press against this projection the wires are horizontal. If the vertical thread is then inclined, as shown by the plumb line attached to the rod, it must remain so.

To make the axis of the level parallel to the upper surface of the rings, it is necessary to make the vertical planes passing through them parallel and to make them equally inclined to the horizon.

To make the lateral adjustment, raise the clips fastening the level to the telescope, and revolve the level about the telescope a short distance each side of the vertical. If the bubble runs in opposite directions when on opposite sides of the vertical, the level is to be adjusted by means of the opposing horizontal screws at one end of the level until such is not the case.

To make the vertical adjustment, raise one of the clips and read the level in its direct position and also when it is reversed on the telescope. The difference between the differences of end readings in each position is four times the error of adjustment, and is to be corrected by the opposing vertical screws at one end

of the level case. The error of adjustment must not be allowed to exceed two divisions of the level. Care must be taken that the telescope rings are free from dust when adjusting the level. After having made the vertical adjustment, it will be necessary to examine the lateral adjustment again, since making one of the adjustments affects the other.

To make the level and vertical axis of revolution perpendicular to each other, loosen the small clamp screw at one end of the horizontal bar fastened to the vertical axis, and by means of the elevating screw raise or lower that end of the upper horizontal bar until the telescope can be rotated 180° from any position, and have the level reading the same in both positions.

To adjust the level attached to the rod, set up the rod in its tripod in such a position that when a plumb line is attached to the small hook near the top of the rod, the point of the plumb bob shall coincide with the point of a small cone attached to the rod near its foot. Now bring the level bubble to the centre by means of the levelling screws. In making this adjustment, the rod should not be exposed to the wind, as the plumb line is influenced thereby. This adjustment will be made at least once each day.

Each time that the instrument is placed on a station its axis will first be made vertical by means of the levelling screws in such manner that the telescope may be turned around the horizon without the bubble of the level running a great number of divisions. The telescope is finally made horizontal by means of the elevating screw. The inclination at the moment of observing must not ordinarily exceed three divisions of the level and never five divisions.

The instrument when in use ought always to be sheltered from the sun and wind. It is carried from station to station without being dismounted, but the level should be taken off and carried in the hand. The small clamp screw at the end of horizontal bar and the large screw which fastens the instrument immovably to the tripod should both be turned tight before moving the instrument.

The rods must be placed on the plates which accompany them, and held in a vertical position as indicated by the spherical level attached. It is advisable to always use the same rod with the same foot-plate. In placing the foot-plates, great care should be taken that they be horizontal, on firm ground, and not liable to change. The surface of the ground, if not firm or level, should be removed.

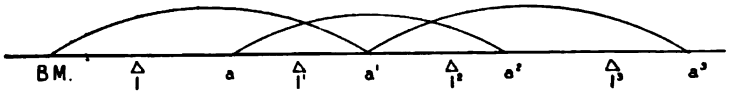
The errors of adjustment will be determined at beginning and end of each series of observations: that is to say, after having mounted the instrument and before dismounting it, and in all cases at least once each day. If the instrument has been deranged by a jar, the corrections must be determined anew.

The error of collimation will be determined by two readings of the rod at a distance of 50 metres when the telescope is in its normal position and two when it is rotated 180° in the wyes. The difference between the means of the two readings, after being corrected for the inclination of the level, must not exceed 2.5 millimetres at that distance, and commonly should not exceed 1 millimetre. The error of the adjustment of the level will be determined by reading the level four times when direct and four times when reversed on the telescope, reversing it between each reading.

The error of adjustment must not exceed two level divisions, and commonly should not exceed one. All the details of the determination of the errors of adjustment must be entered in the note-book in their proper place. It is always advisable to have the errors of adjustment as small as possible, and necessary that they be well determined. The time of making these determinations will be recorded in the note-book.

In all work along the main line of levels each observer will duplicate his own work by running over the line in opposite directions, preferably under similar conditions as to illuminations, &c.

While connecting two bench marks the order of using the rods will be as follows:—



In the above figure let 1, 1^1, 1^2, &c., represent the successive stations occupied by the instrument; BM, a^1, a^3, &c., the positions occupied by rod 1; and a, a^2, &c., the positions occupied by rod 2. The instrument having been set up at 1, rod 1 is placed on BM, and rod 2 at a, making distance $1-a$ equal to 1 – BM. Rod 1 is then read and immediately afterward rod 2. The time elapsing between these readings commonly will not exceed 1 minute, and should not exceed 5 minutes. The instrument is then carried to 1^1 and rod 1 to a^1, the distances $a-1^1$ and 1^1-a^1 being equal. Rod 2 will then be read, and immediately afterward rod 1. The instrument will then be taken to 1^2, and the rods read in the order 1, 2. Work will be continued in this manner until the other bench mark is reached. Rod 1 must be placed on this bench mark, which will be the regular order if there have been an even number of instrument stations. If there have been an odd number of instrument stations, at the last station use rod 1 for both back sight and fore sight. When levelling the rate of progress in favourable weather will be about 1 kilometre per hour. After having properly levelled the instrument at any station, and having made the vertical thread coincide with the centre line of the rod, *the observation will be made and recorded in the following order*:—First the level will be read, the tenths of the division being estimated; then the position of the threads on the rod will be read, the millimetres being estimated; and finally the level will be read again. The observer will then read the rod a second time to make sure that no error has been made. The recorder will then take the differences between the readings of the middle and extreme wires to guard against errors, and if these differences denote any error, the observations must be repeated. If an error exists it will be shown by too great a difference between the differences. This is a most important check, and must not be neglected. These differences will also serve as a check upon the distances between the instrument and rods.

The recorder should also check the level readings to make sure that errors of whole divisions have not been made. This may be done by summing up the readings and noticing the length of the bubble. In reading the level by means

of the mirror, care should be taken that the position of the eye is such that there will be no parallax. Such positions can be determined once for all when the mirror is at its greatest angle of elevation, by a second person reading the level directly while the observer finds the position from which the reading of the level in the mirror is the same. The notes will be kept in the form given in note-books. When once a number has been written down it must not be erased or made illegible. If wrong, a line will be drawn through it and the correct number written above.

The length of sights taken will depend on the condition of the atmosphere, but the rods should always be near enough to be seen distinctly. It will be seldom that lengths of sights greater than 150 metres can be taken. The back sight and fore sight corresponding to any instrument station must not differ in length by more than 10 metres, and the sum of the lengths of the back sights and fore sights between any two bench marks should be equal.

Whenever it is necessary that the line of levels should cross a river or other wide obstruction, a narrow place should be chosen. Firm points should be set upon the two banks; levels in good adjustment are set upon posts about 10 metres from each bench mark, and both levels go through the same operation.

The error of adjustment is first accurately determined. Call one of the levels A. A first reads on the bench mark near it, once with the telescope normal and once with the telescope inverted, then on the rod across the river five times with the telescope normal and five times with the telescope inverted. The error of adjustment of the level is again accurately determined. The rod across the river will need an extra vane. B performs the same operation simultaneously. A and B change places and repeat the observation at these new stations. The simultaneous levels eliminate refraction, the change of station eliminates curvature and small instrumental errors. Unless good results are obtained the levels should be repeated. If but one level can be used, the operation will be performed in the same order, but the time occupied in crossing must be as small as possible. With a single Kern level this process has given for a river 815 metres wide five results, the mean of which has a probable error of ± 0.5 millimetre.

Permanent bench marks will be established at intervals of 3 miles along the river, and 5 miles on lines connecting the river line proper with the other levels or bench marks.

These bench marks will consist of a thoroughly verified tile $4 \times 18 \times 18$ in. placed 3 ft. below the surface of the ground, and surmounted by a 4 in. wrought-iron pipe as a surface mark. The tile should have time to settle before levelling to it. Both tile and pipe will be suitably marked to designate the character of the point. In the centre of the upper surface of the tile a copper bolt will be leaded, the upper surface of which will be the point of reference. These bench marks will be placed where they can be easily found, and where they will not be disturbed. Property corners should be utilised where practicable. In addition to the above, benches should be established on permanent brick or stone structures by leading into them a horizontal copper bolt, with the letters U.S.P.B.M. and the number of the bench mark cut near it. A small hole in the centre of the bolt will be the point of reference.

In connecting with a bench mark, if the bolt is vertical the foot of the rod

is placed directly upon it. If the bolt is horizontal in the wall of a building or other structure, it may be best connected in the following manner:—Set up the instrument in such a position and at such an elevation that the small hole in the bolt may be bisected by the middle thread without displacing the level by more than five divisions, using the elevating screw for making this bisection. Since the instrument can be raised or lowered about 2 centimetres by means of the levelling screws, the instrument can be placed in such a position by two or three trials.

Now bisect the bench mark with the telescope normal and also inverted, noting the reading of the level. Read the rod on the plate with the telescope in both positions. It is necessary to eliminate collimation by inverting the telescope, since the collimation of the middle wire is not the same as that of the three wires. The distance of bench mark from instrument must be determined and recorded.

Whenever work is stopped, at least two temporary bench marks should be established. These will consist of large nails or spikes driven their entire length vertically into the base of trees, or in the tops of sound stumps. When not in the vicinity of trees or stumps, wooden posts may be firmly set in the ground with their tops flush with the surface, and nails driven into them. When near the river temporary bench marks should be set every 2 kilometres. Every bench mark will be fully described in a note-book kept for that purpose. Its position with reference to the most prominent objects near it should be given by distance and direction. Public buildings, such as depôts, court-houses, churches, &c., are the best positions for permanent bench marks. In a village or town several permanent bench marks should be established to secure some one against loss.

If a railroad is crossed the elevation of the foot of the rail will be determined, and if levelling along a railroad, the elevation of the foot of the rail at depôts will be determined.

The elevation of the zeros of all water gauges and also the gauge bench marks will be determined.

The datum planes of cities along the line of levels will be connected with and their elevations deduced.

Frequent connections will also be made with the United States Engineer bench marks between St Paul and Grafton.

In reducing the observations the nearest tenth of a millimetre will be retained. The distance will be taken out from the table to the nearest metre.

The limit of discrepancy in closing a polygon will be—

$$5 \text{ millimetres } \sqrt{\text{distance in kilometres}}$$

The distance referred to is the entire length of the polygon from bench mark 1 to bench mark 2 and back to bench mark 1, and the limit of discrepancy refers to the polygons between successive bench marks. If the discrepancy exceeds the prescribed limit, then the entire polygon must be re-run one or more times, or until the difference of the means of the direct and reverse results is within the limit.

The notes will be kept in the following form:—

Left hand page.] BACK SIGHT.

Thread Readings.	Mean.	Difference of Threads.	Level.		Rod.	Remarks.
			Eye.	Object.		
7.95 10.02 12.08	1001.7	207 206 413	11.1	11.1	12	

FORE SIGHT. [*Right hand page.*

Thread Readings.	Mean.	Difference of Threads.	Level.		Rod.	Remarks.
			Eye.	Object.		
18.69 20.56 22.42	2055.7	187 186 373	11.4	11.4	10	

CHAPTER IV.

ADJUSTMENT OF INSTRUMENTS.

Chain.—From pulling through fences or other causes the chain is liable to become lengthened or shortened, and must be tested and adjusted from time to time. The best way to test the chain is to compare it with a steel tape, noting also whether the half-chain or 50 links mark is exactly 33 ft. If the chain is too short, it must be lengthened by straightening out any links which may have become bent, or by inserting one or two of the spare rings connecting the links, some of these being always provided on the chain for this purpose. If the chain is too long, some of these rings must be taken out. A Government standard of all kinds of English measures has been established in Trafalgar Square, London, by means of permanent bronze marks let into the granite plinth of the terrace wall in front of the National Gallery. There is also a standard in the Guildhall belonging to the Corporation of London, and similar marks are established in nearly every city or town by the burgh surveyor, in some central position.

Theodolite.—The adjustments of the theodolite may be divided into the *Temporary Adjustments* and the *Permanent Adjustments.*

TEMPORARY ADJUSTMENTS.—These are the adjustments which must be attended to each time the instrument is set up.

1. **To Set up the Instrument over a Station.**—This is effected by means of the plumb bob and plumb line attached to the vertical axis of the instrument, which is brought over the point first roughly by means of the legs and then exactly by the adjusting screws or sliding plate. When there are no adjusting screws or sliding plate the plumb bob is brought over the point by means of the legs only, pressing into the ground one leg or another until

the plumb bob is exactly over the point. In accurate work see that the levelling up of the instrument does not disturb the position of the plumb bob, and if so, readjust and level up again.

2. **To Level up the Instrument.**—By this is meant making the horizontal plates *t* and *y*, Fig. 52, truly horizontal, and the vertical axis truly vertical. It is effected by means of the two spirit levels *v* attached to the upper or vernier plate *t*. Rotate the instrument horizontally until one of the levels *v* is parallel to one pair of the levelling screws *g'*. By means of these screws bring the bubble of this level to the centre of its run, and by the third screw, or if there are four levelling screws, by means of the other pair of screws, bring the bubble of the other level to the centre of its run. Now rotate the instrument horizontally through 180°, and see if the bubbles of the two spirit levels still remain in the centre of their runs. If they do, the adjustment is effected; but if not, then the spirit levels are not parallel to the upper plate *t*. In this case correct *half* the deviation by means of the capstan screws attaching the levels *v* to the upper plate, and *half* the deviation by the levelling screws *g'*, and repeat the operation until the bubbles remain in the centre of their runs while the instrument is rotated into *any* position. The plates *t* and *y* should be clamped together by the screw *a'* during this adjustment.

As this adjustment is so liable to get out, it is here described under the head of temporary adjustments, although the adjustment of the levels *v* themselves parallel to the plate *t* might be called one of the permanent adjustments; the levelling up by means of the screws *g'* being strictly speaking the temporary adjustment.

A more exact adjustment may be made by means of the large level attached to the telescope. Set the vernier of the vertical circle to zero and bring the telescope over one pair of screws. Now level it up by the levelling screws *g'*. Turn the instrument horizontally through 180°, and if the bubble of the telescope level deviates from the centre of its run, correct *half* the deviation by the tangent screw *r* of the vertical circle and *half* by the levelling screws *g'*. Repeat the operation until the bubble remains in the centre of its run, when the instrument is turned through 180°. Now turn the horizontal plates *t* and *y* (clamped) through 90° so that the telescope is over the third screw, or when there are four levelling screws, over the other pair of screws, and again level up

the telescope level by the levelling screws g'. The horizontal plates t and y will then be horizontal and the vertical axis vertical.

3. **Focussing the Object Glass and Eyepiece**, or as it is called, adjustment for parallax. The foci of the object glass and eyepiece glasses must be made to coincide with the cross hairs of the diaphragm. To adjust the eyepiece b, look through the telescope, and push the eyepiece out or in until the cross hairs are seen clearly and distinctly. Then direct the telescope to some well-defined object, and by means of the milled-headed screw at the side of the telescope move the object glass a out or in until the image of the object is seen clearly and apparently coinciding with the cross hairs of the diaphragm.

The adjustment of the eyepiece need only be made once, but the object glass must be focussed for each new object sighted on at different distances. A good test for parallax is to move the head slightly from side to side. If the adjustments are correctly made, the image will appear to coincide steadily with the cross hairs; if not, it will appear to move from side to side, as the head is moved. It is to be corrected by pulling the eyepiece out or in and focussing the object glass as already described.

PERMANENT ADJUSTMENTS.

1. **Adjustment of the Supports of the Horizontal Axis of the Telescope.**—The object of this adjustment is to make the horizontal axis of the telescope truly level when the instrument is levelled up, so that the telescope may revolve in a truly vertical plane.

Direct the telescope to a well-defined object of considerable altitude, such as the finial on the top of a church spire, the theodolite being placed close to the bottom of the church. Direct the cross hairs on to the top of the spire, clamping the horizontal plates t and y, Fig. 52. Now depress the telescope to the bottom of the church, and there mark a point bisected by the cross hairs. Unclamp and reverse the position of the supports by turning the instrument through 180° horizontally or in azimuth, and again direct the cross hairs to the top of the spire. Clamp again, adjust exactly on to the top of the spire, and see whether the cross hairs still cut the same point previously marked on the bottom of the church when the telescope is depressed. If not,

mark where they cut, and also mark the point midway between the two marks. This middle point is the correct point in the same vertical plane with the top of the spire. Correct *quarter* of the total deviation between the first two marks by the screws *ss* at the top of the support and *quarter* by the tangent screw *d'* of the horizontal circle, thus bringing the cross hairs on to the *middle* point. Repeat the operation until the middle point and the top of the spire are both bisected in both positions of the supports, when the adjustment will be correct. Here we observe that the collimation line is again directed exactly on to the top of the spire before being directed to the bottom of the church for the second time. No error other than that due to the supports is therefore introduced, the telescope being merely depressed. This adjustment may also be tried by seeing if the cross hairs bisect an object of considerable altitude and its image as seen reflected in the horizontal surface of a fluid.

2. **Adjustment of the Collimation Line.**—The adjustment of the line of collimation consists in making it exactly perpendicular to the horizontal axis about which the telescope rotates. Let the cross hairs be directed on to some well-defined distant object P, and bring them to coincide exactly with it. In Fig. 158, let *aob* represent the horizontal axis of the telescope, and *cod* the collimation line directed as described on to the point P. If now the collimation line be perpendicular to the horizontal axis, when the telescope is lifted out of its bearings and replaced with the ends of the horizontal axis *ab* reversed into the position *ba*, Fig. 159, the collimation line *cd* will still be directed on to P. If, however, the collimation line *cd* be not perpendicular to the horizontal axis *ab*, as shown in Fig. 160, when the ends of the horizontal axis are reversed into the position *ba*, Fig. 161, the collimation line will no longer be directed on to the point P, but on some other point as P'. In this case the point P'' midway between P and P' is the correct position of the collimation line perpendicular to the horizontal axis *ab*. The method of adjustment is therefore as follows:—Direct the cross hairs on to some distant point P, clamping the upper and lower plates by the screws *a'* and *c'*, Fig. 52, and leaving the vertical arc unclamped, so that the telescope may be easily lifted out of its bearings. Now carefully lift the telescope out of its bearings and replace it with the ends of the horizontal axis

reversed. The cross hairs should still coincide with the object P. If they do not, but intersect some other point as P', Fig. 161, correct *half* the total deviation PP' by means of the diaphragm screws *dd*, Fig. 52, and *half* by means of the tangent screw *d'* of the horizontal circle. The cross hairs should now coincide with the same point P when the horizontal axis of the telescope is reversed. If not, the operation must be repeated until they do. In the transit theodolite, in order to reverse the ends of the horizontal axis of the telescope without lifting it out of its bearings, which is liable to shift the instrument, turn the telescope over vertically, so that the object glass points away from the point P, and then turn the upper part of the instrument through exactly 180° by means of the vernier *w* on the upper plate *t*.

In the plain theodolite the adjustment of the line of collima-

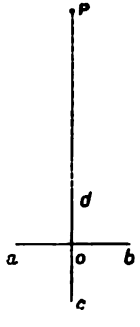


Fig. 158.

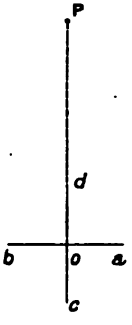


Fig. 159.

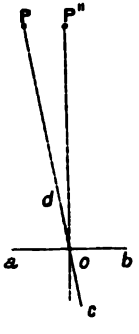


Fig. 160.

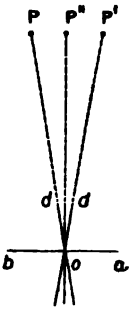


Fig. 161.

Adjustment of Collimation Line.

tion may also be effected by twisting the telescope half round in its Y's about its own axis, and observing whether the cross hairs still continue to coincide with the distant point P. Should they not, *half* the deviation is to be adjusted by the diaphragm screws *d*, *d* and *half* by the tangent screw *d'* of the horizontal plate. By this means the line of collimation is made to coincide with the axis of the Y's, and in this case the adjustment of the axis of the Y's perpendicular to the horizontal axis of the telescope is supposed to be correctly made by the instrument maker.

Accurate Method of Adjusting the Collimation Line of Transit Theodolite.—An objection to the foregoing method is that in lifting the telescope out of its bearings and replacing it the instrument is liable to be moved, unless it is done with extreme care. Also when with a transit instrument to avoid this the telescope is revolved vertically and the instrument turned through exactly 180° in azimuth, any error due to reading the vernier or in the graduations of the horizontal plate, is introduced. The following is the best way to adjust the collimation line of a transit theodolite. Set up the instrument at A, Fig. 162, and direct the cross hairs to a well-defined mark at B. A fine point marked on a peg driven into the ground will do. Turn the telescope over vertically and note where the cross hairs cut a well-defined distant point P. Again direct the telescope on to B, this time with the ends of the horizontal axis reversed. This is effected by leaving the telescope still pointing on P, and turning the whole of the instrument through 180° in azimuth, then accurately bisecting B again by means of the tangent screw *d'*. Again turn the telescope over vertically, and see whether the cross hairs still bisect P. If they do not, but intersect the object say in P', Fig. 162, then the point P'' midway between P and P' is the correct point, *i.e.*, P'' is in line with A and B and BAP'' is a true straight line. To adjust the collimation line, correct *one quarter* the total deviation PP' by the diaphragm screws *d*, *d*, *i.e.*, P'*p*, and *one quarter* by the tangent screw *d''* of the horizontal limb, *i.e.*, *p*P''; in other words, by means of the diaphragm screws *d*, *d* bring the cross hairs to bear on *p* where P'*p* = ¼ P'P, Fig. 162, then by means of the tangent screw *d'* of the horizontal circle bring them to bear on P'' midway between P and P'.

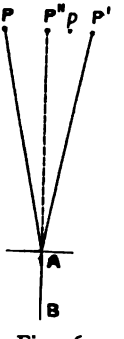


Fig. 162.
Adjustment of Collimation Line.

The cross hairs when the telescope is revolved vertically should now intersect both the point B and the point P'' in *both* positions of the horizontal axis, and the correction must be persevered in until this is effected, when the adjustment will be correct.

By using both *faces* of the instrument in this manner and taking the mean of the two points P and P' each time the telescope is

rotated vertically, a straight line may be ranged out correctly even when the collimation line is out of adjustment.

The points B and P should be selected as nearly on a level as possible. In any case the difference of altitude of B and P should be small, otherwise part of the error may be due to adjustment 1 being out, as the position of the *supports* is reversed. AP should be much greater than AB. Any error will then be increased, and a more accurate adjustment will be effected.

By this method it will be observed that although we may have moved the instrument when rotating it through 180° in azimuth, the cross hairs are again adjusted exactly on to the point B before the telescope is directed on to the distant point P for the second time. The instrument is not liable to be disturbed, therefore, as the telescope has simply to be turned over vertically.

If the ends of the horizontal axis of the telescope be reversed by lifting out the telescope and replacing it, there can be no error due to the supports, adjustment 1, introduced, as the position of the supports is not reversed. It is, however, not easy to lift out the telescope and replace it without shifting the instrument. In this case, however, it is better to reverse the horizontal axis by lifting out the telescope, as the cross hairs are always made to bisect the point B again by means of the tangent screw *d* before the telescope is turned over on to the point P. We may then be sure that none of the error is due to adjustment 1 of the supports being out.

3. **Adjustment of the Level on the Telescope.**—This level should be parallel to the collimation line of the telescope. In the transit theodolite it is to be adjusted precisely in the same way as the level, adjustment 2, page 214. In the plain theodolite, level up the level by means of the tangent screw of the vertical arc. Take the telescope out of the Y's, and replace it with the ends reversed. If the bubble deviates from the centre of its run, correct *half* the deviation by the screws which attach the level to the telescope, and *half* the deviation by the tangent screw of the vertical arc. Repeat the operation until the bubble remains in the centre of its run, when the telescope is reversed end for end in the Y's. This makes the level parallel to the Y's, with the axis of which the collimation line is supposed to coincide. It is, however, more accurate to adjust

in the same way as described for the level, adjustment 2, page 214. This method of adjustment should be used when possible.

4. **Index Error of Vertical Circle.**—Having adjusted the collimation line and made the level attached to the telescope parallel to it, the vernier of the vertical circle should read zero when the bubble of the telescope level is in the centre of its run. If it does not, then the difference of the vernier reading from zero is the index error of the vertical circle which is the correction for each vertical angle observed.

The collimation line may be made to coincide with the axis of the telescope in the same way as described for the level, adjustment 1, page 213. This adjustment, if made, should be done before adjustment 2 or 3.

Besides the foregoing adjustments, the following points should be attended to before making any of the permanent adjustments.

Centring of the Object Glass and Eyepiece.—The centre of the object glass *a* and of the eyepiece *b* should coincide exactly with the axis and collimation line of the telescope. As a rule this adjustment must be left to the instrument maker, as no means is provided for making it. The centring of the object glass is to be effected by focussing the object glass on to a very near point, and then twisting the telescope half round on its axis. If the image of the point continues to coincide with the intersection of the cross hairs, the object glass is correctly centred; but if not, *half* the deviation is to be corrected by moving the object glass, and the operation repeated until the image of the same point continues to coincide with the intersection of the cross hairs when the telescope is twisted round on its axis. The same operation is then to be repeated for a very distant point and again for a near point if necessary until the adjustment is correct. Before centring the object glass the cross hairs should be put exactly in the axis of the telescope.

As usually no means are provided for altering the object glass, it is a good plan to make a continuous scratch on the ring of the glass and its slide so as to be able to see at any time that the glass is always in the same position relatively to its slide. Otherwise if the glass happens to be not quite truly ground and a little eccen-

tric, the adjustments may be put out if it is taken out to be cleaned and not screwed up again to precisely the same position. By means of the continuous scratch the glass may be always screwed up again into the same position.

The glasses of the eyepiece should also be similarly centred to the axis of the telescope so as to see the intersection of the cross hairs precisely in the centre of the field of view.

Adjustment of the Diaphragm Ring so that the Cross Hairs are truly Vertical and Horizontal.—The cross hairs are placed on the diaphragm ring exactly vertical and horizontal by the instrument maker, but sometimes in adjusting the collimation line the ring may be turned a little. To ascertain if one cross hair is vertical (in which case the other being at right angles to it will be horizontal), after having made the temporary adjustments, sight on to a plumb line. If the vertical hair coincides with the plumb line, the cross hairs are truly vertical and horizontal. If not, slightly loosen two *adjacent* screws of the four diaphragm screws *d*, *d*, *d*, *d*, and with a knife, key, or other small instrument, tap gently against the screw heads so as to turn the ring slightly in the telescope, and persevere until the vertical hair coincides with the plumb line. When this is effected, tighten up the screws again. This adjustment must never be made *after* adjustment of the collimation line without again testing the latter, otherwise it may be put out.

When there is no vertical hair in the diaphragm adjust the horizontal hair on to a levelled straight-edge.

To Replace the Cross Hairs.—Pull out the tube of the eyepiece *b* and loosen all the four diaphragm screws, and let the diaphragm ring fall out of the telescope. The best way to put on new cross hairs is to take a thread from a spider's web on a *forked* stick, and having placed a little varnish, glue, or shellac on the diaphragm ring, lay the hair across the ring, and bring it down exactly on the marks on the ring. Then put on the other hairs in the same way. To now replace the diaphragm ring, cut a piece of stick long enough to reach into the telescope to the place where the diaphragm is to be fixed, and point one end so as to fit into one of the screw-holes of the diaphragm. By means of this stick as a handle insert now the ring *edgeways* into the telescope, and

hold it there until two *opposite* screws are put in place and screwed into the ring. Now pull the stick out of the screw-hole of the diaphragm, and with it turn the diaphragm about the two screws already screwed into the ring until the other two screw-holes are in their proper position. Screw in the other two screws, and adjust the diaphragm for verticality of the cross hairs and for collimation. Glass diaphragms will save any of this trouble with cross hairs.

To Replace a Spirit Level or Bubble Glass.—Remove the level from the instrument, pull off its sliding ends, and take out the broken glass. Put in the new one with the graduated side up. Roll some paper round its ends if it fits loosely. Putty or melted beeswax round the ends of the glass will hold it firmly in its tube.

Adjustment of the Sextant.—For adjustments of the sextant, see Chapter X.

Adjustment of the Box Sextant.—When the box sextant is in adjustment both the mirrors are at right angles to the plane of the instrument, *i.e.*, the top of the box, and when the vernier is at zero are parallel to each other. The index glass is permanently fixed by the maker. The horizon glass is adjusted as follows:—Bring the zero of the vernier to the zero of the graduated arc, and look through the eyehole and the unsilvered half of the horizon glass at some distant object. If the instrument is in adjustment this object and its reflection will appear to coincide exactly. If not, the two will appear to be separated either horizontally or vertically, or both, as ⁎*. In this case, apply the key furnished with the instrument to the square-headed screw in the top of the box, and bring the object and its image into a horizontal position, thus * *. Then apply the key to the other square-headed screw in the side of the box, and bring the object and its image to coincide exactly. The instrument will then be in adjustment. See also adjustment of sextant, Chapter X.

Adjustment of the Compass.—The adjustments of the compass are as follows:—

To make the Needle truly Horizontal.—Level the compass, and then see if the needle is level. If not, make it level by moving the wire which is fastened round it towards the high end.

To make the Sights Perpendicular to the Compass Plate.—Hang up a plumb line, and having levelled up the compass sight on to it, see if the slits coincide with it. If the slits should not be in adjustment, they may be rectified by unscrewing the sights, and filing off a portion of the feet on the high side, otherwise they may be wedged up on the low side.

To Straighten the Needle.—Having levelled the compass, note if its two ends continue to point to exactly opposite degrees, while the compass is revolved completely. If they do, the needle is straight, and the pivot is in the centre of the graduated circle; if not, one or both of these are wrong. Level up the compass, then turn it until some graduation (say 90°) comes exactly to the north end of the needle. If the south end does not then point exactly to the opposite 270° division, lift off the needle, and bend the *pivot pin* to make it do so, noticing that every time the point is bent the compass must be turned slightly so as to put the north end of the needle at its 90° mark. Then turn the compass through 180°, or until the 270° mark comes exactly to the north end of the needle. Make a mark where the south end of the needle now is. Then remove the needle and bend it until its south end points *midway between* the 90° and the mark, while its north end is kept at 270° by moving the compass slightly.

Adjustment of the Pivot Pin.—After having straightened the needle, revolve the compass until a place is found where the two ends of the needle coincide with opposite degrees. Then rotate the compass through 90°. If the needle then coincides with opposite degrees the pivot pin is correct; but if not, the pivot pin is to be bent until it does. Repeat the operation until the needle coincides with opposite degrees while the compass is turned through a whole circle.

Pocket Magnifier.—In using a pocket magnifier for the compass readings, take care that it is held with its centre *precisely over* the point to be read and *parallel* to the graduated circle. Otherwise errors of several minutes may be made in a single reading.

Magnetic Attraction about the Person.—No part of the magnifier should be made of iron, as this will attract the

needle. Care should be taken that nothing is carried about the person, as knives, keys, &c., that may attract the needle. Trouble is sometimes experienced from felt hats, which have sometimes an iron wire round the inside of the brim to stiffen them.

Sight Slits.—The sight slits are fixed by the instrument maker in line with the 360° and 180° divisions of the graduated circle. They may be tested by passing a fine thread through them, and observing whether it stands exactly over the 360° and 180° marks.

Remagnetising the Needle.—The needle sometimes loses part of its magnetism and becomes sluggish. It may be magnetised by drawing the north pole of a magnet several times from the centre to the south end of the needle, and the south pole in the same way from the centre to the north end of the needle, rubbing the magnet gently upon the needle. Take the magnet away from the needle while bringing it back to the centre. The needle should be held flat on a smooth hard surface while being magnetised. Bad action of the needle is more often due to defect in the point of the pivot pin. Remagnetising throws the needle off the level, and this is to be adjusted by moving the sliding wire.

Adjustment of the Level.

1. **To make the Collimation Line coincide with the Axis of the Telescope.**—Drive in three pegs as at A, B, C, Fig. 163, an equal distance apart, say 3 or 4 chains. Set up the level at *a* exactly half way between A and B, and read the staff held on the pegs at A and B. Then set up at *b* midway between B and C, and read the staff held on the pegs B and C. Then whether the level is in adjustment or not, because it has been set up midway between A and B, any errors of adjustment will affect the staff readings at A and B equally and similarly; the difference of the readings will therefore give the correct difference of level of A and B. Similarly the difference of the staff readings at B and C will give the correct difference of level of B and C. Now set up at *c*, Fig. 164, as near to A as it is possible to read the staff held at A, and read the staff at A, B, C. From these staff readings compute the differences of level of A, B, C. Compare these with the *correct* differences of level previously found and

compute the errors. Then if the cross hairs are in the axis of the telescope, the error at C will be double the error at B; and if not, the diaphragm carrying the cross hairs must be moved by means of the screws at *c*, *c*, Fig. 104, until the error at C is double the error at B.

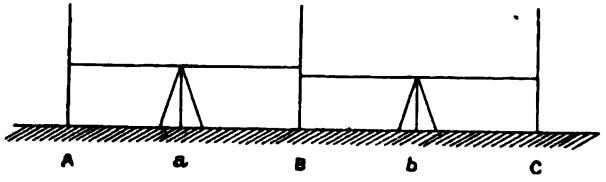


Fig. 163.—Adjustment of Level.

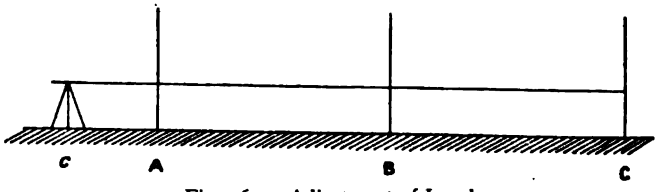


Fig. 164.—Adjustment of Level.

The following table shows the direction in which the cross hairs are to be moved:—

Direction of Error at C.	If Error at C is more than twice Error at B.	If Error at C is less than twice Error at B.
Upward	Move cross hairs upward	Move cross hairs downward
Downward	,, ,, downward	,, ,, upward

2. **To make the Spirit Level parallel to the Collimation Line,** or in other words, to ensure that the collimation line is horizontal when the bubble of the level is at the centre of its run.

This is the most important adjustment of the level, as nearly the whole accuracy of the instrument in practice depends on it. The first adjustment need seldom be made, and when making it

care should be taken that the diaphragm screws are screwed up perfectly tight, and it will then seldom require to be interfered with again.

Drive in two pegs at A and B, Fig. 165, or select two good steady marks at these points, and set up the level at *c* exactly half way between A and B. Then whether the level be in adjustment or not, the staff readings at A and B will be equally and similarly affected, and the difference of these readings will be the *correct* difference of level of A and B. Now set up the level at *d*, Fig. 166, as close to A as it is possible to read the staff, and read the staff held on A. Now knowing the correct difference of level between A and B, compute from the last staff reading at A what

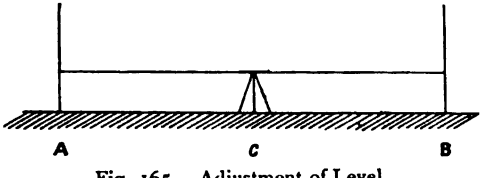


Fig. 165.—Adjustment of Level.

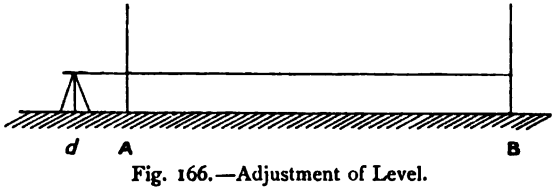


Fig. 166.—Adjustment of Level.

the staff reading at B *should* be. If now the actual staff reading at B does not agree with this, raise or depress the *whole instrument* by means of the levelling screws *s*, *s*, Fig. 104, until the reading at B is correct. Now bring the bubble to the centre of its run by means of the adjusting capstan screws at *e* connecting the level with the telescope. See now if the reading of the staff at A is altered, and if not, the adjustment is correct. If the reading at A is altered, the correct reading at B must be again computed from it, and the instrument made to read this by again depressing or raising it by the levelling screws *s*, *s*; the bubble being then again brought to the centre of its run by the screws at *e*, Fig. 104. Unless the level is very much out of adjustment, the alteration of

the level to the correct staff reading at B will not appreciably affect the staff reading at A, as the distance d_A is very much less than the distance d_B. In any case the *second* adjustment of the level to the proper staff reading at B will probably not affect the reading at A. The adjustment will therefore, as a rule, be effected at the second trial. When finally adjusted see that *both* staff readings at A and B are correct when the bubble is at the centre of its run. The adjusting of the bubble by the screws at *e*, Fig. 104, sometimes moves the level and alters the staff readings, and it is advisable therefore to try *both* staff readings after moving the screws at *e*.

3. **To make the Telescope and Spirit Level together Perpendicular to the Vertical Axis, or to make the Instrument "traverse,"** *i.e.*, so that the bubble remains in the centre of its run in every position of the telescope.

Bring the telescope over one pair of screws and level it up. Now turn it through 180° so as to reverse the ends of the telescope. If the bubble remains in the centre of its run the adjustment is correct; if not, correct *half* the deviation by means of the screws at *t*, *t*, Fig. 104, and *half* by means of the levelling screws *s*, *s*. Repeat the operation until the bubble remains in the centre of its run, when the telescope is reversed end for end. The instrument when levelled over both pairs of screws, or in the three screw instrument over one pair and then over the third screw, will then remain in the centre of its run in every position of the telescope. It is most generally useful to have this adjustment correct when a great many intermediate sights have to be taken, as in cross sectioning; but it is very liable to get out, and seldom remains for long correct. It does not, however, appreciably affect the accuracy of the work; it merely saves time in levelling up the instrument for each sight taken.

As to centring of object glass and eyepiece, horizontality of cross hairs, these adjustments are to be made as described for the theodolite, pages 209, 210.

Focussing the object glass and eyepiece, or adjustment for parallax, comes under the head of temporary adjustments, and are to be made as described for the theodolite, page 203.

Adjustment of the Y Level.—The Y level may be adjusted by twisting the telescope half round on its axis in the Y's, and noting whether the intersection of the cross hairs still continues

to coincide with the same object. If not, *half* the deviation is corrected by the diaphragm screws. This makes the collimation line coincide with the axis of the Y's, with which the axis of the telescope is supposed to coincide.

To adjust the bubble, level up the instrument and turn it end for end by lifting out the telescope and replacing it. If the bubble deviates from the centre of its run, half the deviation is corrected by the screws attaching the level to the telescope and half the deviation by the levelling screws. This makes the bubble parallel to the Y's, and therefore parallel to the collimation line if the collimation line has been previously adjusted to be parallel to the Y's. Any inequality in the size of the rings on the telescope on which it rests in the Y's will, however, affect the adjustment. See Precise Levelling "pivot correction," pages 192, 193, 195.

It is, however, much more accurate to adjust the Y level by the "peg methods" already described. The Y level is made to "traverse" as already described.

See also instructions for precise spirit levelling, pages 194 to 201, for adjustments of Y level.

CHAPTER V.

*RAILWAY SURVEYS AND SETTING OUT.**

Home and Foreign Work.—As railway surveys at home are conducted under very different conditions from similar surveys abroad, this chapter is confined to home surveys for railways and to the setting out and incidental work prior to and during construction, reserving for Chapter XI. railway surveys abroad, in jungle, dense forest, and unmapped country.

Parliamentary Surveys.—The preliminary or first survey for a railway is called the Parliamentary survey, and is so called because its object is the preparation of the plans, sections, and estimates for submission to Parliament in order to get its sanction to the proposed railway.

Ordnance Survey Maps.—On the Ordnance Survey maps of the United Kingdom it is possible to lay down a line between any two points with a very fair approximation to the best location. The 6 in. Ordnance maps are usually used for this purpose, as contours are given on them, while the 25 in. maps have no contours; the 6 in. scale also conforms to the Parliamentary Standing Orders as to scale of deposited plans.

Special Local Considerations.—In home work special local considerations to a very great extent determine the route of the line, which has often to be laid out through expensive cutting to avoid some local landowner's property, &c. &c., while detours have to be made to bring the line within easy reach of towns or villages on the route, or to avoid these when opposition is to be feared, and so on. These matters are usually to be considered in consultation with the promoters of the scheme and the local agent.

Ruling Gradient and Minimum Radius of Curve.—The next point is to fix upon the ruling gradient and minimum radius of curve. For these, see pages 230, 231, 232.

* Practically all this chapter also applies to Road Surveys and Setting Out. For surveys abroad see Chapter XI.

Laying down Line on 6 in. Ordnance Map.—It being supposed that these points have been duly considered and determined, the best procedure is to lay down the line on the 6 in. Ordnance sheets in pencil, as nearly as may be judged from the contours. The bench marks and levels along public roads which are given in the map are also of assistance in doing this.

In laying down the line on the map in this manner, we first fix upon the position and level of the starting point and a gradient, then by scaling the proper distance between adjacent contours we arrive at the position of the line. A good way is to calculate the distance required to rise 50 or 100 ft. on the proposed gradient. For instance, if the gradient is 1 in 50, on this we can rise 100 ft. in a distance of 5,000 ft. Take now 5,000 ft. in the compasses and fit them in between successive 100 ft. contours, marking the points where each contour is intersected. Theoretically the line should follow these points, but this will seldom be possible, and the best that can be done is to put in the curves so as to follow this line as nearly as possible and with as little cutting or embankment as may be. In the hilly parts of the country the contours are only given 100 ft. apart vertically, in flatter parts 50 ft. apart, and in the easiest country every 25 ft. When 25 or 50 ft. contours are given, the distance required to rise or fall 25 or 50 ft., as the case may be, is to be calculated, and the compasses stepped between successive contours, as above described.

Fixing Gradients.—The best gradient, when the ground permits of a flatter gradient than the ruling or maximum gradient, can only be found by trial, and as a rule will not be definitely fixed until the longitudinal section is plotted. For location purposes the gradient must be judged from the configuration of the ground as nearly as can be by estimation with the eye. This is therefore to a great extent a matter of practical experience and outdoor practice. In the first instance, however, the gradients to be tried must be found from a careful study of the map and the levels on it.

Reconnaissance and Sketching in Location on Map.—Having now the line thus laid down on the map, get the sheets pasted together, and cut into continuous rolls 15 or 18 in. wide, and as long as possible, or mounted so as to fold up like a pocket map. Taking this along, now walk carefully over the line and sketch in best location that will fit the ground as nearly as can be

judged, taking the pencil line already laid down as a basis. It is advisable to have a pocket scale and a pair of pencil bows and compasses; a hand level will often be useful on very steep ground for getting approximate differences of level. In locating a gradient up or down the steep hillside of a valley, a good plan is to first fix upon the valley or summit level, then calculate the levels of the gradient proposed to be run, and pencil them on the map at every 10 chains, or at fences approximately 10 chains apart. These figures may be pencilled on along the line already laid down on the map from the contours. The points on the ground where the surface level corresponds with these levels may be found by levelling from the nearest contour or other known level with the hand level or by judging with the eye. These points may then be fixed and marked on the map by measuring or stepping distances from fence corners, buildings, &c., shown on the map. A line joining these points is the line the railway should theoretically follow, and the straights and curves may be drawn on indoors so as to follow this line as nearly as possible.

Plotting Sketched Location.—Having thus gone carefully over the ground and sketched in the proper location by hand, taking care to use the compasses or pencil bows in the field so as to avoid sketching in curves of less than the minimum radius, next draw in the curves and straights to coincide with the line sketched in the field as nearly as possible, and mark on the chainage at every 10 chains along the line. It is also a good plan to scale the chainage of each fence crossed and figure it in pencil on the map, as this will save the leveller a considerable amount of time in taking the levels for the section.

Levelling the Longitudinal Section.—The next operation is to take levels over the line thus located. The leveller must be supplied with a map, with the line marked on it, and he takes levels at the points where the slope of the ground changes along the line, booking the chainage of each point where a level is taken. As a rule these levels will be taken at the points where the line intersects fences, and he will locate the exact position of the line on the ground by measuring the distance scaled off the map along the fences from the nearest corner or by directing the staff-holder to step so many paces from it. Points intermediate of fences will be fixed in like manner by measuring or pacing along the line from

the nearest fence or other object shown on the map. As already remarked, it will facilitate the levelling if the chainage of each fence crossed by the line is pencilled in figures on the map, as the leveller can then work to these chainages. The leveller should also be provided with a scale to scale distances or intermediate chainages on the map.

Owing to the limited time usually available for Parliamentary surveys, and as the location is by no means a final one, while the limits of deviation as a rule leave ample room for alteration in the final location, the location above described is usually all that can be done on a Parliamentary survey. The method of making a proper final location will be found described on pages 225, 226, 227.

Correcting the 6 in. Ordnance.—The limits of lateral deviation prescribed by the Parliamentary Regulations are 300 ft. on each side of the centre line of the railway in the country, and 30 ft. in towns. They are the limits within which the centre line of the railway may be altered when the detailed or working survey is made after Parliamentary sanction has been given to the scheme. The limits of vertical deviation are 5 ft. in the country and 2 ft. in towns. As the Parliamentary plans must show all details correctly within the limits of deviation, it is necessary to pencil on the limits of deviation as well as the centre line, so that any corrections necessary may be made. As the date of the Ordnance Survey map may be a considerable number of years prior to the date of the railway survey in hand, in some cases a good deal of labour is necessary to correct alterations to existing features, such as fences, &c., survey and plot on new buildings, &c. When there is not much alteration the leveller may be able to correct the Ordnance within the limits of deviation while taking the levels, but as a rule it is more expedient for him to devote certain days to this. If there is much alteration, and the time is limited, it will be necessary to have a special assistant to correct the Ordnance. All buildings included within the proposed limits of deviation must be surveyed and plotted to a scale of not less than 400 ft. to 1 in. They are usually plotted on the 25 in. Ordnance sheets if not already shown on these. These sheets should therefore be obtained for those parts of the line where buildings are to be included, and handed to the assistant whose duty it is to correct the map within the limits of deviation.

Referencing.—Every field, enclosure, building, road or path within the limits of deviation must be numbered on the plan and the numbers entered in a book of reference, together with owner or reputed owner, lessee or reputed lessee, and description, *i.e.*, whether arable, pasture, &c. All parish and county boundaries are also to be shown on the plan, as well as the names of all the parishes and counties through which the railway passes. The work of referencing is done by the solicitors to the promoters of the scheme, to whom should be supplied a set of 6 in. Ordnance sheets corrected to date and with the limits of deviation and the centre line of the railway marked on. It will as a rule be necessary for the engineer to send an assistant out with the solicitor's clerk to number and identify fields, enclosures, buildings, &c., on the map.

Preparation of Parliamentary Plans.—The Parliamentary plans are usually prepared by tracing from the 6 in. Ordnance on which the line has been laid down. They are made of a uniform size in sheets each containing 4 miles of the railway. The section of the part on each sheet is shown underneath the plan. Where it is intended to include within the limits of deviation any building, yard, courtyard, or land within the curtilage of any building, or any ground cultivated as a garden, an enlargement of this must be shown to a scale of not less than 400 ft. to 1 in. These enlargements, as already stated, are usually taken from the 25 in. Ordnance sheets. The limits of deviation are shown on the plan by a dotted line. Any enlargements are drawn under or above the general plan at the points where the buildings, &c., to which they refer are situated.

The tracings are handed to one of the lithographic firms which make a specialty of this class of work, and they are retraced by their draughtsman in specially prepared lithographic ink, and the figures and writing neatly printed on in the usual conventional styles of printing. As Parliamentary surveys are often only decided on at the last minute of available time, the engineer and surveyor have not as a rule time to spend in elaborate printing and figuring, &c. The description and figuring on the tracings are therefore simply written on in ordinary plain handwriting, to be copied by the lithographic draughtsman in neat printing. The lithographic tracing having been impressed on the stone, proof sheets are sent for correction, and the corrected lithographs are then prepared and

bound up in proper order. The engineer retains some copies for his own use and sends the remainder to the solicitors for the proposed railway, who deposit copies with the various local authorities &c., as specified in the statutory regulations.

Preparation of Parliamentary Sections.—The section is plotted in the usual manner, the gradients and gradient lines representing the surface of the rails being shown on it, to the same horizontal scale as the plan, and to a vertical scale of 100 ft. to 1 in., which conforms to the Standing Orders. The section must be referred to a known datum which is usually one of the Ordnance bench marks near one end of the line. The rail level at each change of gradient must be figured on, together with the distances in miles and furlongs, point of commencement and termination of each railway and branch, with total length of each in miles, furlongs, and chains. All public roads, railways, and canals which are crossed, how crossed, *i.e.*, whether by a level crossing or by a bridge, whether to be raised or lowered, and if so, how much, and the span and headway of each bridge, are also to be marked on the section. The extreme height of any embankment or depth of any cutting, where these exceed 5 ft., must also be figured on. Public roads crossed by level crossings must be levelled along for 200 yds. on each side of the point of crossing, and a section shown to an enlarged scale of not less than 5 chains to 1 in. horizontal and 40 ft. to 1 in. vertical. When many roads are thus crossed, they add very considerably to the leveller's work. Every public road the level of which is to be altered must have a cross section drawn to the same scales, and the greatest present and intended rates of inclination marked thereon in figures.

As already stated, the section of each 4 miles is shown on the same sheet as the general plan of that part of the line and underneath it. The sections of roads crossed are shown on a separate sheet or sheets placed at the end of the general plan and section. They are, however, sometimes placed above the general longitudinal section at the points where they occur.

Another point to be noted is that when a junction is intended with any existing or authorised railway, a plan and section of the existing railway must be shown for 800 yds. on each side of the point of junction, to the same scales as the general plan and section.

Parliamentary Estimate.—An estimate of the probable cost of the proposed railway has to be made out in a form prescribed by the Board of Trade. This estimate must state the quantities and cost of the various items; cuttings, embankments, bridges, accommodation works, viaducts, culverts and drains, metalling of roads, permanent way and fencing, sidings, junctions, stations, contingencies, land and buildings, &c.

Board of Trade Regulations for Light Railways.—Of late years, in order to avoid the expensive procedure necessary to get a Bill passed by Parliament authorising the construction of a railway, the Light Railways Act has been passed, and the Board of Trade in September 1896 issued their "Rules with respect to applications to the Light Railway Commissioners for orders authorising Light Railways." The engineer should provide himself with a copy of the latest rules, which may be had on application.

Under the Light Railways Act it is not necessary to go to Parliament for sanction to construct a railway if it is made to come under the regulations of the Light Railways Act which chiefly affect the signalling and working of the line. The scheme is considered by the Light Railway Commissioners, at an inquiry held locally, at which the engineer must attend and give evidence. If sanctioned by them, it is then passed on to the Board of Trade for final approval. As far as the engineer or surveyor is concerned, his work is much the same whether the line is an ordinary railway or a light railway.*

Example of Parliamentary Plan and Section of Railway taken from actual practice.—Figs. 167, 168, Plate VIII., are examples of the Parliamentary plan and section of a railway actually constructed and surveyed as already described in this chapter.

Working Surveys.—The Bill having been passed, or the sanction of the Light Railway Commissioners and Board of Trade having been obtained, the engineer is forthwith (let us hope) instructed by the promoters to proceed with the detailed or working survey.

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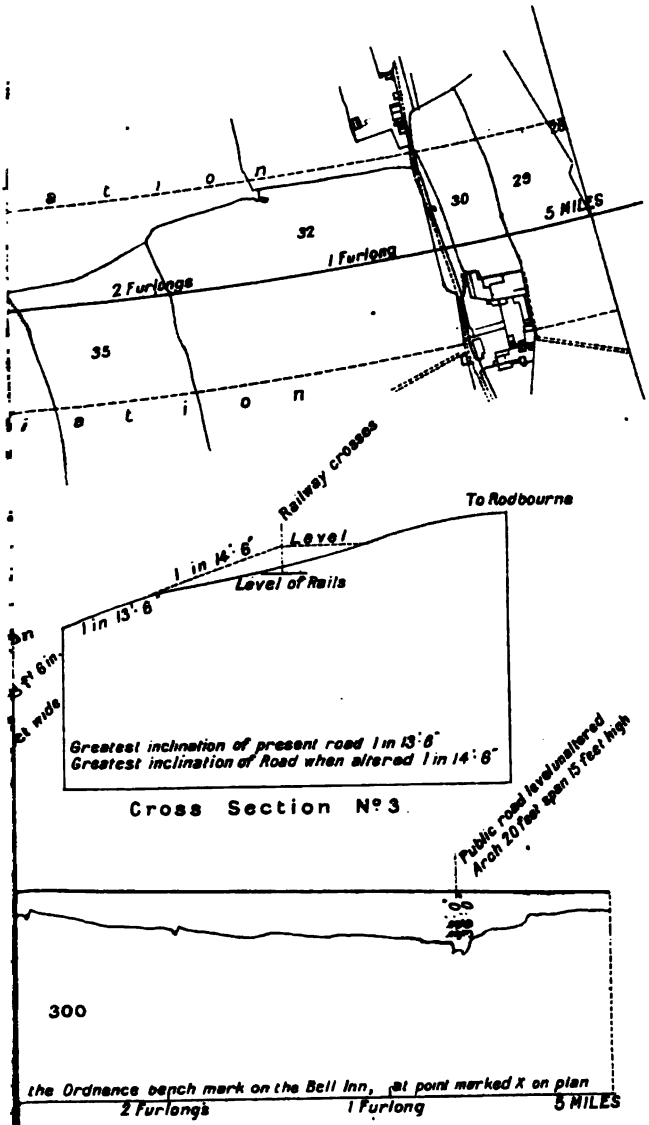
* A copy of the Parliamentary Regulations for Ordinary Railways should be obtained, which may also be had on application.


30
29
5 MILES
32
1 Furlong
2 Furlongs
35
Railway crosses
To Rodbourne
Level
1 in 14·6
Level of Rails
1 in 13·6
Greatest inclination of present road 1 in 13·6
Greatest inclination of Road when altered 1 in 14·6
Cross Section Nº 3
Public road level unaltered
Arch 20 feet span 15 feet high
300
the Ordnance bench mark on the Bell Inn, at point marked X on plan
2 Furlongs
1 Furlong
5 MILES

[*To face p.* 224.

Revising and Improving Parliamentary Location.—The working survey is usually conducted on the 25 in. Ordnance scale, on the sheets of which the Parliamentary line and the limits of deviation should now be pencilled. A careful examination of the Parliamentary section and plan and a comparison of the ground with the larger 25 in. scale map will usually result in various modifications of the line presenting themselves, by means of which cuttings and embankments may be reduced.

As a rule, it will be possible to decide definitely upon these merely by inspection of the ground, but in difficult country it may be necessary to take levels and cross sections and plot one or more longitudinal sections of the proposed alterations before fixing upon the final line.

Final Location of Line in Difficult Country.—The ruling gradient and minimum radius of curve have of course been already fixed on the Parliamentary survey and plans, and the method of locating the line in difficult country is as follows:—Take levels over the Parliamentary line laid down on the map and leave marks where cross sections are to be taken. The points where cross sections are to be taken will be determined by the general configuration of the ground. They should be such that the surface between adjacent cross sections is approximately plane, or roughly speaking, cross sections should be taken wherever the slope of the ground changes. For methods of cross sectioning see Chapter III. As a rule, the clinometer is the most rapid to take cross sections with. Having taken and plotted the cross sections as shown in Fig. 169, the contours are to be taken off them and laid down on the plan as follows:—Draw a vertical AB to represent the centre line of the railway as laid down on the map. Suppose the reduced level of the surface of the ground at the centre line of the railway is 102.80, and that contours are required 5 ft. apart vertically. Then the first contour above the centre line of the railway will be the 105 ft. contour, *i.e.*, the contour whose reduced level is 105.00 and 105.00 – 102.80 = 2.20. Plot up, therefore, 2.20 ft., and *ab*, *bc*, *cd*, each equal to 5 ft. Similarly the first contour below the centre line of the railway will be the 100 ft. contour and 102.80 – 100.00 = 2.80. Plot down, therefore, 2.80 ft., and *ef*, *fg*, *gh*, each = 5 ft. The points *h*, *g*, *f*, *e*, *a*, *b*, *c*, *d*, will then be at the reduced levels 85.00, 90.00, 95.00,

100.00, 105.00, 110.00, 115.00, 120.00 ft. respectively, and the horizontal lines *hs*, *gr*, *fq*, *ep*, *ak*, *bl*, *cm*, *dn*, will give the horizontal distances from the centre line of the railway to the 85, 90, 95, &c., contours. These horizontal distances are to be measured off the centre line of the railway on the plan, and will give the position of the contours at that point.

Each cross section is similarly treated, and the positions of the contours plotted on the plan. By joining these corresponding points on the plan we then get the contours of the surface of the ground. A quicker method is given in Chapter III., graphic interpolation of contours, page 154.

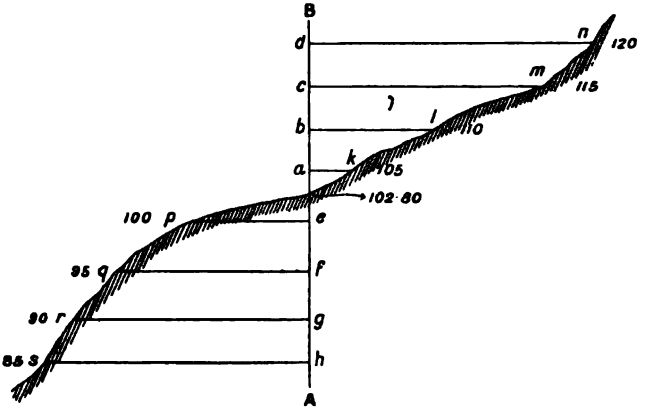


Fig. 169.—Position of Contours from Cross Section.

If the scale of the general plan is too small to plot contours 5 ft. apart vertically, the centre line of that portion of the railway to be cross sectioned and contoured should be laid down on a larger scale on a separate sheet and the contours plotted on it.

Having then decided on a trial gradient, calculate the distance in which the rise or fall of the gradient is 5 ft. if the contours are 5 ft. apart vertically; if 10 ft. apart, the distance for a rise or fall of 10 ft. Take now this distance in the compasses, and step from contour to contour, marking the points where the compass touches each contour. A line joining these points is then the line on which there is neither cutting nor bank, and is called a "surface line." The proper location of the railway is then the line which can be

drawn most closely to agree with this line, and as a rule the smaller the minimum radius of curve the closer will the location agree with the "surface line." A longitudinal section of this proposed location may now be plotted from the contours and examined with a view to adoption or otherwise. If not satisfactory, a new line with cross sections on it may have to be surveyed and the operation repeated until a satisfactory location is obtained.

It is perhaps unnecessary to say that all alterations must be within the limits of deviation as marked on the Parliamentary plans.

As regards sharpening curves and steepening gradients, there are certain specified limits which must not be exceeded, for which see a copy of the regulations. These matters must of course be strictly attended to, as before the inspection or opening of the railway plans and sections of the final and Parliamentary lines must be handed to the Board of Trade for approval. In certain cases, if sufficient cause can be shown, exception may be made to the regulations under special circumstances.

Pegging out Line.—The line having been definitely fixed in this way and laid down on the 25 in. Ordnance sheets, these should be pasted together and mounted, and cut up into continuous rolls convenient for use in the field, or a tracing on linen made for field use. The next operation is now to set out the line by driving in pegs 1 chain or 66 ft. apart along the centre line. The field details of this operation are entered into later (see page 233 *et seq.*).

Pegs.—The pegs should be set out with the theodolite, and should be about 1½ to 2 in. square, and 15 to 18 in. long. At every 10 chains an index peg should be driven in slantwise, slightly to the right hand of the centre line peg, with the chainage marked on it in miles and chains with a blue pencil.

Tangent Points.—Tangent points should be distinguished by pegs driven in slantwise, one on each side of the centre line peg, and to right and left of it.

Chainages and Survey of Existing Detail.—The chainage of all roads, ditches, fences, &c., crossed by the railway must be carefully taken and entered in the field book. Any alterations to fences, boundaries, new buildings, &c., must be carefully sur-

veyed, and the 25 in. map in fact corrected and brought up to date. The survey of existing features should extend for about a chain beyond the probable position of the railway fence as nearly as that can be judged. This may be estimated approximately from the probable height of bank or depth of cutting.

When important works, such as large bridges, &c., occur, it may be necessary to survey the road or river, &c., crossed for some distance on each side of the railway.

Longitudinal Section.—The line having been pegged out, a careful section is next taken along it, levels being taken at every chain peg and also at any intermediate points where the slope of the ground changes.

Cross Sections.—Cross sections are also to be taken wherever the ground slopes at right angles to the centre line, or where a section at right angles to the centre line is rough and broken.

Working Plan.—The working plan is prepared by tracing from the 25 in. Ordnance on which the line was laid down or by replotting from it and the field notes. The chainage should be marked at every chain and numbered at every 10 chains. The radii of the curves, the tangent points and their chainage, the intersection points, the lengths of the tangents, and the intersection angles of the curves, are also to be marked on the plan, together with span, width, height and description of bridges, number of the drawing which shows the details of the bridge, the level crossings, road and stream diversions, accommodation works, culverts, pipes, &c. &c.

The fence lines should also be shown as well as the lines representing the tops and bottoms of the slopes, and the space between the latter, *i.e.*, the excavation or embankment, should be coloured red.

The centre line, fence lines, tops and bottoms of slopes, and in fact all new works, are to be in red ink, all other details existing on the ground being in black ink.

Working Section.—The section is usually plotted to the same horizontal scale as the plan, viz., $\frac{1}{2500}$; the vertical scale is then usually 30 ft. to 1 in. The levels are invariably referred to Ordnance datum, and the datum line may be plotted at any convenient height above or below Ordnance datum, it being so stated on the section.

The line representing the *formation* of the railway, *i.e.*, the bottoms of the cuttings or the tops of the banks, or the "road bed," is drawn on the section in red ink, the surface line being in black ink, as also the datum line. Blue vertical lines are drawn upwards from the datum line, extending as far as the surface or formation line, as the case may be, at every chain, and every 10 chains is numbered along the datum line. All chainages should be in miles and chains.

Fixing Gradients on Working Section.—The gradient line will be limited by the ruling gradient, and the gradients on the Parliamentary plans together with the limits of vertical deviation. It is drawn on the section so as to equalise the cuttings and embankments as much as possible, consistently with at the same time getting economical and uniform working gradients. A good method of fixing the gradient line is to use a piece of fine thread stretched tight, moving it up or down until the cuttings and banks are as nearly as may be judged by the eye equalised. Transparent set squares are also useful for this. In laying down the gradients it should be remembered that the slopes of the embankments will generally be much flatter than the slopes of the cuttings, and that the banks are wider at the top than the formation widths of the cuttings. Thus a rock cutting may have slopes of $\frac{1}{4}$ to 1 or nearly vertical, while the adjacent embankment may have slopes of $1\frac{1}{2}$ to 1, and this is to be allowed for in equalising cuttings and banks on the section.

It should also be remembered that there is shrinkage to be allowed for in the case of earth embankments, while rock expands in bulk when put into bank.

Levels on Working Section.—Having thus decided upon the gradient line, which will be considerably influenced by the necessity of crossing roads on the level and by the headroom required for bridges, the levels at each change of gradient are next to be figured on near the datum line, and the gradients calculated out and printed on in red ink, as 1 in 100, 1 in 50, and so on. The changes of gradient should as far as possible be selected at even chains, and should be indicated by *red* in place of *blue* vertical lines ruled up from the datum line.

The level of the surface of the ground at each chain should be figured on alongside the vertical blue line at the chain to which

it refers, *next* to the datum line and in *black* ink; the level of the formation of the railway calculated from the gradient should be figured on at each chain next above the ground level, and in *red* ink; finally, the difference between the two, being the depth of cutting or height of bank, as the case may be, should be figured above the formation level, in red ink when cutting, and in blue ink when bank.

Cuttings and Banks on Working Section. — The spaces included between the ground line and the formation line of the railway, being the cuttings or banks as the case may be, should be coloured red.

A blue line representing the surface of the rails is also often drawn on parallel to the formation line.

New Works and Existing Details on Working Section.—All new works, such as bridges, culverts, pipes, &c., should be described in writing above the section at the points where they occur in *red* ink, all existing details, such as roads, streams, &c., being similarly described in *black* ink.

The setting out and the preparation of the general plan and section as above described may be said to complete the surveying work proper, the preparation of the detail working drawings of bridges, culverts, accommodation works, &c. &c., the specification and bill of quantities, coming under the head of the engineer's work.

Examples of Working Plan and Working Section of Railway taken from actual practice.—Figs. 170 and 171, Plates IX. and X., are examples of the general working plan and working section of a railway actually constructed, executed as described in this chapter.

Ruling Gradients.—The ruling gradient is the steepest gradient which occurs oftenest on the line. The considerations which fix the ruling gradient are—

1. *Train load.*
2. *Speed.*
3. *Power and weight of locomotive and train resistance.*

1. *Train Load.*—The train load must be fixed from local considerations of traffic, extent of rolling stock, &c. Ordinary 10 ton coal waggons, built for use over main lines to railway

companies' regulations, will weigh about 6 tons each empty. Composite six compartment passenger carriages will weigh about 12 tons each empty.

2. *Speed.*—The most economical speed for goods is 20 miles per hour.

3. *Locomotive Power and Train Resistance.* — The tractive power of a locomotive is given by—

$$T = \frac{D^2PL}{W} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where D = diameter of cylinder in inches.
P = mean pressure of steam in cylinders in lbs. per square inch.
L = length of stroke in inches.
W = diameter of driving wheel in inches.
T = tractive force in lbs.

The tractive power is, however, limited by the adhesion of the locomotive drivers to the rails. In ordinary English weather the adhesion of a locomotive may be taken at 450 lbs. per ton of weight on drivers, or about one-fifth. When the tractive power developed by the cylinders exceeds the adhesion, the adhesion must be taken as the tractive power. In addition to the tractive power the boiler capacity is a dominant factor in determining the working load which a locomotive will take.

The resistance of a train on the level may be found from the following formulæ:—

$$R = T\,(6 + .33V) + .0025V^2A \quad \ldots\ldots\ldots\ldots (2)$$
$$\text{or } R = T\,(6 + .067V) + .00002V^2B \quad \ldots\ldots\ldots\ldots (3)$$

where T = weight of train in tons; V = velocity in miles per hour; A = area of frontage of train in square feet; B = volume of train in cubic feet; R = resistance in lbs. on a level. As compared with experiment, equations (2) and (3) give results too high for low speeds.

The average train resistance may be taken at about 10 lbs. per ton at about 20 miles per hour, on the level.

In addition to the above train resistance on the level, the resistance due to gravity in ascending an incline must be calculated upon. Let G = gradient, so that $\frac{1}{G}$ = inclination of gradient,

i.e., for 1 in 60 $\frac{1}{G} = \frac{1}{60}$, and let W = weight of train. Then the component of the weight of the train due to gravity may be taken as—

$$W \times \frac{1}{G} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

which is the resistance due to gravity to be overcome in ascending an incline *in addition* to the resistance of the train on the level for the given speed.

By equating the train resistance on the level for the proposed speed plus the resistance due to gravity in ascending a gradient in terms of G, equations (2) or (3) and (4), with the tractive power of the locomotive proposed to be employed and solving for G the ruling gradient may be found, or if the ruling gradient is fixed the tractive power of locomotive required may be found. In the latter case sufficient boiler capacity to take the maximum load up the longest steepest gradient at the proposed speed must be stipulated for.

Maximum Gradient.—1 in 30 is about the steepest gradient that may be worked under usual conditions with ordinary locomotives, and this may be taken as an exceptionally steep gradient.

"Pusher" Gradients.—In some cases it may be advisable to adopt different ruling gradients on different portions of the line, working the steeper gradients or "pusher" grades with auxiliary or special engines, or adopting one of the rack systems on the steep sections. This may be adopted on lines on which there are "valley" sections and "hill" sections.

Minimum Radius of Curve.—As regards minimum radius of curve, 10 chains radius is about the sharpest curve that will be used on main line 4 ft. 8½ in. gauge. Rolling stock will go round curves of 5 chains radius and even less in station yards and sidings; 3 ft. gauge lines have been constructed with curves of 145 ft. radius, or 2 chains 13 ft.

Compensating Gradients on Curves.—When gradients occur on sharp curves they should be compensated so as to make the joint resistance due to curve and gradient equal to the resistance due to gradient alone on the straight. This is effected by reducing the gradient where curves occur. The resistance of a 1° curve is estimated as being equivalent to the resistance due

to a gradient of from .025 to .06 ft. per 100 ft. As an average it may be taken as equivalent to a gradient of .04 ft. per 100 ft. For a 1° curve, therefore, the gradient should be reduced by .04 ft. per 100 ft. For any other curve the resistance is proportional to the degree of curve, *i.e.*, for a 6° curve the resistance is $6 \times .04 = .24$ ft. per 100 ft. to be deducted from the gradient. The degree of a curve is the angle subtended at the centre by a 100 ft. chord, *i.e.*, twice the deflection angle of the curve for a 100 ft. chord.

Cost of Line, Light Railway.—£2,700 per mile is about the very lowest figure for which a single line of light railway can be constructed at home, and this in exceptionally easy country, not inclusive of cost of land and not including rolling stock. £3,000 or £4,000 per mile will be a more usual figure.

A single line, capable of handling 1,000 tons of freight per day, can usually be built anywhere in moderately easy country for £4,000 or £5,000 per mile, including equipment and rolling stock. This does not apply to lines abroad neither of whose termini are in connection with a seaport. Mr R. C. Rapier, of Messrs Ransomes & Rapier, in "Remunerative Railways," estimates the equipment of 40 miles of metre gauge single line at £86,708, or £2,168 per mile. This includes 40 lb. rails, wooden sleepers, seven engines 15 tons each, turntables, tanks, water-cranes, weighbridges, sheer legs, signals, 35 passenger carriages and break vans, 150 waggons, workshop fittings and stores.

In another estimate for 40 miles of 3 ft. 6 in. gauge line, Mr Rapier gives the cost of equipment at £2,471 for 45 lb. rails and eight 18-ton engines. As regards the cost of ordinary double line railways, the cost will vary so much with the locality and special circumstances of each case that no general figures can be given.

Details of the Field Work of Working Survey and Pegging out: Instruments.—With reference to the actual field work of the working survey and pegging out, first as regards instruments, the most convenient size of theodolite is a 5 or 6 in. theodolite, which, for the purposes of an ordinary railway survey, need not read closer than to single minutes. American instruments for ordinary railway purposes only read to single minutes, and the angles are read off without the aid of a microscope, a piece of white ivory or celluloid being fixed above the vernier

to reflect the light on it. A 6 in. theodolite of English make usually reads to 20″, and is furnished with microscopes. This is of course useful for accurate work, but it takes more time to read than the American instrument reading to single minutes only, which is really all that is required for ordinary railway purposes.

Steel Band and Tapes.—For chaining out the line the ordinary steel chain 66 ft. long or steel band is used, the latter being the best for chaining out the centre line of a railway. Ten or more of the usual steel chaining arrows should be carried. A tape is also required for measurements from fence corners, buildings, &c., for fixing the position of the line to be run, odd measurements, &c.

Ranging Rods, &c.—Two or three iron-shod ranging poles 10 or 12 ft. long are required for distant points in long lines, and about a dozen ordinary ranging rods.

For driving in the pegs a stout wooden mallet is best, as it does not split and break the heads of the pegs so much as an iron hammer. A couple of small plumb bobs and a stout cord line 1 or 2 chains long, together with some billhooks or knives for cutting through hedges and a small hand hammer, should be carried. It will be found expedient to have hedgers' gloves for the men when quickthorn hedges have to be cut through. An axe will occasionally be required to cut down small trees. One or two stout canvas bags should be provided for carrying these things.

Chainmen.—As regards chainmen, it will be found expedient to have two men for chaining and one for carrying pegs, fetching back flags forward, &c. Three men will usually be sufficient.

Pegging out Centre Line.—The surveyor commences operations by locating the beginning and the end of the first straight line, and fixes its position on the ground by ranging rods. This he will do by scaling off the 25 in. Ordnance map and measurement on the ground from existing objects, such as fences, buildings, &c., unless the exact position is indicated by some object intersected by the line. Having driven in the first peg at the commencement of the first line, set up the theodolite over it, level it up, and direct the cross hairs on to the ranging rod at the extremity of the line, and clamp the instrument firmly in this position. One end of the chain being held on the first peg, the leading chainman now holds a ranging rod at the other extremity

of the chain, and moves it to right or left as directed until it is bisected by the cross hairs, when he inserts the rod into the ground and then drives in a peg at the mark thus made. The surveyor will direct the peg to be knocked to right or left until it is bisected by the cross hairs and is correctly in line, and the leading chainman should then apply the end of the chain to the top of the peg and make a scratch on it along the chain handle with a chaining pin. Very often each peg is merely driven so that its centre is at the end of the chain as nearly as may be. In a long line it will be necessary to shift the instrument forward about every 15 chains or so, otherwise the pegs will get out of line, as the ranging rod and the pegs cannot be seen with sufficient distinctness to get them exactly into line at greater distances. In this case, by a signal prearranged with the leading chainman, the surveyor will range in a chaining pin held on the top of the peg, with which a point mark is made on it, and over which the theodolite may be set up. In this work the leading chainman should be the more intelligent of the two.

Chainages and Survey of Detail.—The chainage of each road, fence, ditch, &c., intersected should be taken and booked, and any alterations from the 25 in. map in existing fences, new buildings, &c., must be surveyed. The box sextant will sometimes come in useful for the purpose of measuring the angles at which fences, &c., cross.

The chainages of roads, fences, ditches, and survey of alteration to existing detail may be taken by recalling the chainmen prior to moving the theodolite forward, and sending one of them forward with it, then with the assistance of the others the fence chainages, &c., are booked, and any details which require to be surveyed are taken. It is a good rule to take everything required as the work proceeds, as going over the ground again leads to loss of time. When, however, surveys of roads, rivers, &c., for some considerable distance on either side of the railway are required, a special survey of these must be made. One or more assistant surveyors should be a help and ought to expedite the work, but it is quite easy for one man to do all that is required if the time permit.

Bridges.—When roads or rivers are crossed on the skew and a bridge is contemplated, the proper angle of skew for the bridge,

which will best suit the ground, should be measured on the ground with the theodolite, and a correct survey of the road or river for some distance on each side of the railway should be made. Levels and cross sections along the road will also be very useful in designing the bridge.

Setting out Curves.*—When the pegs have been driven nearly to the end of the first line, scale off the position of the

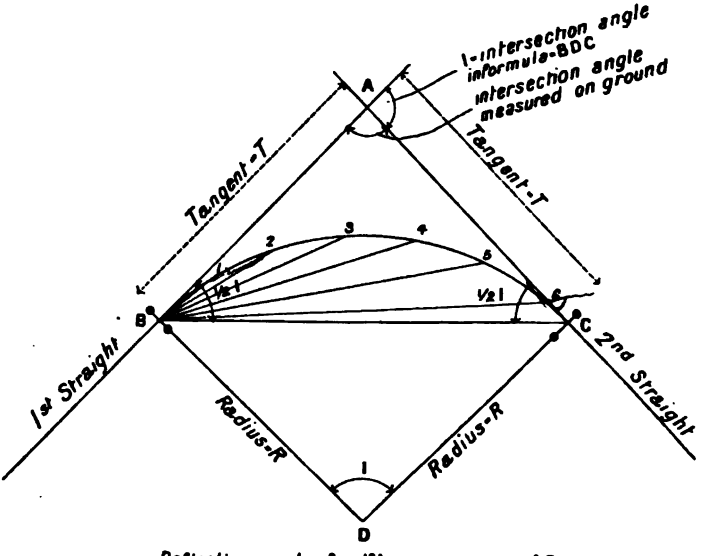


Fig. 172.—Setting out Curves.

tangent point of the first curve as nearly as may be from the 25 in. map on which the line is laid down, and direct the chainman to stop driving pegs when he gets to within a chain or two of where the tangent point will be, and continue the chaining with chaining pins until he gets a chain or two beyond the intersection point, lining in the chaining pins with the theodolite and leaving

* For classification of curves by "degree," see Chapter XI.

them in. Now fix the second straight by a ranging rod or pole near each extremity. Set up the theodolite over the nearest of these points and line in two chaining pins on this line, one on each side of the first line, and as near as possible to it. The intersection point may now be found by stretching a cord line between two of the pins on the first line and another cord between the two pins ranged in on the second line. The required point will then be the intersection of the two cord lines, at which a peg may be driven. The theodolite is then set up over this peg and the intersection angle of the curve measured. Do not forget always to leave a back flag for this purpose at the commencement of the first line, and if there is a sufficient supply of ranging rods, the back flags may be left in and collected by the chainmen as they go out to their work in the morning, thus saving the time required to send a man a long distance back.

Calculation of Length of Tangents.—Having measured the intersection angle, now calculate the length of the tangent which is given by

$$T = R \tan \tfrac{1}{2} I$$

where T = tangent
R = radius
I = intersection angle

Note that this intersection angle is the angle subtended at the centre of the curve, and that it is 180° minus the angle measured on the ground (see Fig. 172). R should be in chains, then T will be in chains and decimals.

Having the theodolite still set up over the intersection point, now direct the chainmen to measure out the length of the tangent from the intersection point, measuring the second tangent first. This is done with chaining pins, and three pegs are driven at each tangent point, each tangent point peg being carefully lined in with the theodolite.

Shift the theodolite to the first tangent point, leaving a ranging rod at the second tangent point, and proceed to set out the curve. Before doing this the deflection angle for each chain must be calculated, as also the total length of the curve.

Calculation of Deflection Angles.—The deflection angle for 1 chain for the given radius of curve is usually taken from

one of the published tables of deflection angles of curves. If there is not one to hand, it may be calculated as follows:—

$$\text{Sin of deflection angle for 1 chain} = \frac{1}{2 \text{ radius in chains}}$$

The deflection angle corresponding to this value of the sine may then be found by looking up a table of sines. This is not strictly correct, although it is generally near enough, as it assumes that the chord of an arc of 1 chain is also 1 chain in length.

Other formulæ are—

$$\text{Deflection angle for 1 chain in degrees} = \frac{28.65}{\text{radius in chains}}$$

$$\text{or deflection angle for 1 chain in minutes} = \frac{1719}{\text{radius in chains}}$$

The two last formulæ are arrived at by taking the circular measure of the deflection angle for 1 chain, which is

$$\tfrac{1}{2} \text{ chain} \div \text{radius in chains, or } \frac{1}{2 \text{ radius in chains}}$$

and multiplying by $\frac{180^\circ}{3.1416}$ which converts the circular measure into degrees; this again multiplied by 60 gives minutes or the second formula.

Exact Formula for Calculation of Deflection Angles.—The exact deflection angle of any arc $= \frac{\text{arc}}{2 \text{ radius}}$ in circular measure; this converted into degrees, minutes, and seconds gives the deflection angle to any required degree of accuracy. In accurate work, as for instance curves in tunnels, this formula must be used, and in the case of sharp curves it will also be necessary to calculate the length of the chord for any given arc; thus the chord of a 1 chain arc on a sharp curve may be 65 ft. 11 in. and an odd fraction of an inch.

Calculation of Length of Curve.—Next to get the length of the curve in chains, divide half the intersection angle by the deflection angle for 1 chain. For more accurate work reduce the angle subtended at the centre of the curve to circular measure and multiply it by the radius.

Chainage of Tangent Points and Deflection Angle for each Peg on Curve.—Now put in the pegs on the first straight

up to the first tangent point, and measure the distance from the last peg to the tangent point, which gives the chainage of the first tangent point. Add to this the length of the curve and we get the chainage of the second tangent point, *i.e.*, the end of the curve.

The next step is now to note down the deflection angles for each chain round the curve. As the first tangent point or the beginning of the curve is not generally at an even chain, the deflection angle for the first peg on the curve is usually some fraction of the deflection angle for 1 chain. For instance, if the first tangent point is at 0 miles 25.28 chains, then the distance from it to the first peg on the curve at 0 miles 26 chains is 1 chain – .28 chain = .72 chain or 72 links, and the deflection angle for this peg is therefore .72 of the deflection angle for 1 chain. Add now to this angle the deflection angle for 1 chain and we get the deflection angle for the second peg on the curve, and if we add again the deflection angle for 1 chain we get the deflection angle for the third peg on the curve, and so on to the last peg on the curve.

Check on Calculation of Deflection Angles.—As a check on the calculation, add to the deflection angle for the last peg the fraction of the deflection angle for 1 chain corresponding to the distance from the last peg to the second tangent point or the end of the curve, and we get the deflection angle for the second tangent point, or in other words, the deflection angle for the whole curve, and this should be equal to half the intersection angle, or at all events within a few seconds of it, the difference being due to neglect of one or two units, as the case may be, in the last decimal place to which the deflection angle for 1 chain has been worked out.

As each deflection angle is calculated it should be entered in the field book opposite the proper chainage, and each angle should be checked off in the field book as it is set off with the theodolite, to avoid confusion.

Laying off the Deflection Angles and Putting in Pegs on Curve.—To set out the curve, the theodolite being set up over the first tangent point and the vernier set to zero, direct the cross hairs on to the back flag at the beginning of the first straight, and reverse the telescope and see whether the cross hairs also bisect the intersection point. Now set the vernier to the deflection angle for the whole curve, and see whether the

cross hairs also bisect the second tangent point; if so, the work is so far correct. Now set the vernier to the deflection angle for the first peg on the curve and drive that peg, then lay off the deflection angle for the second peg, and so on until all the pegs have been driven.

Check on Laying off the Deflection Angles.—Now lay off the deflection angle for the whole curve again; the cross hairs should again bisect the second tangent point, otherwise the instrument has been shifted or some slip has been made.

Check on Chaining Round Curve.—Measure now from the last peg to the second tangent point or the end of the curve. This measurement should agree with the chainage of the second tangent point, as calculated from the length of the curve, otherwise the work is not correct, either the chaining of the tangents or the setting out or chaining round the curve being at fault, provided that there is no error in the calculations.

In practice, with fairly good chaining and on ordinary ground, the error in chainage should not be more than half a link on a long curve.

Final Error in Setting out Curve.—When the deflection angle for the whole curve is set off the cross hairs should not be more than 1 or 2 in. at the outside off the second tangent point. In a long curve, in which the theodolite has to be shifted several times, 1 or 2 in. of error is excusable, and may be adjusted by altering slightly the last three or four pegs on the curve.*

The vernier should now be turned back again to zero, when the cross hairs should again bisect the intersection point and the back flag.

Continuing Work after Setting out Curve.—To continue the work, set up the theodolite over the second tangent point, and direct the cross hairs on to the flag ahead on the second straight, turn the telescope over vertically and set off the deflection angle for the whole curve. The cross hairs should now bisect the first tangent point. This is as a check on the tangency of the second straight to the curve set out. The cross hairs are now

* These remarks as regards error apply only to field work of ordinary railways. In the case of curves in tunnels, &c., much greater accuracy is obtained by more accurate measurements, and by adopting special precautions in alignment (see Chapter VII.).

again directed on to the flag at the extremity of the second straight, the pegs on which are lined in at every chain in the same way as on the first straight until the second curve is reached, which is treated in exactly the same way as the first curve already described.

Chaining to be "through."—It is perhaps unnecessary to say that the chaining must be *through* or continuous from the commencement of the railway to its termination, the tangent points being put in at whatever odd points on the chainage they may occur, but the chain pegs must follow each other consecutively 1 chain apart whether tangent points intervene or not. In fact, the first peg on the curve may be measured, not from the tangent point, but from the last even chain peg behind it.

The formulæ already given are all that are necessary to set out a curve, but the surveyor should have a thorough knowledge of trigonometry and the properties of circular curves, as many problems may arise, such as inaccessible intersection or tangent points, obstacles in the curve itself, &c. &c. For the many field problems of this nature the reader is referred to one of the various engineers' field books which are devoted to these matters, and which usually contain in addition concise tables of logarithms, logarithmic sines and tangents, natural sines and tangents, deflection angles for curves, &c. &c. Fig. 172 will, however, illustrate the ordinary process of setting out and the principal properties of a circular curve required.

Shifting the Instrument forward to Intermediate Point on Curve.—When the curve is long and obstacles intervene so that the second tangent point cannot be seen from the first tangent point, we cannot check the accuracy of the work before beginning to set the pegs on the curve, by laying off the deflection angle for the whole curve to see whether we strike the second tangent point correctly. The theodolite has also to be shifted forward along the curve as the work proceeds, but the deflection angle for the whole curve should always be laid off as soon as the end of the curve is visible, to see that no error has occurred. As a rule, on a long curve the theodolite should be moved forward about every 10 chains, otherwise the pegs will get out of line, especially on a sharp curve.

When it is necessary to shift the instrument forward the process

is as follows:—Having moved the theodolite forward, look back on the first tangent point and set off the deflection angle for the next chain peg, which is the total deflection angle for that peg, then proceed as before, laying off the angle for each peg in succession, all as originally calculated and entered in the field book opposite the chainage of each peg. This virtually amounts to laying out a tangent to the curve at the point at which the instrument is set up, and from that tangent laying off the deflection angle for the next chain ahead, only of course we do not stop to actually lay out the tangent to the curve, but continue to turn the instrument round to the angle of the next peg; in other words, for the first peg beyond the point to which the theodolite is shifted we have to lay off so much deflection angle *behind* for the part of the curve behind us plus the deflection angle for 1 chain *forward*. Beyond that we have to increase the angle by the deflection angle for 1 chain for each peg in succession, all of which is already calculated and entered in the field book, as originally directed.

When it is necessary to move the theodolite forward a second time we have to look back on the furthest back point of the curve which it is possible to see. Suppose this is 10 chains back, then for the next peg we have to lay off the deflection angle for 11 chains, *i.e.*, 10 chains behind and 1 chain in front. We have then to add to this the deflection angle for 1 chain for each peg in succession. It will be seen that in this case fresh calculation of the deflection angles is necessary, these being no longer continuous round the whole curve; but the angles which have to be calculated are merely the deflection angles for even numbers of chains, with the exception of the fraction of a chain from the last chain peg on the curve to the second tangent point.

Curve to Left.—When the curve curves to the right hand the deflection angles are laid off in a right-handed direction round the graduations of the theodolite. When, however, the curve curves to the left hand the angles have to be laid off in a left-handed direction or contrary to the way in which the theodolite is graduated. In this case we must either subtract each deflection angle from 360° and set the vernier to that angle, or we must use the vernier backwards and read off each angle directly. To do this, set the *end* of the vernier to 360°, not the zero as is usual, the zero being regarded as the end of the vernier for the time being. The degrees

are then counted off in the left-handed direction, for instance 10° will be at 350°, and the minutes are read off the vernier in its reversed position. With a little practice it is almost as easy to use the vernier one way as the other.

Transition Curves.—The object of a transition curve is to make a gradual change from a straight line to a curve. As on every curve the rails must have a certain amount of "cant" to counteract the centrifugal force of the train, the transition curve also serves to put the cant on the rails gradually. In addition to this there is the "widening of the gauge on curves"; this also may be gradually done on the transition curve. In the absence of transition curves both the cant and the widening of the gauge are usually effected on the straight, so that there is the full cant and extra width of gauge at the commencement and throughout the whole of the curve. It is obviously a very incorrect proceeding to put cant and extra width of gauge on a straight, while equally obviously the change from a straight to a curve ought to be made gradually, the radius of curvature decreasing from infinity at the commencement of the curve of adjustment to the same radius as the circular curve at its termination.

For these reasons it is the universal practice to put in curves of adjustment on the Continent, and the practice is also general in the United States and South America. Of late years they have been adopted to a small extent by English engineers in home work, and their universal use on railways in this country will no doubt not be long delayed.

Various transition curves have been proposed, among them being the cubic parabola and the compound transition curve. The latter is formed of successive circular arcs increasing in curvature a certain amount for each chord.

Froude's Curve of Adjustment.*—The best practical form of transition curve is Froude's curve of adjustment, which approaches nearly a cubic parabola and is readily calculated and laid out on the ground.

To allow of the cant being gradually applied with a gradient of

* Minutes of Proceedings, Institution of Civil Engineers, vol. cxxxiii., "The Field Practice of Laying Out Transition Curves," by Mr John Robinson, M.Inst.C.E.

1 in 300, which is the usual gradient and that recommended by Froude, the length of the curve of adjustment will be = 300 × cant.

In Fig. 173, let FI, LI be two straights to be united by a circular curve with transition curves, and let the intersection angle I = central angle of curve, be measured as usual. Let AB represent the ordinary circular curve, having its tangent points at A and B. The method of introducing transition curves is to "shift" the curve AB to CD, AB and CD being concentric; the ordinary curve AB must be therefore considered as being of a radius = proposed radius of

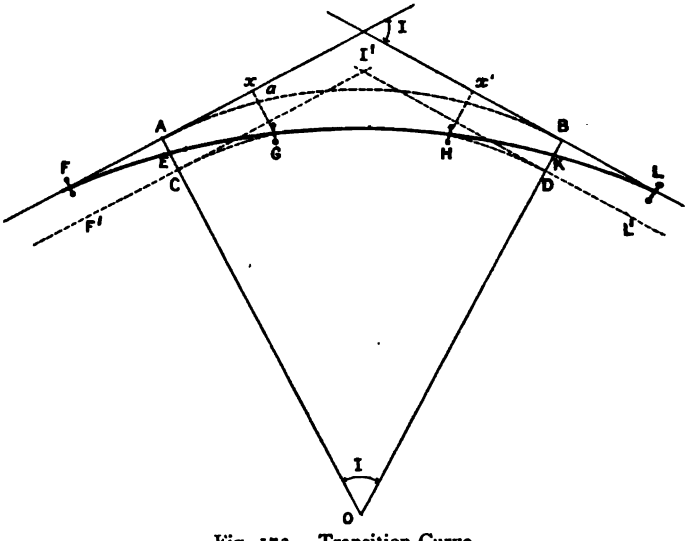


Fig. 173.—Transition Curve.

circular curve *plus* the "shift" AC. Let R = radius of curve CD, then radius of curve AB = R + S where S = shift.

The length of the tangents AI, BI is then given by

$$AI = BI = (R + S) \tan \tfrac{1}{2}I.$$

To get s we have length of curve of adjustment = 300 × cant = l say. Then—

$$S = \frac{l^2}{24R}$$

The distances IA and IB are now to be measured along the

tangents, and pegs put in temporarily at C and D, making AC and BD each = S.

The transition curves will bisect AC and BD at E and K.

The points of commencement of the transition curves are now to be marked by three pegs at F and L, the distances AF and BL being made $= \frac{l}{2}$, *i.e.*, half length of transition curve.

(This is on supposition that AF and BL measured on the tangents do not appreciably differ from EF and KL measured on the curves, which is in practice the case.)

The terminations of the transition curves and their points of junction with the circular curve are at G and H, EG and KH being each $= \frac{l}{2}$

These points are fixed by means of the offsets xG and x'H measured at right angles to the tangents FI, LI.

These offsets xG and x'H are each = 4S, *i.e.*, 4 times the shift.

They are also equal to the offset xa to the circular curve + the shift S or to

$$\frac{\text{chord Aa}^2}{2(R+S)} + S$$

Setting out Transition Curves.—The transition curves are set off by means of offsets measured square off the tangents FI, LI. These offsets are proportional to the cubes of their distances from the commencement of the transition curve, and having the offset xG = 4S at a distance l from F the offset at any other point is easily calculated. Otherwise take the offset AE $= \frac{S}{2}$ at a distance $\frac{l}{2}$ from F, and calculate the others from this. The last offset at x should then be = 4S.

Having thus set off the transition curves by means of their calculated offsets from the tangents, set ranging rods in the lines F′I′, L′I′, these lines being parallel to the original tangents and at a distance from them = shift. The circular curve GH may now be set out in the usual manner, by setting up the theodolite at C and working off the tangent F′I′. On laying off the deflection angle for the chord CG the point found should coincide with G as previously fixed by measuring the offset xG from FI. Similarly the termination of the curve should coincide with H as fixed by measuring x'H from LI.

The length of curve CD = circular measure of angle I × R.
Length of curve GH = CD − *l*.
Total length of curve from F to L = GH + 2*l*.
Or total length of curves from F to L = (circular measure of angle I × R) + *l*.

These lengths of curves are on supposition that EG and CG are equal. In practice they will not differ appreciably.

The portions CG and DH of the circular curve are of course not staked out, and the temporary pegs at C and D are removed, after setting out the curve GH.

The radius of the above transition curve at G and H is equal to R, the radius of the circular curve.

"**Cant.**"—The length of the transition curve above described will be proportional to the cant, which again depends on the radius of curve and speed of train. The following formula gives the cant:—

$$\text{Cant in inches} = \text{gauge in feet} \frac{V^2}{1.25R}$$

where V = velocity in miles per hour, R = radius of curve in feet.

For a 10 chain curve on 4 ft. 8½ in. gauge at 40 miles per hour the cant is about 9 in.; at 1 in 300 this gives a length of transition curve of 225 ft. For curves of greater radius the cant is less, and length of transition curve therefore less. The length of transition curve is, however, often made greater for curves of larger radius. When sharp curves follow each other closely there is sometimes little room to get in the transition curves, but in practice transition curves from 180 to 400 ft. long will usually cover all cases. Curves above about 40 chains radius do not usually require transition curves, the small amount of cant and widening of gauge being effected on the straight.

Transition Curves between Reversed Curves.—When there is not room to get in a piece of straight between two reverse curves, the change of cant may be readily effected by uniting the two reversed curves by a transition curve as shown in Fig. 174.

Let ABC be two reverse curves reversing at M. Shift each curve the proper amount to GF and DE. Calculate the length of transition curve, which will be in this case 300 × *sum of* cants.

The middle point of the transition curve will bisect the total shift DF at M, and the ends of the transition curve will be at H and K, MK and MH being each = half length of transition curve. Set off the curve KM by offsets from KF, making each offset proportional to the cube of its distance from K, and the curve HM similarly. The final centre line is then GKMHE.

Vertical Curves at Changes of Gradient.—For uniting steep gradients parabolic vertical curves should be used. Fig. 175 shows a ready method of calculating the levels of a parabolic vertical curve. Let EF be the datum line on the section, and AB, BC the gradients to be united by the parabolic vertical curve

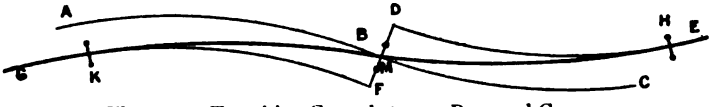


Fig. 174.—Transition Curve between Reversed Curves.

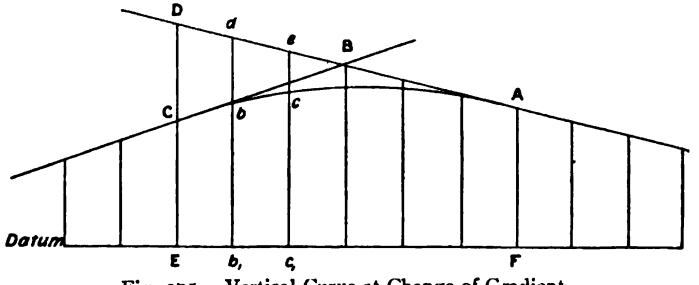


Fig. 175.—Vertical Curve at Change of Gradient.

AC. Produce one of the gradients to D, and measure the offset DC at the proposed termination of the vertical curve. All the other offsets from A to D, as *db*, *ec*, &c., may be readily calculated, as they are simply proportional to the square of their distances from A measured along the datum line, *i.e.*, to $(Fb_1)^2$, $(Fc_1)^2$, &c. From these offsets the levels of the points *b*, *c*, &c., may be calculated and figured on the section as usual.

Setting out of Work during Construction: Transferring Tangent Points.—The first thing the engineer in charge of the setting out of work on a railway must do is to carefully

transfer the tangent points of the curves. In doing this he must calculate on the amount of cutting or bank at the tangent point in question, so as to transfer the points to beyond the slope of the cutting or bank, where they will not be disturbed by the work.

The best way to transfer the tangent points is to drive in two pegs just beyond where the top of the slope of the cutting or the bottom of the slope of the bank will come, and on the same side of the railway. Then measure with a steel tape the exact distance of each of these pegs from the tangent point. It is also advisable to put in a third peg on the other side of the railway and in line with the tangent point and one of the first two pegs. Thus in Fig. 176 having the pegs *a*, *b*, and the distances *a*P, *b*P, the point P may be fixed by these measurements in the bottom of the

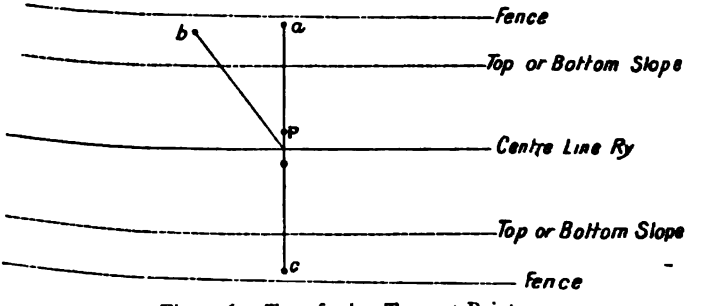


Fig. 176.—Transferring Tangent Points.

cutting or on the top of the bank after the original tangent point has been excavated away or buried in the bank. The third peg *c* serves as a check on the measurements, or comes in useful if either of the other pegs is lost. Lining in between *a* and *c* is also of assistance in refixing the tangent point.

The object of the above is to get the centre line pegged out again for the rail laying, &c., after the cuttings and embankments have been made and the original centre line pegs excavated or buried, and also for the alignment during their construction. Without the means of relocating the tangent points this might be a somewhat troublesome business.

Setting out Fence Widths.—The fence width is usually pegged out at every chain, and at any intermediate points necessary.

A list of fence widths should be made out. They are calculated as follows:—Suppose the depth of cutting is 10 ft. and the formation * width 28 ft., the slope being 1 to 1; then for fence width we have 14 ft. half formation width + 10 ft. slope + 6 ft. allowance from top of slope to fence = 30 ft. fence width, *i.e.*, from centre line of railway to fence is 30 ft. This supposes the ground to be level across. When the ground is irregular or sloping transversely the fence widths may be scaled off the cross sections. The fence widths are measured from the centre line pegs before the work of excavation or embankment is commenced, and pegs are driven in, one on each side of the railway, at every chain, to which the fence is erected.

Levelling in Tops and Bottoms of Slopes. — The slopes of cuttings are usually worked up from the bottom and trimmed off to the correct slope by the foreman with a batter rule and plumb bob. In banks the material is usually tipped and left to find its own natural slope, being afterwards trimmed off with the batter rule, working down from the top.

In deep cuttings or high banks, however, it is sometimes necessary to peg out the top or bottom of the slope, and the engineer may occasionally be called upon to do this. When the cross sections are reliable and the ground is not very irregular, possibly the widths from the centre line to the top or bottom of the slope may be scaled off the cross sections. The best way is, however, to level them in. As this is a tentative process, it often presents some little difficulty to the beginner. The procedure is as follows:—Take the depth of cutting or height of bank on the centre line from the longitudinal section at the given chainage. From this calculate the distance from the centre line to top or bottom of slope, which, on the supposition that the ground is level, will be half formation width plus slope. If now the ground slopes transversely to the centre line, add to or deduct from this distance to allow for the slope of the ground as much as may be judged by estimating the rise in the ground by the eye. Now direct the staff-holder to hold the staff at that distance. (Another chainman to hold the tape on the centre line is required in addition to the staff-holder.) Read now the level of the ground at

* "Formation" is the bottom of cutting or top of bank.

this point and find the difference between it and formation level, which will be the depth of cutting or height of bank at that point. From this depth or height recalculate the slope width, which is as before half formation width plus the slope due to the height or depth as given by the level. If this agrees with the actual width from the centre line to the point where the staff was held, then that point is the correct top or bottom of slope, and the peg may be there driven in. If not, the operation must be repeated until a point is found which is correct both as to level and corresponding distance out from the centre line. Much depends on the judgment of the engineer, but with a little practice the first or second trial will usually be sufficiently close unless in the very roughest description of ground. Scaling off the cross sections may be of assistance for the first trial point.

Setting out Bridge and Culvert Foundations.—The following is the best practical method of setting out the bridge foundations shown in Fig. 177. Drive in a peg at O, the centre of the bridge, and setting up the theodolite over this peg, range in chaining arrows at as many points as necessary on the line OA, OB, so that a cord may be stretched between them. The lines AOB, COD are the centre lines of the road or stream and of the railway, respectively. On rough ground several pins will be required to enable the cord to be stretched out in the straight line. Set off OC, OD at right angles to OA, OB, and similarly range in chaining pins and stretch a cord on this line. The engineer should be provided with a tracing of the plan of the foundations of the bridge, as shown in Fig. 177, and should have drawn on it the co-ordinates of each of the angular points *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, referred to the centre lines AOB and COD. Thus the co-ordinates of the point *a* are 9*a* and 1*a*; of *c*, 4*c* and 7*c*, and so on. The distances 07, 08, 09, 010, 011, 012, 013, 014, are to be marked on the tracing, and these are measured along the cord line COD, and chaining pins are inserted at the points 7, 8, 9, 10, 11, 12, 13, 14. Similarly the distances 01, 02, 03, 04, 05, 06, along the line AOB are marked on the tracing, and these are measured and chaining pins inserted at the points 1, 2, 3, 4, 5, 6. Now take two tapes, and putting the rings together, direct the chainman to pull them tight while the distances 9*a* and 1*a* are read on the tapes held at 9 on the line COD and at 1 on the line AOB re-

spectively by the engineer and his assistant. This obviously fixes the point *a*, and a peg is then driven in. The other points are all similarly fixed by their co-ordinates, and pegs driven in at each point.

The cord lines should now be taken up and passed round the periphery of the figure *cabdfhgec*, and a rut dug along this line. The other abutment is treated similarly, the pegs being of course

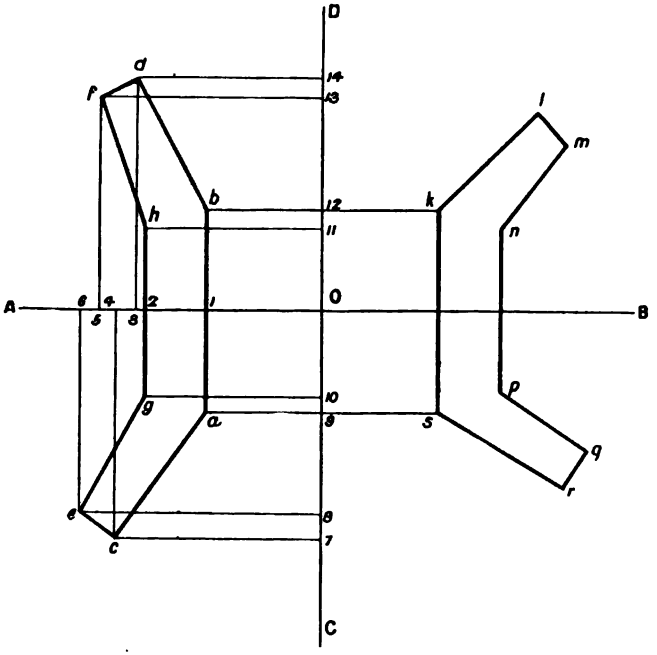


Fig. 177.—Setting out Bridge Foundations.

put in at the same time as those for the first abutment, and before taking up the cord lines. The outline of the excavation for the foundations of the bridge is thus clearly defined on the ground, so that the foreman can make no mistake in his digging.

On one of the pegs at the end of each wing as *c* and *d*, a level should be taken, and a note of the depths from the pegs to the bottom of the excavation should be given to the foreman. Levels on the ground at all the pegs should also be taken for the sake of

getting the correct quantity of excavation. When the bridge is on the skew the procedure is the same, the lines AOB, COD being set out at the proper angle with each other.

Setting out Culverts.—Usually only type drawings of culverts are furnished, showing the cross section of the culvert and the plan and elevation of the front and wing walls, and it devolves upon the engineer to calculate the length of the culvert and make a correct plan of it on the basis of the type drawing.

The first thing to do is to take levels in the bed of the stream for which the culvert is intended, and decide upon the invert level at each end. From the invert level we get the level of the top of the outside of the arch (the extrados at the crown), and the difference between this and the formation level of the railway is the height of bank above the culvert. Half the length of the culvert is then equal to half formation width plus the slope as calculated from the height of bank above the culvert, plus any allowance for face walls, &c., according to the type drawing. When the culvert is on the square, a rough sketch may usually be made from these figures sufficient to set it out, but when on the skew it may be necessary to draw out a plan of it. Having got out the plan, the process of setting out is exactly similar to that described for setting out a bridge.

Setting out Bridge Abutments.—When the foundations of a bridge have been laid in, the corners of the abutments should be accurately set out with steel tapes. This is done by co-ordinates in the same manner as already described for setting out the foundations. When one abutment has been built or set out the corners of the second abutment should be set out by measuring from one centre line only, the *span* as measured from the first abutment being used as the other co-ordinate. This will ensure the span being correct.

Other Setting out: Centre Lines and Levels in Cuttings and Banks.—During the progress of construction the engineer will continually be called upon to give centre lines and levels in the cuttings and on the banks as the work proceeds. Having all his tangent points transferred, he will usually have no difficulty in doing this, the work simply consisting in setting up the theodolite and ranging in a few pegs on a straight or turning

off a few angles on a curve. The levels required are put in by levelling from the nearest bench mark; see also To Locate a given Level, page 253.

"**Boning Rods.**"—When a cutting is started the foreman is given a note of the depths at each peg, but when the cutting is deep and there are lifts or benches in the face he is unable to measure down these depths after the cutting has gone in a little way. The engineer must therefore give him two pegs, 1 or 2 chains apart, driven in so that their tops are at formation level. The foreman places his *boning rods* (see page 161) on these pegs, and *bones in* his levels in front as the work proceeds. Level pegs must be put in as often as necessary to keep them within a chain or two of the face of the excavation.

"**Crossheads.**"—In the case of embankments the foreman is given a note of the heights at each chain peg, and in this case he puts up what are called "crossheads" to indicate the height of the bank. A "crosshead" is simply a post with a crosspiece about 2 ft. long nailed across the top of it, and at right angles to it. The "crosshead" is erected so that this crosspiece is at the given height above the chain peg plus 3 ft., the height of the *boning rod*. Having erected two or three crossheads, by using his *boning rod*, which is 3 ft. long, and sighting on to them, the foreman is enabled to keep his bank to the correct level.

Allowance for Shrinkage and Settlement.—In banks tipped with soft material an allowance of about 1 in. per foot of height must be added to the height of the bank to allow for shrinkage and settlement. In high banks this allowance becomes excessive, and a smaller percentage must be added. The allowance to be made of course depends on the nature of the material, and is a matter of experience.

Bench Marks and Checking through Levels.—Before doing *any* setting out on a railway for construction purposes the levels should be carefully checked through from end to end of the railway, and bench marks established not more than ½ mile apart and at each bridge or other important work.

To Locate a given Level.—In order to put in a peg or mark a point at any given reduced level it is simply necessary to

deduct this reduced level from the height of the instrument, and the difference is the "staff reading." The peg has then to be driven in until this reading is observed, when the staff is held on it. In the case of making a mark at any given reduced level on a wall, &c., the staff is to be placed against the wall and moved up or down until the correct "staff reading" is observed. The mark is then made at the foot of the staff.

CHAPTER VI.

TACHEOMETRY OR STADIA SURVEYING.

Tacheometry.—The term tacheometry is derived from the Greek *tacheos* (quickly) and *metreo* (I measure), and therefore signifies the art of rapid measurement. It is, however, now confined to distance and height measurement by telescope.

Principle of the Stadia.—The principle of tacheometry in its simplest form is supposed to have been first used by Mr W. Green in 1778. He used a simple tube containing three horizontal wires, and on the supposition that rays of light travel in straight lines from the object observed to the eye, the distance of the object is proportional to the extent of it apparently intercepted between the cross wires. Thus in Fig. 178, let A be the eye, *b*, *d*, *c* the cross wires, BC and B′C′ the extent of the observed objects apparently intercepted by the straight lines A*b*, A*c* produced. If the tube is held so that the line A*d* from the eye to the central wire is horizontal, then AD and AD′ are the distances of the observed objects from the eye, and BC and B′C′ being vertical will be bisected by A*d* produced in D and D′. The triangles BDA and B′D′A being similar, we have—

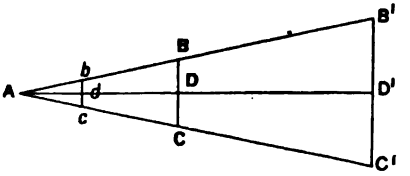


Fig. 178.—Principle of the Stadia.

$$\frac{AD}{AD'} = \frac{BD}{B'D'}$$

and as BC and B′C′ are respectively double BD and B′D′, also

$$\frac{AD}{AD'} = \frac{BC}{B'C'}$$

i.e., the distances from the eye are proportional to the amounts intercepted by the cross wires.

If now the wires are so arranged that when the distance AD is 100 ft., BC is 1 ft., then if B′C′ is 2 ft., the distance AD′ will be 200 ft., and so on, or in other words the distances will be 100 times the intercepted amounts. Obviously then by placing a staff at C and C′, the distances AD and AD′ will be equal to the staff readings with the decimal point removed two places to the right.

The horizontal wires at *b*, *c*, *d* are called *stadia* wires, and the above is the principle of the *stadia*. The word stadia is Italian, and was originally used to mean the staff. Stadia work is simply another name for tacheometry.

To look at the matter in another way, referring to Fig. 178, we see that the triangles ABC, AB′C′ are each similar to the triangle A*bc*, and we have—

$$\frac{AD}{BC} = \frac{AD'}{B'C'} = \frac{Ad}{bc} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

If now $\frac{Ad}{bc}$ be put equal to K = 100 say, then from (1)—

$$AD = \frac{Ad}{bc} BC = K \times BC = 100\ BC$$

$$\text{and } A'D' = \frac{Ad}{bc} B'C' = K \times B'C' = 100\ B'C'$$

That is, the ratio $\frac{Ad}{bc}$ being made 100 so that A*d* is 100 *bc*, then the distances AD and AD′ are 100 times the intercepted amounts BC and B′C′.

Tacheometry simply consists in applying this principle to the telescope, by means of which the staff may be accurately read at considerable distances.

Distance and Height Measurement by Simple Vertical Angles.—Before describing the tacheometer proper the following method of distance and height measuring by means of simple vertical angles and staff readings may be described:—

In Fig. 179, let T be the theodolite and ABC the staff held at C. Let the horizontal distance TD = H be required. Measure the vertical angles DTA and DTB, and let them be θ and ϕ respectively; also let the distance intercepted on the staff be *s*, being equal to the difference of the staff readings at A and B, which is equal to $b - a$ in Fig. 179.

Now in the figure—

$$b = \text{H} \tan \theta \quad \ldots\ldots\ldots\ldots\ldots (1)$$

$$\text{and } a = \text{H} \tan \phi \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Therefore $(b - a) = s = \text{H} (\tan \theta - \tan \phi)$

whence—

$$\frac{s}{\tan \theta - \tan \phi} = \text{H} \quad \ldots\ldots\ldots\ldots (3)$$

or "the horizontal distance is equal to the difference between the staff readings divided by the difference of the tangents of the observed vertical angles." The computation of H is facilitated by making s an even number = 10 say.

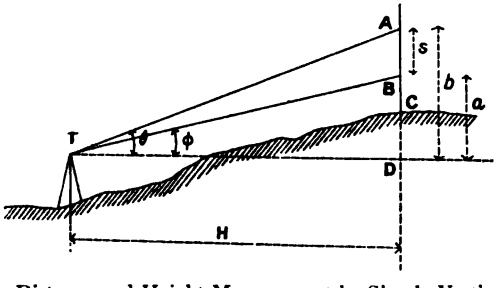


Fig. 179.—Distance and Height Measurement by Simple Vertical Angles.

The levels of optical axis of instrument or of staff station are easily found by taking the staff reading at either A or B along with the corresponding vertical component b or a, and treating these as described on page 274.

Bell-Elliott Tangent Reading Tacheometer.—An instrument specially adapted for measuring distances as above described has been patented by Mr G. J. Bell of Carlisle, and is made by Messrs Elliott Brothers, London. The instrument is shown in Fig. 180, and is called the Bell-Elliott tangent reading tacheometer. It is described in *Engineering* as follows:—"In order to measure distances with this instrument a level staff is erected at the distant point, and the instrument having been carefully levelled, two readings are taken on points of the staff, say 10 ft. apart. Then if θ be the inclination of the upper line of sight to the horizontal in making the upper reading, and ϕ the inclination in

making the lower reading, we have obviously the horizontal distance of the staff—

$$H = \frac{10}{\tan\theta - \tan\phi}$$

In Mr Bell's instrument a ready means is provided for reading off these tangents direct from the instrument without referring to tables. Subtracting the two readings from each other, and multiplying the reciprocal of the result by 10 gives at once the horizontal distance. The instrument is essentially an ordinary theodolite fitted with a trough compass, which can be used in the ordinary way. The two attachments which fit it specially for tacheometry consist of, in the first place, a very accurately divided scale rigidly secured to the upper parallel plate, and fitted with a micrometer adjustment, by which it can be shifted with certainty through a distance of $\frac{1}{1000}$ in. up to $\frac{1}{32}$ in. in the direction of its length. This scale can be read through a microscope fixed at right angles to the telescope as shown, and moving with the same, so that its axis makes the same angle with the prime vertical as the latter does with the horizontal plane of the instrument. A total reflection prism is used to deflect the line of vision through the microscope through 90°, so as to bring the eyepiece into a convenient position for use. In making an observation the telescope is first aligned on the upper of the fixed marks on the level staff. The micrometer is then set to zero, and the division of the scale which is nearest to the cross wire of the microscope is read off through the latter. Say this was 19, then the natural tangent of the angle the line of sight makes with the horizontal is 0.19 ± a correction to be obtained from the micrometer head, which is by means of a vernier divided into 500 parts. This head is now rotated till the cross hair of the eyepiece exactly cuts the scale mark, and the reading of the head is then noted. Assume this to be 125. Then the natural tangent of the angle in question is 0.19125. On making similar readings for the lower mark on the staff the numbers might be, say, 17 and 823, then we should have $\tan\theta - \tan\phi = 0.01302$. The reciprocal of this is 76.805, and the corresponding distance between staff and instrument would be 768.05 ft. This perhaps would be rather a long sight, and the accuracy of the instrument is of course the greater the nearer the staff."

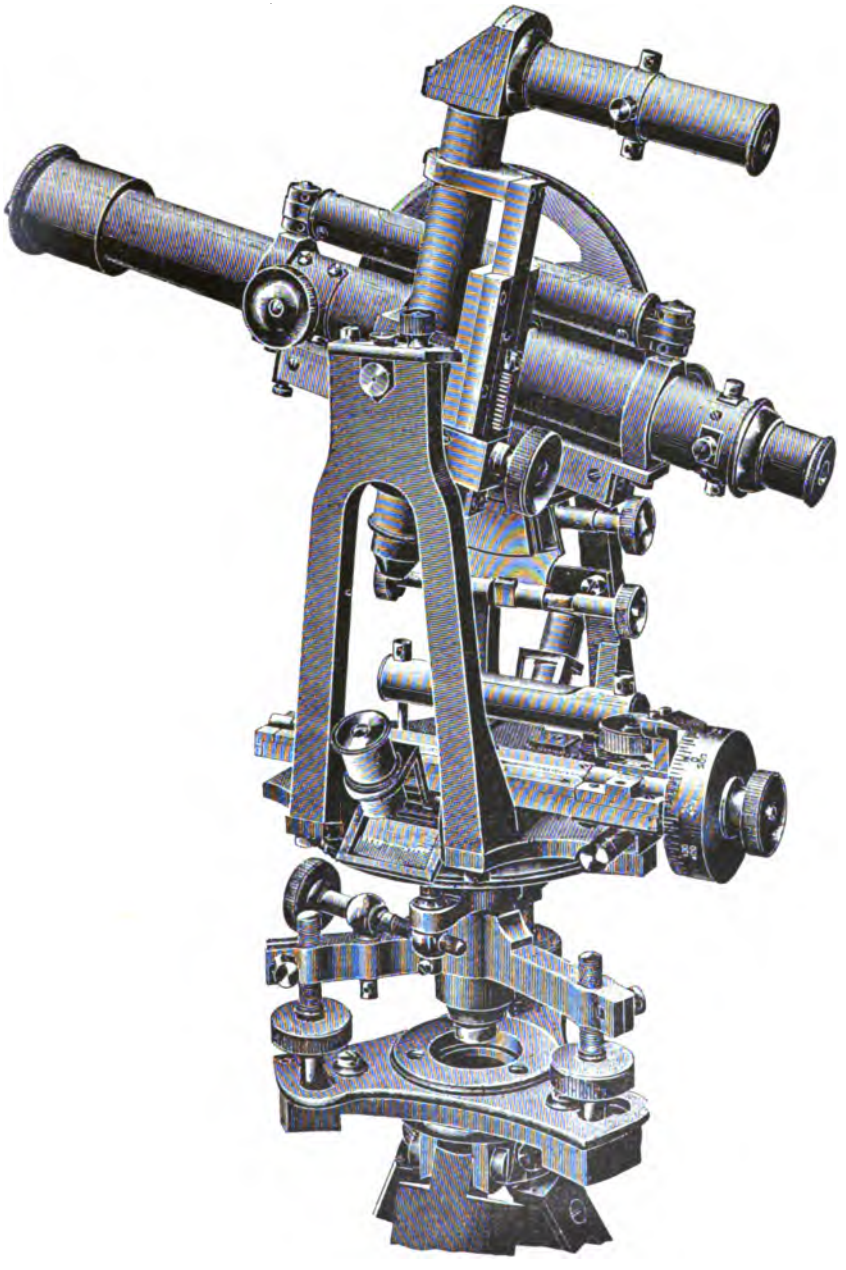


Fig. 180.—The Bell-Elliott Tangent Reading Tacheometer.

Barcenas' Tangential Method.—In Fig. 179 we have—

$$b = \text{H} \tan \theta$$

and $a = \text{H} \tan \phi$.

Therefore $(b - a) = \text{H} (\tan \theta - \tan \phi)$.

That is $s = \text{H} (\tan \theta - \tan \phi)$.

If now the vertical angles θ and ϕ are such that $\tan \theta - \tan \phi = .01$, we have—

$$s = .01\text{H}$$

or $100 s = \text{H}$.

Barcenas' method consists in setting the vertical arc to two angles, the difference of whose tangents is equal to .01, and observing the staff readings at A and B for each angle. The distance TD = H, Fig. 179, is then simply 100 times the difference of the staff readings at A and B. For example, the difference of the tangents of the following consecutive angles is equal to .01 :—

Angle.	Tangent.
0° 34′ 23″ - - - -	0.01
1° 8′ 54″ - - - -	0.02
1° 43′ 6″ - - - -	0.03
2° 17′ 36″ - - - -	0.04
2° 51′ 45″ - - - -	0.05
3° 26′ 1″ - - - -	0.06
&c. &c. &c. - - -	&c.

The vertical arc being set at any consecutive two of the above angles, *the distance of the staff from the instrument is then simply* 100 *times the difference of the staff readings*, or the difference of the staff readings with the decimal point removed two places to the right. In Fig. 179, suppose that the vertical angles used are 1° 8′ 54″ and 1° 43′ 6″, whose tangents are .02 and .03 respectively, then for the vertical component a we have—

$$a = \text{H} \tan 1° 8′ 54″$$
$$= 100 s \times .02 = 2s$$

and for the vertical component b we have—

$$b = \text{H} \tan 1° 43′ 6″$$
$$= 100 s \times .03 = 3s.$$

The vertical components are therefore simply 2, 3, 4, *or* 5, *&c., times the space s intercepted on the staff.* In reducing the levels

care must be taken to use the proper staff reading corresponding to the vertical component. Thus if the vertical component $a = 2s$ is computed, the staff reading at B is to be used in finding the reduced level of the point C, and if $b = 3s$ is computed, the staff reading at A is to be used in finding the reduced level of C, Fig. 179.

It will thus be seen that the reductions of the sights are so simple that they may be performed mentally. No tables, no diagrams, no slide rules are required, and in point of simplicity this method cannot be equalled.

As, however, on the ordinary graduations of the vertical arc of a theodolite the vertical angles cannot be set off with sufficient accuracy, it is necessary to have a specially graduated vertical arc, each division representing one of the angles given on page 260,

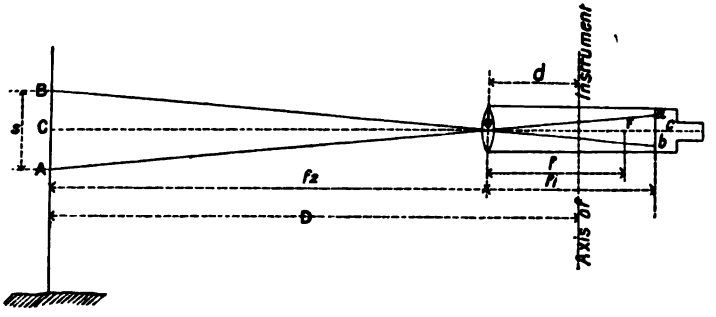


Fig. 181.—Principle of the Tacheometer.

0° 34′ 23″, 1° 8′ 54″, 1° 43′ 6″, &c. &c., the difference of whose tangents is .01. An ordinary theodolite may be made available for this method by having an additional set of the above graduations made inside the ordinary graduations of the vertical arc and an additional zero point on the inside of the vernier.

The necessity for special graduations on the vertical arc of the theodolite is probably the reason that the method has not come into general use. It is supposed to have been first given by Barcenas, a Spanish writer on tacheometry.

Principle of the Tacheometer.—In Fig. 181, let a, c, b be stadia hairs placed in the diaphragm of an ordinary theodolite telescope. The image of the intercepted portion of the staff AB is found by drawing lines from A and B through the centre of

the object glass O, meeting the stadia hairs of the diaphragm at *a* and *b*.

Let F be the position of the image for parallel rays, *i.e.*, for an object at an infinite distance.

Then we have OF = focal length of telescope = f

$$\left.\begin{matrix} Oc = f_1 \\ OC = f_2 \end{matrix}\right\} \text{conjugate foci}$$

Also put distance apart of stadia hairs $ab = i$

intercepted portion of staff AB = s

distance of object glass from centre of instrument = d

The triangles AOB, aOb are similar, and

$$\frac{f_2}{s} = \frac{f_1}{i}$$

$$\text{whence } f_2 = \frac{f_1}{i}s \quad \ldots\ldots\ldots\ldots\ldots (1)$$

This gives the distance f_2 in terms of f_1, i, and s, of which i and s are known quantities; but f_1 is unfortunately variable, being greater as the distance of the staff is less and *vice versâ*, the object glass O being screwed out or in in focussing the object.

However, from the law of lenses we have—

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

$$\text{whence } f_1 = \frac{ff_2}{f_2 - f}$$

Substituting this value of f_1 in (1) we get—

$$f_2 = \frac{sff_2}{i(f_2 - f)}$$

$$\therefore = \frac{f}{i}s + f \quad \ldots\ldots\ldots\ldots\ldots (2)$$

In other words, we have two equations $f_2 = \frac{f_1}{i}s$ and $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$ and two unknown quantities f_2 and f_1. These equations are solved for f_2, and we get $f_2 = \frac{f}{i}s + f$, as given in (2) above.

Equation (2) gives the distance of the staff from the object glass

in terms of the known quantities f, i, and s. To get the distance of the staff from the centre of the instrument the constant quantity d must be added, and for the whole distance D we have—

$$D = \frac{f}{i}s + f + d \quad \ldots\ldots\ldots\ldots\ldots (3)$$

As the object glass is moved out or in in focussing, d may vary an inch or two, and the distance from the centre of the instrument to the mean position of the object glass may be taken. This only affects the calculated distances 1 or 2 in., and the error is not appreciable in practice in tacheometric work.

Examining the expression for f_2 given by equation (2), we see that—

$$\frac{f_2 - f}{s} = \frac{f}{i}$$

That is, the ratio of the distance of the staff measured to a point at a distance f in front of the object glass, to the staff reading, is equal to $\frac{f}{i}$, which is constant, f being the focal length of the telescope and i the distance apart of the stadia hairs; also $f_2 - f = \frac{f}{i}s$, that is, the distance of the staff measured to a point at a distance f in front of the object glass is to be found by multiplying the staff reading by the constant $\frac{f}{i}$.

The whole may therefore be represented as in Fig. 183, where F is a point at a distance $f + d$ from the centre of the instrument, the angle AFC is constant, the ratio of the distance BF to the staff reading A′C′ is constant, and this distance BF = staff reading A′C′ × constant $\frac{f}{i}$.

Distance apart of Stadia Hairs.—In using an ordinary theodolite for tacheometry two methods may be adopted. The stadia hairs may be put in at a certain distance apart so that the constant $\frac{f}{i}$ may be an even number, usually 100. In this case as $\frac{f}{i} = 100$ we have $i = \frac{f}{100}$, that is, the distance apart of the stadia hairs must be $\frac{1}{100}$ of the focal length of the telescope. The other

method is to use stadia hairs put in at any distance apart, and determine their distance apart by observing the staff reading on a known carefully measured distance. This latter method is the better of the two, and gives the best results in practice for two reasons. First, because it is difficult to put in the stadia hairs exactly to a given interval apart; and secondly, because the interval between the stadia hairs may be determined in the field by taking the mean of several observations at different times of the day, and thus a result according as nearly as possible with the actual conditions under which the survey is carried out may be obtained. To avoid systematic errors and obtain any degree of accuracy it is most important to determine the stadia hair interval by several observations in the field as above stated. The effect of refraction is much greater close to the ground in the strata of air passed through by the lower line of sight than it is in the strata passed through by the upper line of sight. This difference in the refraction alters at different times of the day, and gives a slightly different staff reading for the same distance.

General Rules in Stadia Work.—The following rules should be observed:—

Each observer should determine for himself the stadia hair interval. The stadia hair interval should be determined for various distances between the limits of the distances to be met with in the survey and at several different hours of the day distributed throughout several days, on lines which approximate to the nature of the country to be surveyed.

In the case of a change of field conditions or change of season the stadia hair interval should be redetermined.

Avoid when possible reading the lower stadia hair near the ground.

If these rules be observed the accuracy attainable in the survey will be much greater.

Methods of Determining Distance apart of Stadia Hairs and other Constants.—In order to determine the interval between the stadia hairs the focal length of the telescope f and the distance d must first be measured.

To find the focal length of the telescope, focus the telescope on to a distant point, the moon or a star, and measure the distance

from the object glass to the diaphragm carrying the stadia hairs.

The distance d is the distance from the axis of the instrument to the object glass, and is to be measured on the instrument.

Carefully measure a base, set up the instrument over one end of it, and take the staff reading held on the other end.

Then from equation (3) we have—

$$D = \frac{f}{i}s + f + d$$

$$\text{whence } \frac{sf}{D - f - d} = i$$

A more accurate determination of the constants may be made as follows:—Carefully measure two bases differing considerably in length, as say 50 ft. and 500 ft. Set up the instrument over each in turn, and make several observations of the staff reading on each at different times throughout the day. The mean of each set of observations is taken as the staff reading for each base. We have then two equations, viz.—

$$D_1 = \frac{f}{i}s_1 + (f + d)$$

$$D_2 = \frac{f}{i}s_2 + (f + d)$$

From these two equations the two unknown quantities $\frac{f}{i}$ and $(f + d)$ may be found, D_1 and s_1 and D_2 and s_2 being known.*

Another method is also useful when a traverse is run by the stadia method between two points of a triangulation whose distance apart is accurately known from the triangulation observations and calculations. Assume the constant multiplier $\frac{f}{i}$ to be 100, and calculate the distance between the triangulation points from the lengths and bearings of the stadia traverse. Suppose this is 14,540 ft., while the true distance from the triangulation is 14,720 ft., then the true value of $\frac{f}{i}$ is $100 \times \frac{14720}{14540} = 101.24$.

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* This is much the best method, as by it there is no need to measure f and d on the instrument. As a rule it is not easy to measure these accurately.

In Messrs Troughton & Simms' tacheometer, described on this page, the constant of the instrument is 50. The distance is, however, made equal to 100*s* by making the divisions on the staff twice their proper size. In a 14 in. telescope d may be about 8 in. and the constant $f+d$ to be added is therefore 14 + 8 in. = 22 in. = 1.84 ft. As plans of preliminary location surveys are usually plotted to a scale on which the nearest foot only can be scaled, in practice the even 2 ft. is usually added. Similarly with a 10 in. telescope in which $d=5$ in. the constant to be added is 10 + 5 in. = 15 in. = 1.25 ft., and in practice an even 1 ft. is added. These constants are easily added mentally, and need entail very little extra trouble. If much accuracy is desired in the *levels* the *exact* value of $f+d$ should be used in computing the *levels*.

Porro's Stadia Telescope.—In order to do away with the necessity of adding a certain amount to each horizontal distance, in 1823, Porro, a Piedmontese officer, introduced between the object glass and the diaphragm a third lens, called an anallatic lens, so placed that its focal length and that of the object glass overlap each other slightly, while the principal interior focus of the object glass coincides with the vertical axis of the instrument. The apex of the measuring angle, therefore, coinciding with the vertical axis of the instrument, the necessity of adding $f+d$ to each horizontal distance is obviated. It is found in practice that the inconvenience arising from having to add a constant usually an even 1 ft. or 2 ft. is trifling, and that the additional lenses and complications adopted by Porro are therefore quite unnecessary.

Troughton & Simms' Tacheometer.—Nevertheless many engineers prefer to have a special lens introduced, and Fig. 182 shows a tacheometer of this description as made by Messrs Troughton & Simms. As long as the object glass and the anallatic lens are in the same position relatively to each other the measuring angle is constant. Also by altering the distance apart of the stadia hairs in the diaphragm any other measuring angle may be formed, its apex still remaining in the vertical axis of the instrument. A detachable key fits in at C and actuates the arrangement for adjusting the measuring angle. A key fitting at B adjusts the vertical circle. There are two sets of horizontal hairs in the diaphragm, the measuring angle formed by the furthest apart set

corresponds to the angle twice the tangent of half of which is 0.02, while the measuring angle formed by the other set corresponds to that angle twice the tangent of half of which is 0.004. When the

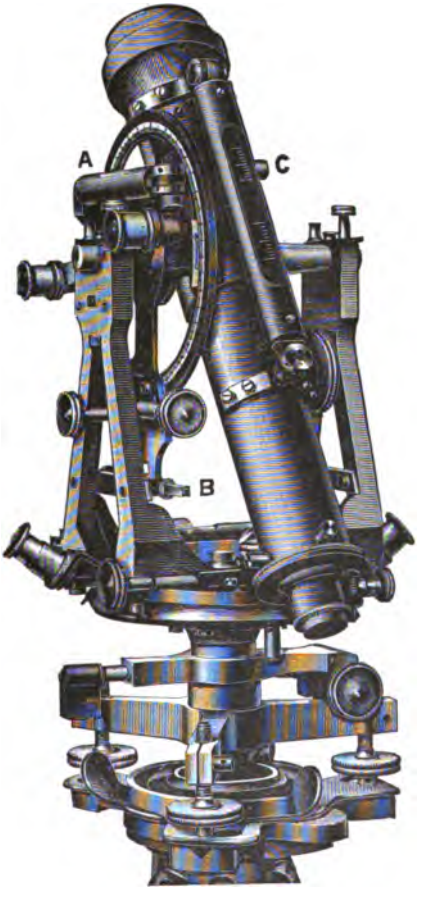

Fig. 182.—Troughton & Simms' Tacheometer.

line of sight of the furthest apart set is obscured the other set is to be used, the staff reading being multiplied by 5 as $.004 \times 5 = .02$; the .02 set being as a rule used. The eyepiece can be moved up

and down by the pinion shown so as to obtain a clear sight of the extreme cross hairs. The bubble at A is connected to the vernier arms of the vertical circle. For this instrument the size of the divisions on the staff must be .02 of the unit of measurement, *i.e.*, .02 ft. or .02 metre as the case may be. For all practical purposes, however, an ordinary theodolite, having cross hairs fitted so that the constant $\frac{f}{i}$ is approximately $\frac{1}{.01} = 100$, while the divisions of the staff are also .01, *i.e.*, .01 ft. or .01 metre, which is the ordinary division of the levelling staff, is all that is necessary for tacheometry work.

Specially Graduated Staff.—A staff may be graduated to suit the stadia hairs of any particular theodolite, so that if the constant multiplier $\frac{f}{i} = 103.20$ say, using an ordinary staff graduated to feet and hundredths, the multiplier is exactly 100 using the specially graduated staff. This is of course effected by making the size of the divisions of the special staff to suit the instrument in question. This system has been found less accurate than the system which uses the ordinary levelling staff graduated to feet and hundredths, and is objectionable for the following reasons:—The cost and trouble of regraduating and repainting the staff when the constant of the instrument changes, as when new stadia hairs are put in or for other reasons; the staff cannot be used with any other instrument except that for which it is made; such a staff could not be used for obtaining the levels of the various points observed without calculating laborious corrections.

Stadia Tables for Horizontal Sights.—Many engineers have objected to using the stadia system on account of the labour involved in reducing the field observations. It is, however, easy to prepare a table for any instrument by means of which the labour of reducing the field observations is much diminished. Suppose for the theodolite in use $\frac{f}{i} = 104.80$, $f + d = 2$ ft., such a table would be as under:—

$$\frac{f}{i} = 104.80 \quad (f+d) = 2.00$$

Staff Readings.	Distance.	Staff Readings.	Distance.	Staff Readings.	Distance.
.01	1.05	.10	10.48	1.00	106.80
.02	2.10	.20	20.96	2.00	211.60
.03	3.14	.30	31.44	3.00	316.40
.04	4.19	.40	41.92	4.00	421.20
.05	5.24	.50	52.40	5.00	526.00
.06	6.29	.60	62.88	&c. &c.	...
.07	7.34	.70	73.36	up to	...
.08	8.38	.80	83.84	14.00 or	...
.09	9.43	.90	94.32	16.00	...
.10=12.48		.40=43.92		.70=75.36	
.20=22.96		.50=54.40		.80=85.84	
.30=33.44		.60=64.88		.90=96.32	

Suppose the staff reading is 4.73, from the table we have—

4.00	=	421.20
.70	=	73.36
.03	=	3.14
4.73	=	497.70

That is, the distance corresponding to a staff reading of 4.73 is 497.70. This is obtained from the table by inspection and the simple addition of three quantities.

In preparing the table the constant $(f+d) = 2.00$ is to be included in the distances for the even feet staff readings only, *i.e.*, in the last column only. If the $(f+d)$ were included in the distances for the tenths and hundredths of a foot also the result would be that the $(f+d)$ would be added in twice or three times in place of once only, in taking out any distance from the table.

For staff readings less than 1 ft. the subsidiary table for tenths given under the principal table is to be used. This is simply the distances given in the fourth column of the first table with the constant $(f+d)$ added to each. The parts for the hundredths of a foot are taken from the first table as before. For example, for a staff reading of 0.99 we have—

From subsidiary table	.90 =	96.32
From first table	.09 =	9.43
	.99 =	105.75

For staff readings less than .10 the table does not apply unless the constant $(f+d)$ be added to each of the distances in the second column of the first table. Of course another subsidiary table might be prepared for the hundredths of a foot staff readings, but as .10 corresponds to 12.48 ft. distance readings will not be taken nearer than this, and it is not required.

A table like that given above may be prepared in less than an hour if we remember that omitting the $(f+d)$ the distances are simply proportional to the staff reading.

With such a table and a little practice the field observations of a whole day may be reduced in about fifteen minutes.

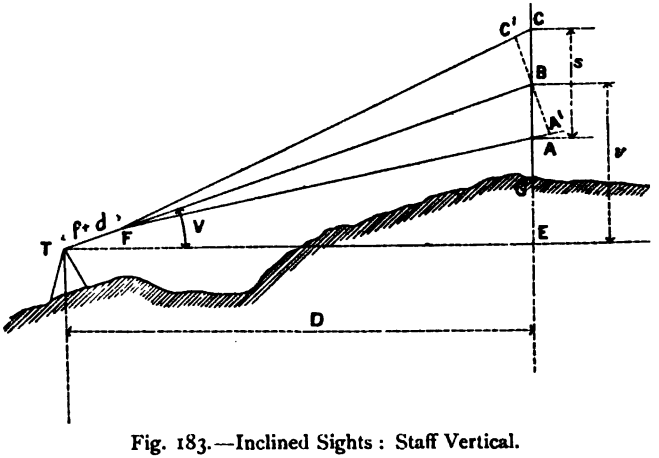

Fig. 183.—Inclined Sights: Staff Vertical.

Inclined Sights: Staff held Vertical.—We have hitherto supposed that the collimation line of the telescope is horizontal. In practice, however, it may be inclined at any angle as shown in Fig. 183.

In this case the staff reading AC must be reduced to A'C' at right angles to the collimation line FB. The angle CBC' is equal to the vertical angle V, and the triangle CBC' may be considered to be right angled at C' without appreciable error, therefore we have—

$$A'C' = AC \cos V = s \cos V$$

Therefore distance $FB = 100s \cos V$, if the constant of the instrument is 100. To this we add $(f+d)$ say 2 ft., and we get—

$$TB = 100s \cos V + 2$$

Also D = TB cos V

$$= 100s \cos^2 V + 2 \cos V \quad \ldots\ldots\ldots\ldots (1)$$

Also for EB = v, we have—

$$v = TB \sin V$$

$$= 100s \cos V \sin V + 2 \sin V$$

It is, however, usual to deduce v from D, and we then get—

$$v = D \tan V \quad \ldots\ldots\ldots\ldots\ldots (2)$$

In the above formulæ the constant of the instrument has been taken as equal to 100, *i.e.*, $\frac{f}{i} = 100$ and $(f + d) = 2$. The most general form of the formulæ is $D = \frac{f}{i} s \cos^2 V + (f+d) \cos V$ and $v = \frac{f}{i} s \cos V \sin V + (f+d) \sin V$.

Inclined Sights: Staff held Perpendicular to Line of Sight.—In place of holding the staff vertical it may be held perpendicular to the line of sight, as shown in Fig. 184. This is effected by having a sliding sight on the staff and at right angles to it. The staff-holder inclines the staff until he sights the instrument through the sights. The collimation line being then directed to the sight, the staff will be at right angles to the line of sight TB.

In this case if $f + d = 2$, and the constant of the instrument is 100, TB = $100s + 2$, and it is usual to neglect BB′. As B′GB = V, BB′ = GB sin V, and is usually small.

D is then = TB cos V

$$= 100s \cos V + 2 \cos V = (100s + 2) \cos V \ldots (3)$$

For v we have v = TB sin V

$$= 100s \sin V + 2 \sin V = (100s + 2) \sin V$$

v is more conveniently deduced from D, and we have—

$$v = D \tan V \quad \ldots\ldots\ldots\ldots\ldots (4)$$

Errors in Distance and Level with Staff held Perpendicular to Collimation Line.—Taking V = 30°, sin V = $\frac{1}{2}$, s = 6 ft., and distance D therefore about 500 ft., the staff reading at B being 5 ft., we get—

BB′ = $5 \times \frac{1}{2} = 2.5$ ft. error in distance.

B′G = BG cos V = $5 \times .87 = 4.35$ ft.

$5 - 4.35 = .65$ ft. = 8 in. nearly error in level.*

* In finding the level of the ground at G the staff reading BG is deducted from the level of the point B or B′; as it is really B′G which should be deducted the error in level of the point G is therefore BG − B′G, as above.

In this case, therefore, it is advisable to take the through levels of the main stations of the traverse with the ordinary spirit level, the leveller following up the tacheometer. The tacheometer levels will be near enough, however, for the spot levels for contours, &c.

Errors of 2.5 ft. in distance and 8 in. in level in a distance of about 500 ft. due merely to the reduction of the sights are very considerable, and to attain anything like accuracy the correction BB′ must be calculated and applied to the distance and the staff reading BG must be reduced to B′G, the latter being used in com-

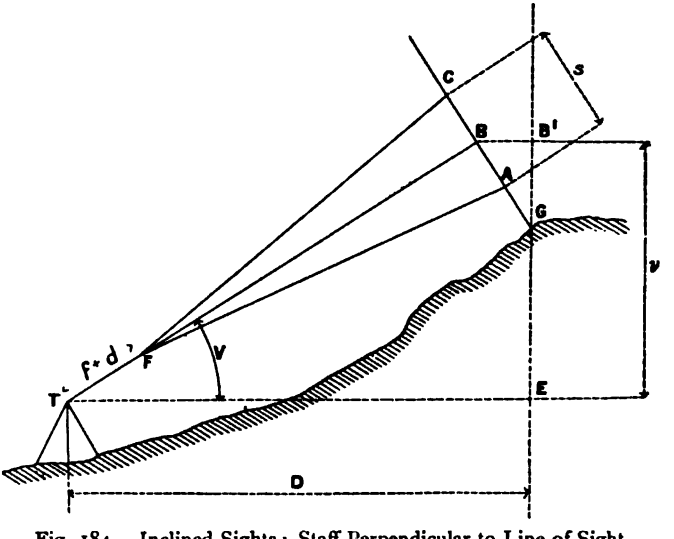

Fig. 184.—Inclined Sights: Staff Perpendicular to Line of Sight.

puting the level of the point G. These calculations render the reductions laborious and troublesome, and the system of holding the staff perpendicular to the collimation line is not much used. The system of holding the staff vertical is that most generally adopted.

Error from badly held Staff.—The advantage of holding the staff at right angles to the collimation line is that a small error in the position of the staff does not affect the results so much as when the staff is vertical. Thus at a distance of about 600 ft. on the collimation line when $v = 30°$ a deviation of 3° from the vertical position of the staff gives an error of 15 ft. in horizontal

distance, while a deviation of 3° from the perpendicular position of the staff gives only about 1 ft. of error in horizontal distance; for an instrument in which the stadia hairs are so placed that $\frac{f}{i} = 100$. When using a staff held vertically, it should therefore always be provided with a spirit level to ensure perfect verticality, otherwise the staff-holder must carry a plumb bob. In level country the staff is of course read with the collimation line horizontal, which saves trouble in the reduction of the distance. When possible the lower hair should be directed to an even foot on the staff; this simplifies the subtraction of the lower staff reading from the upper to get s the space intercepted. It is usually more convenient, however, to set the vertical arc to an even minute, so as to get the reductions directly from the tables.

Tables for making the Reductions.—When the staff is held at right angles to the collimation line as in Fig. 184, the reductions may be simply performed with the ordinary tables of sines, cosines, and tangents. When the staff is held vertical, however, to get D we have to multiply by $\cos^2 v$ and this again by tan v to get v. This would be too tedious without special tables. In making the reductions of inclined sights a table similar to that given on page 269 should be prepared, but *without* adding in the constant $(f+d)$. The distances as obtained from this table are then to be multiplied by $\cos^2 v$ and the constant $(f+d)$ cos v added to each. When $v = 5° 44'$ the reduction of the inclined sight to the horizontal is *one per cent.* exactly, or 1 ft. per 100 ft. Where an error of 1 in 100 is permissible the reduction to the horizontal need not therefore be made for vertical angles less than 6°. For obtaining the elevations of the main stations of the traverse the reduction must, however, be made in all cases if the elevation is deduced from the distance, *i.e.*, from $v = D \tan v$.

Stadia Table for Inclined Sights.—Table IV. will be found very convenient for reducing inclined sights in stadia work. This table gives values of $\cos^2 v$ and cos v sin v for every 2 minutes of vertical angle up to 31°. At the foot of each page values of $(f+d)$ cos v and $(f+d)$ sin v are given for $(f+d) = 0.75$, 1.00, and 1.25.

To reduce any inclined sight it is only necessary to multiply the value of $\cos^2 v$ or cos v sin v given in the table by the space

intercepted on the staff and by the constant of the instrument $\frac{f}{i}$; the value of $(f+d)$ cos v or $(f+d)$ sin v is then to be added. By preparing a table similar to that on page 269, but *without* adding in the constant $(f+d)$, the values of $\frac{f}{i}s$ may be obtained from it. These are then to be multiplied by the values in Table IV. and $(f+d)$ cos v or $(f+d)$ sin v as given in Table IV. added.

Reducing the Levels.—Care must be taken in reducing the levels.

For Back Sights we have—

When the vertical angle is plus L.I. = L.S.S. + B.S. − V.C.

„ „ minus L.I. = L.S.S. + B.S. + V.C.

For Fore Sights.

When the vertical angle is plus L.S.S. = L.I. + V.C. − F.S.

„ „ minus L.S.S. = L.I. − F.S. − V.C.

These apply whether staff is held vertical or perpendicular to collimation line.

L.I. = level of axis of instrument.
L.S.S. = level of staff station.
B.S. = back sight (as read by axial hair).
F.S. = fore sight („ „).
V.C. = vertical component = v in formulæ and figures.
Plus vertical angle = angle of elevation.
Minus „ = „ depression.

Field Book.—The following is a good form of field book :—

Station of Instrument.	Height of Axis of Instrument above ground.	Staff Station.	Bearing.	Vertical Angle.	Readings of Stadia Hairs.	Reading of Axial Hair.	Distance on Collimation Line.	Horizontal Distance, D.	Vertical Component, v.	Rise.	Fall.	Reduced Level.		Remarks.
												Of Axis of Inst.	Of Station.	

Field Work.—The tacheometer is chiefly useful for preliminary location surveys of lines of communication, such as roads or railways. The first point is to fix upon the main stations of the traverse on *each* of which the tacheometer is to be set up. These stations may be selected at the most convenient points for commanding the strip of country to which the survey is to be confined, and need not necessarily coincide with the final location of the centre line, although it is advisable to make them do so as nearly as possible. The staff must first be held upon the bench mark, or on a point whose level is known or assumed as a datum for the levels. Book the vertical angle and the staff readings. The staff reading of the axial hair should be taken. Also measure the height of the instrument's axis above the ground or peg over which it is stationed. From these data the reduced level of the axis of the instrument and the level of the first station A is deduced as explained on page 274. It is not of course necessary to do this in the field, as the reductions may be performed in the office. The staff-holder now proceeds to all the points in succession the levels and positions of which are to be taken. It will of course save time if there are several staff-holders, if there is a special assistant to book the readings at the instrument. An expert tacheometer man can do with as many as six staff-holders. The bearings, vertical angles, stadia and axial hair readings of each point are to be noted. A good plan is to denote the main stations by the capital letters of the alphabet, as A, B, C, &c., the subsidiary stations or spot levels observed from station A being denoted by A_1, A_2, A_3, &c., those observed from station B by B_1, B_2, B_3, B_4, &c. When the capital letters are exhausted we may use the small letters *a*, *b*, *c*, &c., then the capital letters may be used over again, and so on.* In this manner all the data necessary to plot the contours, positions of roads, streams, rivers, buildings, boundaries, &c., visible from the first station A are booked.

The next thing is to book the vertical angle, bearing, and staff readings of station B. From these the position and level of B are obtained by the reductions performed in the office. The tacheometer is now shifted to B, and the first sight taken is the back sight to A. From this the distance BA and level of axis of instrument is obtained.

* Gribble, Preliminary Survey.

Measure also the height of the axis of the instrument above the peg or ground at B.* The height of the axis of the instrument as deduced from the level of B + height of instrument above ground or peg should agree with height of axis of instrument as deduced from vertical angle and staff readings observed in the back sight from B to A. The distance AB should also agree with the distance as deduced from the fore sight A to B. If these do not agree, the means may be adopted for the plotting in the office. Now all the lateral and intermediate points visible from B are to be observed and the readings booked. Any very important points previously observed from A may also be observed from B as a check and the mean values adopted. Next the station C is observed, and the readings booked. Then shift to C and take a back sight to B, then take all detail visible from C, and so on, until the whole route has been traversed. Special side stations may be used when it is desired to see over an area not visible from the main traverse. In case of any obstacle to reading of extreme stadia hairs or in case of distances so great that both extreme hairs cannot be brought on the staff, use one extreme hair and the axial one and deduce the other extreme hair reading.

Micrometer for Long Sights.—When the length of the sights exceeds the power of the instrument to distinguish the figures and divisions of the staff, the micrometer must be used. This consists of an extra pair of movable hairs actuated by a micrometer screw near the eye end of the telescope. These movable hairs are moved apart or closer together until they coincide with the extremities of two discs on the levelling staff or on a special staff. The distance apart of the discs is usually 10 ft. From the micrometer reading the distance is deduced. As a rule the working limit of the length of the sights will be about 10 chains, beyond which a micrometer should be used.

Best Class of Instrument for Tacheometry.—As the ordinary 20-power telescope will not read .01 ft. on the staff or to 1 ft. of distance at 10 chains, a 40-power telescope should be stipulated for in the theodolite, either with ordinary stadia

* It is a good plan to have the extremity of the plumb-bob chain exactly 2 ft. below the axis.

hairs to read 1 per cent. or 1 ft. per 100 ft., or with an extra anallatic lens.

Office Work.—The first work to be done is to reduce the distances and levels in the field book. When the staff is held at right angles to the collimation line, ordinary tables of sines, cosines, and tangents will suffice. Crelle's tables, which give at one inspection the products of three figures by three figures, will also be of assistance. With the staff held vertical, Table IV. will be useful. The traverse lines are first plotted in the usual manner. The details are next to be plotted by means of radiating lines from each of the traverse stations. Each of these radiating lines will be the bearing of the point where the staff was held. The distances are scaled off, and the reduced levels of the points figured on the plan alongside each. From these reduced levels all the contours are drawn in by interpolating between them. See graphic interpolation of contours, Chapter III. From the contours the proper location of the line is laid down in the usual way (see Chapter V.).

Special Protractor.—In order to avoid unnecessary ruling of radiating lines at each main station and consequent disfigurement of the plan, a special protractor has been devised. It is semicircular in shape, the scale of the plan being marked along the line of its diameter. It rotates round a needle passing through a hole in the protractor and through the main station. The protractor being set with the bearing of the line in coincidence with the meridian line through the main station, its zero will then be on the bearing of the line—that is, the diameter of the protractor will coincide with the position of the line desired to be protracted. The point may therefore be protracted by simply scaling off the distance along the scale on the protractor, after it has been set to the proper bearing.

Accuracy of Tacheometer or Stadia Surveying.—When through levels of the main stations are taken with the spirit level, the resulting longitudinal section of the line plotted from the contours will agree very closely with the actual working section. Cross sections may be plotted from the contours, and the quantities of earthwork will also agree closely with the final quantities.

When through levels are taken with the ordinary spirit level the level of the axis of the instrument is of course deduced from these levels + height of axis above peg or ground, and not from the back or fore sights.

There can be no doubt of the rapidity of tacheometry in the field work, while much more detail is likely to be surveyed with the tacheometer than by any of the older methods of surveying. Its use is, however, of course confined to open country. On the Continent the tacheometer is supplanting every other method of making preliminary surveys for public works, especially among Italian, French, German, and Spanish engineers.

Every engineer should at least stipulate for stadia hairs in the telescope of his theodolite.

Mr R. E. Middleton, M. Inst. C.E.,* in order to get practical data of the accuracy of tacheometric distances and levels, measured a base on a plot of ground near Wimbledon, the probable error of which was under $\frac{1}{4}$ in. He had a number of stations set out trigonometrically, the lengths of the lines being calculated from the measured base, while the levels were determined by levelling with the spirit level. All the stations were then surveyed and levelled by tacheometry with a 5 in. theodolite divided to 20′. Two systems were employed, the first being the tangential system by simple vertical angles, described on page 256, and for which the Bell-Elliott tacheometer (Fig. 180) is specially adapted; the second being the tacheometer system proper with stadia hair readings of an ordinary levelling staff divided into feet and hundredths of a foot. The latter is called the subtense system by Mr Middleton. On the back of the staff black lines $\frac{3}{100}$ ft. wide and 10 ft. apart were painted as sights for the tangential system. In the observations of the tangential angles both faces of the instrument were used, two verniers being read at each observation, thus making four readings for each angle. The observations were made by an assistant, so as to represent the accuracy attainable in practice. Observations on the tangential system were also taken to two lines only 5 ft. apart on the staff, and also on the tacheometer system proper, with the interceptions of only two stadia hairs in place of three. The results of the 10 ft. base and three stadia hairs only are given here:—

* Practical Observations in Tacheometry, Minutes Proc. Inst. C.E., vol. cxvi.

TANGENTIAL SYSTEM.

Length of Line.	0 to 100 ft.	100 to 200 ft.	200 to 400 ft.	400 to 600 ft.	600 to 800 ft.	800 to 1,000 ft.	Above 1,000 ft.	Total
Number of lines -	5	12	32	28	20	10	13	120
Average length per line - - -	Feet. 55.94	Feet. 164.23	Feet. 323.42	Feet. 498.36	Feet. 682.23	Feet. 902.60	Feet. 1388.42	Feet. 560.62
Average error per line* - - -	0.16	0.15	0.84	1.91	3.13	4.28	17.17	3.43
Proportional average error per 1,000 ft.†	−2.93	−0.25	+0.97	−0.91	+0.64	−0.36	+2.44	+0.68

Average length, 479.88 ft. ; average error, +0.116 ft. ; total length, 48944.88 ft.; number of lines, 102.

SUBTENSE SYSTEM.

Length of Line.	0 to 100 ft.	100 to 200 ft.	200 to 400 ft.	400 to 600 ft.	600 to 800 ft.	800 to 1,000 ft.	Above 1,000 ft.	Total
Number of lines -	5	13	41	39	31	10	1	140
Average length per line - - -	Feet. 55.94	Feet. 163.01	Feet. 328.88	Feet. 498.96	Feet. 676.97	Feet. 912.49	Feet. 1136.19	Feet. 475.64
Average error per line* - - -	0.20	0.57	0.72	1.22	1.51	2.82	2.07	1.15
Proportional average error per 1,000 ft.†	+2.18	−0.81	+0.30	−1.49	+0.09	−2.35	+1.82	−0.65

Average length, 452.00 ft. ; average error, −0.43 ft.; total length, 56047.72 ft. ; number of lines, 124.

The following are the conclusions arrived at by Mr Middleton. It may be seen that the shortest lines, taken separately, are more accurate than the longer lines. The short lines are not, however, so accurate as the long lines when taken in the aggregate and proportionally.

Comparison of the two Systems.—A comparison shows that the limit of accuracy is reached in the tangential system when the staff is 1,000 ft. from the instrument; as in lines between 100 and 1,000 ft. long the average error per 1,000 ft. is +0.116 ft.,

* These figures are arrived at by dividing the totals of the errors by the number of lines.

† These figures are arrived at by dividing the final error by the number of lines, and reducing to a length of 1,000 ft. in simple proportion.

while in lines above 1,000 ft. long the average error is +2.44 ft. In the subtense system the limit of accuracy appears to be reached at 800 ft. Between 100 and 800 ft. the average error is −0.43 ft. per 1,000 ft., while beyond that distance it increases to −1.97 ft. (see Tables). Within the above limits the results are decidedly in favour of the subtense system as regards individual errors, and are not much affected by distance.

The degree of error of the subtense system may be represented by a probable error of ± 1 ft. per 2,000 ft., which compares very favourably with careful chaining, and is much superior to chaining on rough ground. As regards the reducing, there is not much difference between the labour involved, but with proper tables the advantage would probably lie with the subtense system. As regards manipulation there does not appear to be much difference between the two systems, as each is easily mastered. For greater lengths than 1,000 ft. in the one case and 800 ft. in the other the tangential system is best, especially if sighting discs are attached to the staff. With these by the tangential system distances up to a mile may be measured with some degree of accuracy.

In using the tacheometer, the notion of English engineers that the instrument is the responsible post must be abandoned, and the chief must devote himself to directing the staff-holders and recording the positions where the staff is held, leaving the manipulation of the instrument and the recording of the observations in the field book to assistants.

Levelling.—In levelling with the theodolite the two systems are identical, and the results attained the same. In 160 continuous observations, the average error per observation was +0.152 ft. The closing error in a length of 7,247 ft. was ±0.54 ft.; in a length of 23,744 ft. it was ±0.65 ft. This makes the probable error per mile 0.40 ft. in one case and 0.30 ft. in the other. If the error of each station be examined independently, and the results averaged, the error in 160 observations, the total length of the lines being 92,269 ft., and the average length of each line 576 ft., is at the rate of 0.0067 ft. per observation, and 0.02 ft. per mile.

Of lines between 0 and 800 ft. in length, the number is 131, with an average length of 436 ft., the error per observation being 0.017 ft., and the probable error per mile 0.06 ft.

For the longer lines, their average length is 1,172 ft., the error per observation 0.04 ft., and the probable error per mile 0.085 ft. The effect of long lines in producing inaccuracy is therefore small. At one point the instrument was out of perfect adjustment, and in more experienced hands the errors above recorded might have been materially reduced.

American Practice in Tacheometry or Stadia Surveying.*—Tacheometry is much used by American engineers. American experience in this work shows that the accuracy attained depends on the care taken in finding the constants, as explained on page 264, and upon the roughness of the country surveyed. Results of surveys give more accuracy in moderately rough country where the line of sight is at a considerable height from the ground, thus avoiding the errors of difference of refraction of the upper and lower lines of sight.

In general, under the most unfavourable circumstances, a limit of error of 1 in 300 is easily reached, while in favourable circumstances the error may be reduced to 1 in 2,000 or less.

On the United States Lake Survey the mean error of 141 traverse lines was 1 in 650. The lengths of the lines averaged from 800 to 1,000 ft., the maximum length being 2,000 ft. The officially stipulated for limit of error was 1 in 300.

On the Mexican Boundary Survey the stadia method was used for the survey of the detail of 1,750 sq. miles, and also for the measurement of the boundary line, the conditions under which the work was done being very unfavourable to accuracy.

On this survey, on fourteen traverses averaging 8.2 miles each in length, the closing error in level was 0.17 ft., or 2 in. per kilometre (.6 mile). The closing errors in level increased rapidly as the average vertical angle increased, and in the roughest country seventeen traverses averaging 4.1 miles each in length, with a large average vertical angle, had a closing error in level of 0.59 ft., or 7 in. per kilometre. The average error in distance was 1 in 752.

On the St Louis Topographic Survey a traverse over 40 miles in length was run, and the mean error in distance was 0.18 ft. or 2 in. per mile. The total closing error in level for the 40 miles was 0.64 ft. At a point 20 miles from the commence-

* Johnson, Theory and Practice of Surveying.

ment the error in level was zero, while at 27 miles it was more than twice the final closing error, which shows that in stadia work the errors compensate.

Surveys under the Mississippi River Commission attained the following results:—Thirty-six traverses of a mean length of 1.7 miles each were carried over rugged country ranging in level from 100 to 300 ft. The average error in level was 0.59 ft. per mile. Seventy-four other traverses 1.8 miles each in average length were run over nearly level ground, and the average error in level was 0.31 ft. per mile. The maximum error in level was 2.5 ft. in a traverse 2.7 miles long. The minimum error in level was 0.03 ft. in another traverse of about the same length.

There can be no doubt that the tacheometer or stadia system is the best for preliminary surveys undertaken for the location of a road or railway or other line of communication, and also in many cases for the details of surveys of more or less extensive areas; it is the most rapid and cheapest, and more detail is likely to be surveyed by the stadia method than by any other system. Also the distances on the main traverse lines by the stadia method will be much more accurate than chaining as usually executed on such a survey. On the Mexican Boundary Survey, in a test measurement of 100 miles by stadia work and by chaining, the stadia was found compared with the true triangulation distance to be much more accurate than the chain. The stadia system can only be used in *open* country, but even in dense forest or jungle the *main traverse line* may be surveyed by the stadia method. The *final* setting out of the line of a railway or road must, however, be done with the theodolite, level, and chain, pegs being put in at every chain, as described in Chapter V. Owing to the general use of the Ordnance maps for home work, tacheometer or stadia work will be chiefly useful abroad.

Tacheometry was first used in Italy about 1820. It was first officially used in the United States in 1864, when it was used on the United States Lake Survey.

Example of Tacheometer or Stadia Survey.—Fig. 184A, Plate XI., is an example of part of a tacheometer survey for a railway executed by Mr Neil Kennedy, M. Inst. C.E. This survey was for the location of the railway, and the dotted line ABCDE is the line of the traverse, A, B, C, D, E being the

stations on which the instrument was set up. All the details and contours shown in Fig. 184A were taken by sights from these stations. The continuous thick black line shows the centre line of the railway as located from the contours and details of the plotted survey. Fig. 184B is a section of the part of the railway shown in Fig. 184A, and the continuous line is the longitudinal section of the surface obtained by levelling along the located line after it was set out, while the dotted line is the longitudinal section along the same line as obtained from the contours and levels of the tacheometer survey (Fig. 184A). It will be seen that these agree quite nearly enough for the purposes of the preliminary location and preliminary estimates.

CHAPTER VII.

TUNNEL ALIGNMENT AND SETTING OUT.

Surface Alignment.—The centre line of a tunnel must first of all be carefully ranged out on the surface from end to end. In any case for at least each portion between contiguous shafts a connected line must be ranged out on the surface extending across both shafts, before driving can be begun. In tunnelling through very high snow-clad mountain ranges it is often impossible for obvious reasons to range out the line of the tunnel on the surface directly along the centre line to be followed. In such cases the shafts or extremities of the tunnel must be connected by a triangulation or a traverse which must be conducted with the greatest possible care and checked to ensure accuracy. When the line is of great length it is not possible to measure the lengths of the lines of a traverse with the required accuracy, and in that case a triangulation is made from a carefully measured base. The alignment of the great Alpine tunnels was effected by triangulations (see page 307).

Surface Alignment over Hills.—When it is possible to range out the line directly along the centre line and over hills or mountains of moderate height, very long sights of 2 or 3 miles may sometimes be necessary. In this case the ordinary small theodolite will not suffice, and a more powerful instrument must be used. The line is first of all ranged out as nearly as possible with the theodolite, and two points in this line are selected as fixed points for the final alignment. Observatories are then erected at suitable places where the slope of the surface changes, and where they will command a clear view of the shafts. A transit instrument is placed in these observatories, and by means of it the line is finally ranged out, the two fixed points on the theodolite line being taken as the base. These observatories are simply brick, concrete, or masonry piers carefully built so that the transit instrument when placed in its bearings on the piers

may be accurately adjusted to the centre line. In the transit instrument (see Figs. 187, 188) the vertical arc and horizontal limbs are dispensed with, and the telescope is levelled by means of a striding level placed across its horizontal axis.

Example of Surface Alignment over Hills: Alignment of the Totley Tunnel.—The following description of the setting out of the Totley Tunnel on the Dore and Chinley Railway* is a good example of surface alignment over hills. The Totley Tunnel is the longest tunnel in the United Kingdom with the exception of the Severn Tunnel.

"*The Alignment Aboveground.*—The greatest precautions were taken to secure the accurate setting out of the centre line. As the longitudinal section, Fig. 185, Plate XII., shows, the profile was favourable to this work, distinct changes in the surface taking place at convenient distances, and high ground beyond each extremity of the tunnel accommodated terminal stations which could be seen from the summit observatory; there was no need to reverse the transit instrument except at that point. The line having been fixed with as much accuracy as could be obtained with a 6 in. theodolite, brick observatories were built at the extreme stations (Bradway and Sir William), and at each end of the changes of the ground surface over the tunnel. In addition to these, an observatory (No. 3 west) was also built beyond the entrance at Padley, at a level to command the heading on the 1 in 1,000 gradient; and a station was fixed at the foot of the hill beyond (No. 4 west), to enable these two points to be seen from within the heading whenever necessary.

"*The observatories* (Fig. 186) were built hollow of brickwork in cement and capped with stone. A large flat cast-iron plate, having a hole 6 in. wide in the centre, was let into the cap and run with cement. Upon this the transit instrument rested. A brass scale, $1\frac{1}{2}$ in. wide, divided into inches and twentieths of an inch, was fixed across this central hole in the plate, and a plumb line from the centre of the instrument could thus be let down through the hole in the plate to touch the side of the scale. The transit instrument was of the fixed type, with a 3 in. object

* Vol. cxvi. Minutes Proceedings Inst. C.E. Mr Percy Rickard, M. Inst. C.E., "Tunnels on the Dore and Chinley Railway." Given by permission of the Council of the Institution of Civil Engineers.

glass and a 30 in. telescope (Figs. 187 and 188). In order to enable it to be used with facility at different observatories as required, an extra cast-iron base A was added, resting upon three

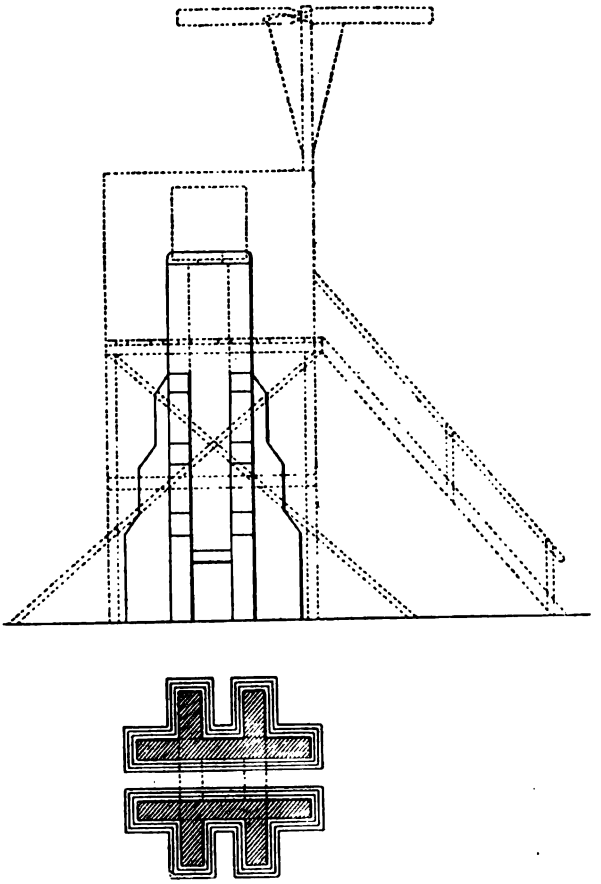

Fig. 186.—Observatories.

levelling screws, and upon it the ordinary standard rested; the latter was pivoted at one end and was secured between two slow-motion adjusting screws at the other. A hole 3 in. in diameter was made in this base plate to allow the plumbing hook to pass

freely through it from the bottom of the standard to which it was attached. This extra base plate enabled the instrument to be levelled with accuracy, and also provided a slow horizontal movement similar to that of an ordinary theodolite.

"*In setting out the line*, two points in that set by the small instrument were taken as fixed, viz., the summit and No. 1 west (Fig. 185), and from the summit observatory the line was set upon the extreme observatories east and west and upon No. 1 east. The instrument was then removed to No. 1 west, and, with Sir William observatory as a fixed point, the line was set on No. 2 west. The instrument was then removed to No. 2 west, and the line was in the same way set upon No. 3 west, and similarly on Nos. 2, 3, and 4 east. The instrument was subsequently set up at Bradway and Sir William observatories, and the centre lines of No. 4 east and No. 3 west were checked. No. 4 west was then set out from No. 3 west, and checked from No. 2 west, and the external line was complete.

"*The objects found most easily distinguishable for sighting upon in the open* were:—(1) A board with a 3 in. central white line painted upon a black ground, fitted with a plummet and fixed by guy ropes (Fig. 189). A large white calico screen was fixed behind the board, and a few feet away so as to avoid shadow, and was inclined towards the sun. This arrangement was used at the terminal stations, which were each more than 3 miles from the summit, but could only be clearly distinguished at that distance so long as the sun was in front of the screen. (2) An iron tripod, 6 ft. high (Fig. 190), with adjustable screwed legs, from which was suspended a heavily-weighted fine steel wire. On this wire was centred a 1 in. blackened tube, 5 ft. long. This instrument was used against the sky, as at the summit observatory, the adjustment screws being used to bring the fine wire exactly to the right division on the scale. (3) A broad-arrow board (Fig. 191), 2 ft. by 1 ft. 6 in., faced with white cardboard, on which was drawn a broad arrow with varying widths of shaft. This was levelled with a spirit level and supported from the back with light iron stays, and was used for short distances.

"*For transferring the centre line down the shafts* the apparatus shown in Figs. 192 and 193 was used. It consisted of a winding drum carrying the wire, mounted upon an iron frame with a ratchet and pawl to secure it in any position. The wire passed over an

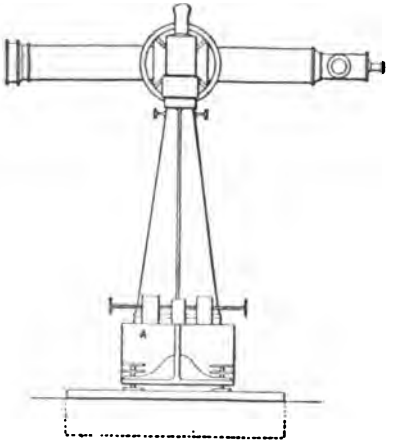

Fig. 187.—Transit.

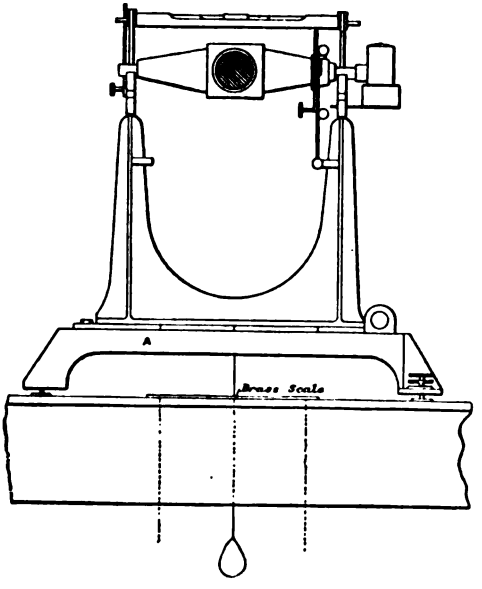

Fig. 188.—Transit.

adjusting screw and was brought into line by turning the screw in either direction as required.

"Great difficulty was experienced at the outset in finding favourable weather for fixing the line upon the terminal stations, as it was essential that the atmosphere should be clear, sufficiently cool to prevent aberrations due to heat, and yet still enough for the observatory to be free from vibration. It was also necessary that the time of day should be such that the sun would illuminate the front of both screens behind the objects to be sighted. The only times when the weather answered all these requirements

Fig. 189.
Sighting Board.

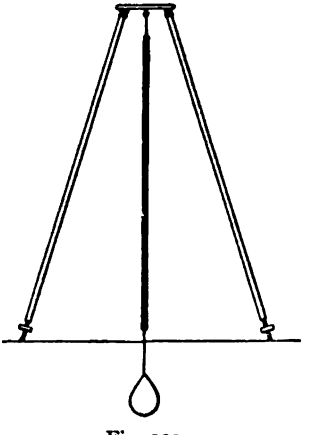

Fig. 190.
Sighting Tripod.

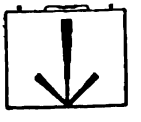

Fig. 191.
Sighting Board.

were rare occasions in the spring and autumn, between the abatement of a high wind and a fall of rain; and as these could not be predicted beforehand, and a day's preparations were necessary, much time was wasted. The greatest difficulty was found in sighting across the Derwent valley westwards, but excellent opportunities for sighting east could always be obtained at sunset after a warm summer's day.

"*The Alignment Underground.*—After the centre line had been fixed upon the observatories at the surface, the positions of the four shafts at Totley were set out from them; and when the shafts had been sunk, the centre line for the headings was trans-

ferred below, in the ordinary way, by weighted wires suspended from the top, the lines being produced underground by a small theodolite until the headings met between the shafts. The brick lining was then proceeded with, and the centre line was again carefully transferred below upon byats fixed securely into the brickwork at No. 4 shaft and at B shaft. With this bearing the line was produced by the large transit instrument westwards into the heading as required. At Padley the line was produced

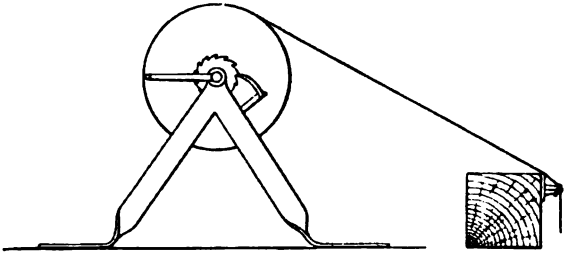

Fig. 192.—Apparatus for Transferring Line down Shafts.

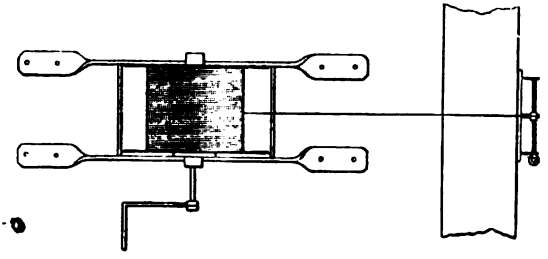

Fig. 193.—Apparatus for Transferring Line down Shafts.

into the heading direct from the observatory (No. 3 west). When used underground, the large transit instrument rested upon a balk of timber, which was supported at each end so as to clear the temporary road. The extreme range of the instrument below ground, when the air was clear, was about $\frac{3}{4}$ mile; but as the headings advanced not more than 10 or 15 chains could be seen under the most favourable circumstances, and the small instrument was then used in preference. The line was marked with a file

upon iron dogs, driven into the byats or head-trees in the usual way; and to avoid instrumental errors, the line was set out twice, the telescope being turned over transversely in the bearings between the operations. The mean of the two results was the centre line adopted. The objects used for sighting upon underground were:—(1) For long distances, a large circular-wick oil lamp of 40 candle-power (Fig. 194), fitted into a circular wrought-iron frame which was suspended by a wire; and (2) for short distances for use with the smaller instrument, a carriage candle, fixed in a weighted frame and suspended in the same way (Fig. 195). For signalling long distances with the large instrument an electrical signalling apparatus was employed. It consisted of two similar instruments, in each of which a 7 in. single-beat bell was mounted, with a battery enclosed beneath, together with ¾ mile of single guttapercha-covered cable wound on a drum, mounted on a portable frame. The cable was thus readily paid out from the trolly on which the instruments were conveyed every time it was used, and the return was made to earth through a galvanised-iron plate temporarily sunk into the ground. With this apparatus messages could be sent in either direction, and to prevent misunderstandings all signals were repeated by the receiver, and any error in transmission could then be corrected by the transmitter. The advantages of electrical signalling were incalculable, for, besides overcoming the difficulty of setting out at such long range in a narrow heading, it saved that straining of the eye for signals on the part of the operator which is so trying under ordinary circumstances, and thus favoured better work. For signalling short distances with the small instrument, a red, white, and green hand lamp was used. When the headings met, the difference between the centre lines of the two headings was found to be 4½ in., and the difference between the levels was 2¼ in."

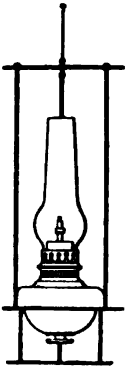

Fig. 194.
Sighting Lamp.

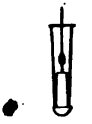

Fig. 195.
Sighting Candle.

Surface Alignment in Towns.—In towns the centre line

of the tunnel is usually approximately the centre line of the street or streets. Cases, however, often arise when it is either not possible or inconvenient to follow the line of the streets. When the line follows the approximate centre of the streets, boreholes may be put down from the surface from time to time. The underground line may then be checked and corrected by these, and in this manner a line having curves in it may be driven. For very long distances between shafts, however, such as across wide rivers, on which no intermediate boreholes can be put down as checks, the centre line is usually made straight. When buildings, large warehouses, &c., intervene as obstacles, they must be carefully traversed round, and in all cases there should be a check traverse made.

Instruments for Setting out Tunnels.—As regards the instruments to be used, all the setting out of some of the largest tunnels in this country has been done with a 6 in. theodolite reading to 20 seconds, but for ordinary tunnel work a 7 or 8 in. instrument reading to 5 or 10 seconds is more suitable. Fig. 196 shows Troughton & Simms' 7 and 8 in. instrument. With micrometer microscopes 1 second may be obtained by estimation on these instruments. Micrometers are shown in illustration of 12 in. theodolite in Chapter XII.

Setting out Curves in Tunnels.—When curves occur in tunnels the curve itself is not ranged out on the surface; the intersection angle only is measured and the distances to intersection point. If the intersection point is inaccessible, measurements are taken from which the intersection angle and distances to intersection point are calculated. From these data the position of the tangent points is calculated, and the curve set out underground in the usual manner (see Chapter V.). In sharp curves the difference between the chord and the arc is to be allowed for.

Taking Measurements. — All measurements should be taken with the steel tape, and it will be found best to measure along the surface of the streets, afterwards taking levels to reduce the measurements to the horizontal. The best way to mark the end of the tape is to make a mark with a piece of chalk, and then accurately mark on this the measurement with a fine-pointed lead pencil. All linear measurements should be repeated, and should agree to a small fraction of an inch.

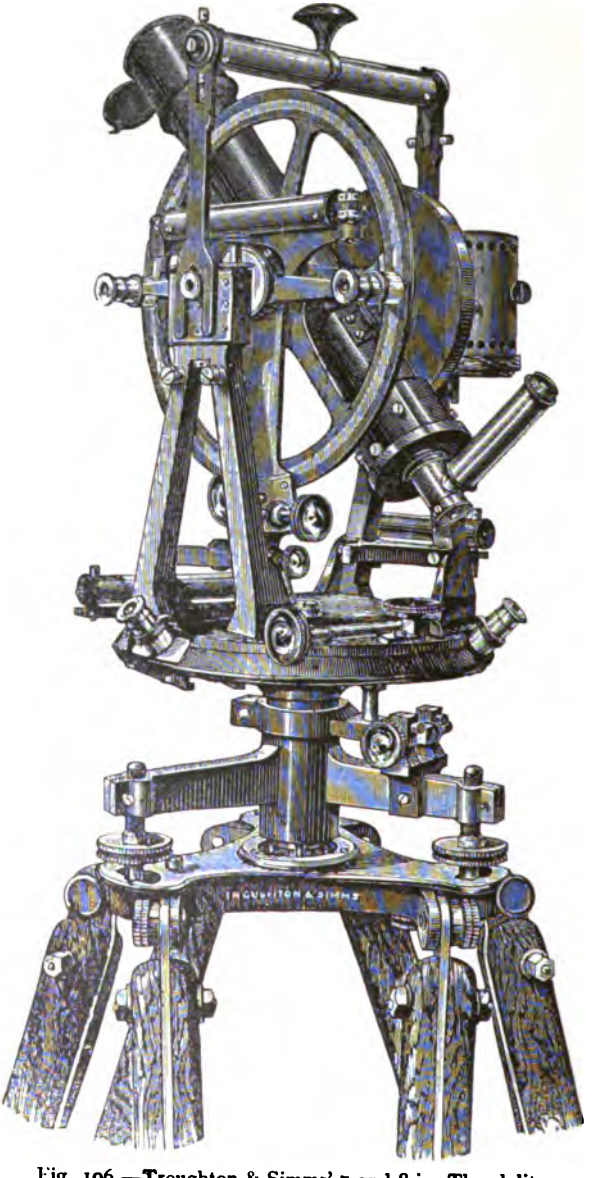

Fig. 196.—Troughton & Simms' 7 and 8 in. Theodolite.

Observing Angles.—Angles should be observed three times, all three verniers of the instrument being read at each observation. This is generally sufficient, but in special cases it may be necessary to repeat and check observations oftener.

Marking Centre Line on Surface.—The centre line is to be marked on the surface when in towns by driving spikes or wedges of iron between the setts of the roadway, the centre line being marked on these with a steel punch. Measurements are taken from the corners of buildings to locate these points as described in Chapter II., Fig. 84.

In the country the surface line may be marked by stout pegs

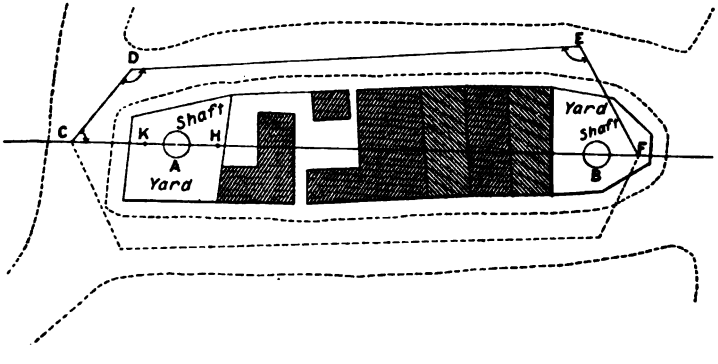

Fig. 197.—Traversing between Shafts.

having brass nails driven into them, on which punch marks are made. In special cases brick piers may be built round the pegs.

The theodolite should be kept in very accurate adjustment, and "both faces" (see Chapter II.) should be used whenever possible.

It is perhaps needless to say that in busy towns on account of the traffic nearly all the surface work must be done during the night with the aid of lamps.

Traversing between Shafts.—Suppose, for instance, that obstacles as shown intervene between shafts A and B, Fig. 197. In this case the line may be carried round by the traverse shown. Thus the angles at C, D, E having been measured, and also the distances CD, DE, the total distance CF may be calculated, as also the distance EF and the angle EFC; from these the point F may

be located, and the line laid out over the second shaft B. The point F should be checked by a similar traverse along the other street as shown by the dotted lines. In many cases when tunnelling through towns, where the line does not follow the streets, it is possible to range out the line directly along the centre line by erecting temporary platforms on the roofs of the houses, on which the instrument may be set up. This is usually more accurate for the alignment than any traverse unless in situations very much exposed to wind and vibration.

Transferring Surface Line down Shafts.—Points having been conveniently fixed on the centre line at each shaft, the next

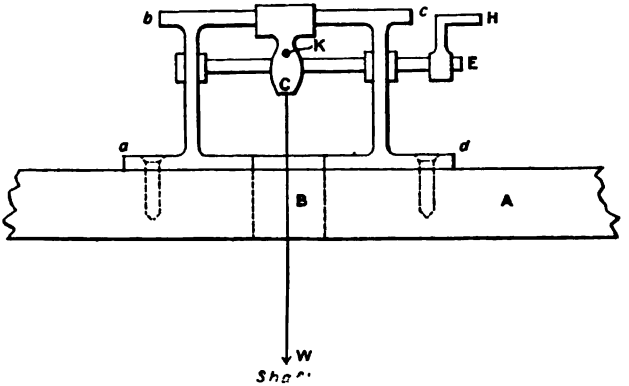

Fig. 198.—Arrangement for Transferring Line down Shafts.

operation is to transfer the line down below. This is usually effected by means of plumb bobs suspended by fine steel wire and accurately ranged in on the centre line. The arrangements used on the Totley Tunnel have already been described. Fig. 198 shows a very handy stand for suspending the plumb lines. The gun-metal stand *abcd* is about 3 or 4 in. high by about 6 in. long, and is fastened by four screws to the timber A. This is a stout timber placed across the shaft at right angles to the centre line of the tunnel, and must be strong enough, and fixed so as to be perfectly steady and free from vibration. The line is first roughly ranged over the shaft, and the hole B is cut out of the

timber. The bottom of the stand has also an oval hole in it, and it is screwed down to the timber centrally over the hole B.

The next operation is to fix the wire to the clip arrangement C. This piece is in two parts, and fits round the head *bc* of the stand, and is screwed to fit the shaft E which is turned by the handle H. The clip C is first loosely put on, and the wire inserted from underneath. It is then tightened up by the small bolt and nut K so as to grip the wire firmly. The plumb bob may now be suspended from the bottom of the wire. Then on turning the handle H, the rotation of the screw shaft E causes the clip C, together with the wire and plumb bob, to travel in either direction along E. The wire W being viewed by the telescope of the theodolite, by means of this arrangement may be brought to accurately coincide with the cross hairs of the telescope. The arrangement permits of very exact adjustment of the wires, and is simple and quick. The preliminary work of fixing the timber and boring the hole in it may be done at any time during ordinary working hours, and the stand also placed on it and screwed down. The stand is then taken off, and when the actual plumbing down of the centre line is done (usually on a Sunday) it is only necessary to fit the stand over the holes and screw it down to the timber. The wire then being inserted in the clip, the whole arrangement is ready for the theodolite in a very few minutes.

A similar arrangement is fixed on each side of the shaft, the stands being placed as far apart as possible, consistently with ensuring that the wires do not come in contact with the sides of the shaft, slides, &c. This is of course to get as long a base line as the diameter or width of the shaft will permit.

As a rule in an ordinary shaft it is not possible to get more than about 6 ft. of base between the plumb lines.

The plumb bobs are usually of lead, and 35 to 40 lbs. weight. They are suspended in buckets of water placed at the bottom of the shaft in order to diminish the oscillations and bring them to rest as soon as possible.

Setting out Underground Line.—Having ranged in the plumb lines by setting up the instrument above ground accurately in the centre line and as close as it is possible to see the wires, the wires should then be carefully examined by travelling slowly down the shaft in the cage or by the ladder, inspecting them by

the light of a good lamp to see that they are everywhere free from contact.

The theodolite is now brought down to the bottom of the shaft and set up in line with the two wires. In doing this the theodolite should be set up as *near* as it is possible to focus the telescope on to the nearest wire. The writer has seen the theodolite set up *2 or 3 chains back from the wires* and there got into line with them. This is a great mistake, as an error in bisecting the wires with the cross hairs which would be imperceptible on the wires at that distance would mean a not inconsiderable error in the position of the instrument itself. On the other hand, if the theodolite is set up as *near as possible* to the wires, they may be bisected by the cross hairs of the telescope with as great accuracy as is possible with the power of telescope used, and the error in the position of the instrument itself will consequently be the least which the circumstances permit of. The instrument should have adjusting screws for getting it into line, see *m'* and *n'*, Fig. 51. These are most useful in tunnel work, and should always be provided.

Marks for fixing Underground Line.—The line is now to be permanently fixed underground. A common way of doing this is to mark it on byats of timber fixed across the tunnel so as to be clear of the horses, &c. Nails, dogs, or spikes are driven into these, and the line marked on them by a steel punch mark or a file mark. A steel punch makes a more accurate mark than can be made by filing. In iron-lined tunnels, wooden wedges may be driven into the joints between the segments in the roof, and nails then driven into the wedges. The wedges are, however, apt to get loose and come out, and file marks or punch marks on the iron segments themselves are better. A good way of fixing the line is to drive stout pegs 3 in. square and about 3 ft. long into the formation or invert of the tunnel. These are surrounded by brickwork about 3 ft. square, parged over with cement flush with the top of the peg. After the cement is set, brass nails may be driven into the pegs, and punch marks made on them. The point made by a punch is readily seen when brass nails are used. When the line can be fixed in this way it is more convenient for setting up the instrument on it and for fixing the lights to be sighted on than when it is marked on the roof or on byats placed across the tunnel.

Transferring Surface Line down Shafts when Plumb Bobs cannot be used.—When on account of wetness of the shaft, vibration and jar from pumping engines, &c., it is not possible to get plumb lines satisfactorily down the shaft, the method adopted on the Severn Tunnel may be employed. In this case the heading was 2 miles long, and the base line 12 ft. A large transit instrument was set up at the top of the shaft and accurately in the centre line. The heading having been driven for a short distance on approximate lines, a wire 100 yds. long was stretched in the bottom, one end being at the side of the shaft, and the other at a point 100 yds. up the heading. The ends of the wire were stretched over the threads of horizontal screws, and the wire was stretched by means of weights hung from the ends. By this means on turning either screw lateral motion was given to the wire. When the wire was lighted by electric light a length of 14 ft. was seen from the transit instrument at the top of the shaft. The furthest points visible on either side of the shaft were carefully adjusted to the line by the transit instrument, and by rotating the screws; a base 100 yds. long was thus secured. When junction was effected the headings met without error.

Underground Sights and Lamps.—A very good form of sight is a large tin lamp about 12 in. square, having the front made of ground glass. If the handle when let down projects slightly beyond the face of the lamp, it is very convenient for attaching the plumb bob, the string of which is distinctly visible against the illuminated ground glass front of the lamp. The arrangement is as indicated in Fig. 199, and the lamp being placed on the brick-work behind the peg and the plumb bob attached to it, it is then adjusted by moving the lamp until the plumb bob is exactly over the punch mark on the nail.

A piece of oiled paper or tracing paper held behind a plumb line and illuminated by candles or a lamp placed *behind* the paper is a good sight for short distances. If the light is held in front of a paper the shadow of the plumb line may lead to error.

For greater distances at which a plumb line is not visible a lamp or carriage candle must be used similar to those shown in Figs. 194 and 195.

The theodolite then having been got into line with the wires suspended down the shaft, plumb lines illuminated as above

described or for greater distances lamps or carriage candles, are carefully ranged in. The points of suspension of the plumb lines or lamps are then marked, and these points mark the centre line of the tunnel permanently underground. From these the brickwork or iron lining of the tunnel is ranged in as the work proceeds.

The operation of transferring the line down the shaft to permanent marks underground should be repeated until results are obtained agreeing sufficiently closely to guarantee the meeting of the lines from each end of the tunnel or between shafts, to the required degree of accuracy.

More accuracy will be attained by making the sights underground of moderate length than if very long sights are used;—at least for fore sights.

Illuminating Cross Hairs of Telescope.—In sighting on to distant points the cross hairs will not be visible unless illuminated. In some instruments this is effected by having the horizontal axis of the telescope hollow, the end being closed by a piece of glass. On the upright supports of the horizontal axis of the telescope there is attached a special lamp provided with the instrument, for illuminating the cross hairs. The writer's experience is that this is not a satisfactory arrangement, as too much light on the cross hairs makes the distant light's image very faint, while with too little light the cross hairs cannot be distinctly seen. As the lamp lighting up the cross hairs through the axis of the telescope does not permit of ready adjustment of the amount of light thrown in on the cross hairs, a candle or lamp simply held in the hand in front of the object glass and slightly to one side of it is more satisfactory. For reading the vernier also a candle or lamp must be used.

Arrangement for Sighting on Wires Suspended in Shaft.*—There is sometimes a difficulty in distinguishing the two plumb line wires down the shaft. To obviate this, Mr F. W. Watkins devised the following illuminated slip apparatus for use in the Croton Aqueduct, New York. Two vertical strips of brass 3 in. long are attached to separate horizontal bars moving in guides and provided with a tangent screw motion, by means of which either one or both strips are moved so as to make the vertical aperture

* Minutes Proceedings Inst. C.E., vol. cxvi.

between them as small as required. One of these instruments is screwed to a plank bracket close behind each plumb line, and so placed that the further can be seen through the telescope just above the other. When the slits are adjusted exactly to the plumb line wires the latter are removed and lights are placed behind the slits. Thus two fixed illuminated lines are substituted for the wires. The results of the alignment (see page 303) showed great accuracy with the use of this apparatus.

White Reflectors to Verniers.—American theodolites for ordinary field use generally have a piece of white celluloid or ivory fixed above the vernier to reflect the light on to it. This will be found most useful for underground work especially. Pieces

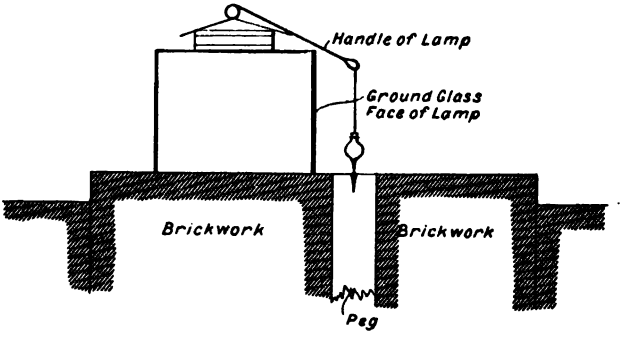

Fig. 199.—Sighting Lamp.

of white glazed notepaper will do if nothing else is at hand. If the light is not vertically over the vernier, errors of reading will be caused, without these shades.

Alignment of the Totley Tunnel.—See page 286.

Alignment of the Mersey Tunnel (Fig. 200).—The distance across the river was about $\frac{3}{4}$ of a mile. The base line obtainable between the plumb lines down each shaft was 6 ft. On the Liverpool side the shaft was on the centre line, but large warehouses and other buildings intervened between A and B. On the Birkenhead side the shaft was off the centre line as shown. It was therefore necessary to measure DC and DCA on the surface, transfer the line down below, range out DC, measure DC, and set off DCA down below.

All the alignment was done with a 6 in. Cooke theodolite reading to 20″. When the headings met near the centre of the river the alignment was found to be correct to a fraction of an inch, both for centre line and level.

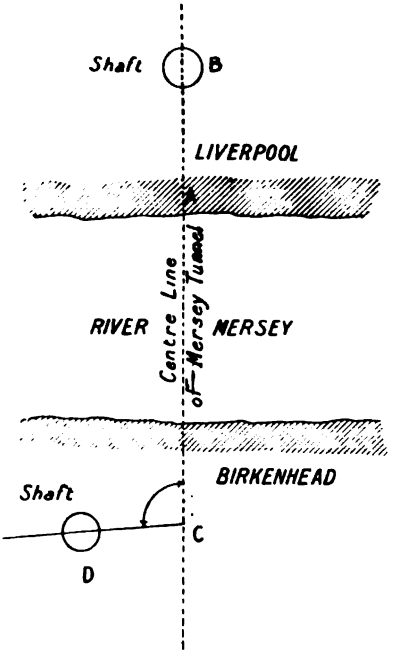

Fig. 200.
Alignment of the Mersey Tunnel.

Alignment of Dunmail Rise Tunnel on Thirlmere Aqueduct * (Fig. 201).—The base line in the shaft was 6 ft. 9 in. between the plumb line wires. The length driven from the shaft to point of junction was 1,518 yds. or 675 times the base. From the south end nearly a mile —1,715½ yds.—was driven, then the angle at A was turned, then nearly another ½ mile was driven to point of junction. To fix the position of A, 1,715½ yds. had to be measured on the surface, which fell 250 ft. in this distance. This measurement was made with rods 10 ft. long, set horizontal with a hand level, and a plumb rule similar to a mason's, specially made for the purpose.

An error of $\frac{1}{80}$ in. in the base was sufficient to account for an error of 8½ in. at the point of junction. The error at the point of junction was 8½ in. in line and 2¼ in. in level. The total length of this tunnel was nearly 3 miles, the section being 8 ft. 6 in. × 7 ft.

Examples of Errors in Alignment of Long Tunnels.—The following shows the results of the alignment in some of the longest tunnels constructed.

* Minutes Proceedings Inst. C.E., vol. cxvi.

Name of Tunnel.	Length.	Error at Junction.	
		In Alignment.	In Level.
	Feet.	Inches.	Inches.
St Gothard * - - - - -	48,872	12.99	1.97
Mont Cenis - - - - -	40,081	Nil.	12.00
Hoosac, Massachusetts - -	25,031	0.03	0.23
Ernst-August Adit, Hartz - -	23,760	1.20	0.09
Totley - - - - - -	18,687	4.50	2.25
Cowburn - - - - - -	11,106	1.00	...
Croton Aqueduct, New York -	6,400	0.09	0.01
Nepean, New South Wales - -	4,341	0.42	0.25

Levels.—The longitudinal section of the surface must be levelled from end to end of the tunnel and bench marks fixed at

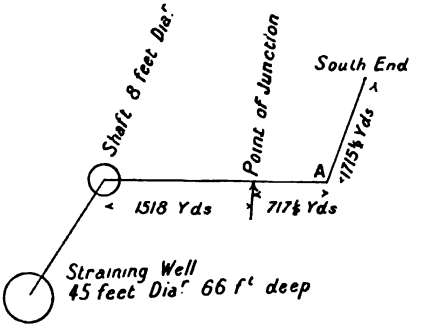

Fig. 201.—Alignment of Dunmail Rise Tunnel, Thirlmere Aqueduct.

convenient points close to each shaft. The levels of the bench marks must be very carefully checked before proceeding to transfer the levels underground.

Transferring Levels down Shafts.—To get the levels down each shaft the best plan is to accurately determine the level of a point marked on one of the slides or guides in which the cage travels, by levelling from the bench mark established at that shaft. Another point is now marked on the guide near the bottom of the shaft and the distance between these two marks is then measured with a steel tape. When the depth of the shaft exceeds 100 ft., so that this distance cannot be measured at one operation, it must be

* Minutes Proceedings Inst. C.E., vol. cxvi.

measured in tape lengths by descending in the cage 66 ft. or 100 ft. at a time, temporary platforms being fixed in the shaft to enable the assistant to hold the end of the tape. When wire rope guides are used for the cage it becomes necessary to make the measurements on the side of the shaft itself, in which case care must be taken that the tape is held vertical at each operation. The whole measurement must be carefully repeated and the results should practically coincide if the measurements are made with sufficient care.

Underground Bench Marks.—Having thus obtained the level of the mark at the bottom of the shaft, the staff is held on it, the level being set up near the bottom of the shaft, and a permanent bench mark is established.

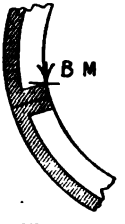

Fig. 202.
Underground Bench Mark.

Iron wedges or nails driven into the side of the tunnel at a convenient height form good bench marks, and should be marked by a broad arrow in white or red paint so as to be easily found. When the centre line is fixed by pegs surrounded by brick piers as described on page 298, these may also be used as bench marks. In the case of iron-lined tunnels the flanges of the segments form convenient points for the bench marks, as shown in Fig. 202.

Checking Underground Levels and Centre Line through between Shafts.—As the tunnelling proceeds new bench marks are conveniently fixed near to the working face of the tunnel from time to time, and the levels are thus carried forward until the faces of the headings meet, when the levels may be checked through from each shaft and the error ascertained. As headings are usually driven forward a certain distance ahead of each face, any small discrepancy between the levels from each shaft may be adjusted by putting in a "junction gradient" between the already executed portions of the tunnel. This also applies to the centre line, any error in which is rectified by putting in an extremely flat curve, or when the error is inconsiderable by putting in a short straight to join up the two lines.

Setting Ribs and Profiles.—In the case of brick-lined tunnels when there is an invert and side walls there are profiles

erected to guide the bricklayers in laying the brickwork. These have to be carefully set to the correct level by the engineer, and are adjusted to his direction by the foreman in charge. The operation of setting these profiles simply consists in levelling from the nearest bench mark and directing them to be raised or lowered until they are at the correct level, but it requires considerable care, as any mistake leads to very serious results. In the case of a new "break up," as the commencement of the widening out of the heading at any point to full tunnel section is called, a mistake is specially liable to be made as there is no adjacent finished work to check from. In setting profiles a level should always be taken on the corresponding part of the last executed portion of tunnel as a check.

When the tunnel is in good rock, invert and side walls are sometimes dispensed with, and in that case it is the ribs of the centring of the arch which are to be set. The ribs are supported on wedges by means of which they are raised or lowered to the desired level. *Both* springings of the ribs are to be levelled, and the level of the crown of the rib should also be taken.

As regards setting ribs and profiles to correct *line*, the centre line should be ranged out with the theodolite and points marked on the centre line opposite each rib or profile to be set. The latter are then adjusted to line by measuring the proper half width from the marks with a steel tape. After each alteration of the rib or profile for *level* it should be tested for *line* and *vice versa*.

Setting Segments of Iron-lined Tunnels.—In the case of iron-lined tunnels each ring is bolted in succession to the last executed portion, and is therefore to a certain extent fixed by what has gone before. Any adjustment required is made by *packing* in between the rings with wood packing. Thus if the last ring is slightly too low or too high, or an alteration in the gradient occurs, the rectification is effected by packing in the bottom or in the top. Similarly if the last ring is slightly off the centre line, or if there is a curve in the centre line, the adjustment is effected by packing in the left side or in the right side as the case may be.

In the case of iron-lined tunnels, therefore, the engineer may test the last executed ring of the tunnel for centre line and level, and order packings accordingly. As regards testing for centre line, the best plan is to string a line across the tunnel, fasten-

ing it to the bolts of the segments, and then range in a plumb bob hung from this line, as shown in Fig. 203. The steel tape is then stretched across a horizontal diameter and the measurements to the plumb line will indicate whether or not the position of the iron lining is correct.

As regards the levels, the crown and invert of the last rings may be levelled and packings ordered accordingly.

Taking Level of "Crown" of Tunnel.—In taking the level of the "crown" of a tunnel (the highest point of the soffit),

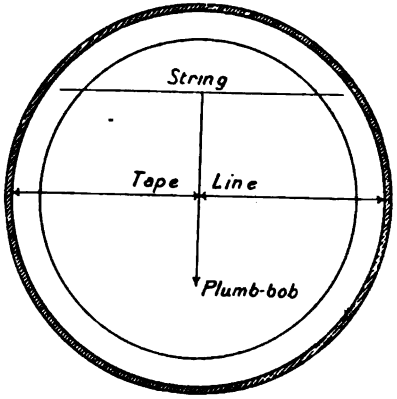

Fig. 203.
Checking Segments of Iron-lined Tunnels.

hold the staff upside down on the "crown," and book with a plus sign in the level book.

Alpine Tunnels:* Mont Cenis Tunnel.—The first long tunnel through the Alps was the Mont Cenis Tunnel. It was driven in a straight line from end to end with a gradient of 1 in 43½ on the French side, and gradients of 1 in 2,000 to 1 in 1,000 on the Italian side, rising on the French side to attain the higher level of the Italian end, and on the Italian side only to get a fall for the drainage. The length of the straight tunnel was 7.6 miles, and this was increased to 7.97 miles by two curves subsequently made to connect with the approaches at each end. The curve

* Minutes Proc. Inst. C.E., vol. xcv., "Alpine Engineering," by L. F. Vernon-Harcourt, M.A., M. Inst. C.E.

at the French end was 464 yds. long, radius 24.8 chains, on a gradient of 1 in 43½. The curve at the Italian end was 891 yds. long, part of it being the same radius and on a gradient of 1 in 33⅓. The object of driving the heading right through on the straight from end to end was of course to ensure accuracy in the alignment. The exact line of the straight tunnel was fixed by very careful triangulation and marked by permanent marks on the surface. The line was ranged into the headings from observatories erected on the line on the further side of the valleys opposite each end of the tunnel in the same manner as described on page 286.

The work of driving was commenced at both ends towards the end of the year 1857, and occupied thirteen years and one month. When the headings met the tunnel proved to be 45 ft. longer than calculated, while the heading on the French side was 1 ft. too high in level, probably owing to the miscalculation of length. The direction of the line was, however, quite correct, proving the care with which the triangulation and the alignment of the headings was effected. The average daily progress was 2.57 lin. yds., and the total cost of the tunnel £3,000,000, or about £224 per lineal yard. The tunnel was opened for traffic towards the end of 1871, or nearly fourteen years after its commencement.

St Gothard Tunnel.—This tunnel was driven in a straight line from end to end, and constructed for a double line throughout. The gradients were 1 in 172 on the north side and 1 in 500 to 1 in 2,000 on the south side, the gradient on the south side being simply to ensure drainage. The length of the straight tunnel was 9.26 miles, but with curves afterwards put in at the ends to connect with the approaches the finished length of the tunnel is 9.31 miles. The driving of the tunnel was commenced in September 1872, and dynamite was used for blasting, whereas gunpowder only was used in the Mont Cenis Tunnel. The headings met on the 29th February 1880, and the length driven from the northern end was 631 yds. more than from the southern end. The driving of the headings occupied seven years and five months, or little more than half the time of the Mont Cenis Tunnel. The average daily progress was 6.01 yds. as compared with 2.57 yds. in the Mont Cenis. When the headings met the error in the line was 13 in. and the error in level 2 in. The length of the tunnel was 25 ft. less than calculated. The greatest depth of the tunnel

below the surface was 5,733 ft. as against 5,076 ft. in the Mont Cenis Tunnel. The temperature of the rock in the tunnel rose gradually as greater depths were reached, and the highest temperature observed towards the centre of the tunnel was 87.4°, the average being 86.7°, in 1880. These temperatures corresponded very closely with the predicted temperature. There was, however, a greater difference between the underground and surface temperatures under the high plateaus than under the high peaks. In May 1882 the temperature of the rock near the centre of the tunnel had fallen to 74½°, or about 12° in two years four months, and it was expected to eventually fall to 68°. The cost of the St Gothard Tunnel was £2,327,000, or about £142 per lineal yard.

The Arlberg Tunnel.—This tunnel is 6⅓ miles long, and is straight from end to end. The gradient on the east side is 1 in 520 to provide for drainage, and 1 in 72 on the west side to reach the higher level of the east end of the tunnel. The driving of the headings began in July 1880, and junction was effected on 13th November 1883, the average rate of progress being thus about 2 miles a year. The cost of this tunnel was £1,209,400, or about £107. 18s. 2d. per lineal yard, being less than the cost of the St Gothard Tunnel by £34 per lineal yard, and less than half the cost per lineal yard of the Mont Cenis Tunnel. The progress of the Arlberg Tunnel headings was half as rapid again as the St Gothard headings, and the completed tunnel followed much closer on the heading in the Arlberg than the St Gothard. The Arlberg Tunnel therefore showed considerable advance on the St Gothard and Mont Cenis Tunnels both in point of speed and cost of construction. The length of the Arlberg Tunnel and the internal temperatures were, however, less than in the case of the St Gothard.

Projected Alpine Tunnels.—In 1898 there were projected three great tunnels under the Alps, viz., the Mont Blanc, the Great St Bernard, and the Simplon.

Mont Blanc Tunnel.—The proposed Mont Blanc Tunnel was at least as long as the St Gothard, and in most schemes about 2 miles longer, or over 11 miles, while the greatest depth below the surface would be at least 9,800 ft. It would, however, be possible to sink a ventilating shaft 1,550 ft. deep from the Veni

Valley 4 miles from the Italian entrance. The cost was estimated by Mr Bonelli at £145. 4s. per lineal yard, or about £2,938,000.

Great St Bernard Tunnel.—This tunnel was nearly 6 miles long, and its greatest depth below the surface would be 3,478 ft. The gradients would be slight, as there is a difference of level of only 7 ft. between the Italian and Swiss sides, the Italian side being the higher. It was proposed to sink three shafts, 722, 754, and 410 ft. deep respectively.

Simplon Tunnel.—This route has been long projected, and was first proposed by Mr Flachat in 1859. The work of constructing this tunnel was commenced in 1898 both on the Italian and on the Swiss sides. It will be the longest tunnel in the world, viz., 12½ miles long; the St Gothard being 9.31 miles, and the Mont Cenis 7.97 miles. It is proposed to make it in two straight lines from end to end, meeting at an angle near the centre, so as to avoid the highest ridges of the mountain, with the object of avoiding excessive depth below the surface and consequent very high temperature in the tunnel.

Notwithstanding, the depth of the tunnel below the surface will be 6,895 ft. at the deepest point, or 1,162 ft. more than the St Gothard. After careful consideration, however, it has been concluded that, although the temperature of the rock in the Simplon Tunnel may exceed the maximum temperature of 87.4° in the St Gothard Tunnel, for the centre portion of 2½ miles, and even reach 100.4° to 104° for a portion of 1¼ miles, it will be possible to execute the work by employing special means of ventilation and means for cleansing and cooling the air at the working faces. This route will put Central Europe and West and Central Switzerland in communication with the Mediterranean port of Genoa. The Simplon lies between the Mont Cenis and St Gothard Tunnels, and is likely to be a great rival. Its low summit level is specially favourable for quick through traffic, and as it will shorten the distance from Paris and Boulogne to Brindisi, it will probably secure the carriage of the Eastern mails. The Swiss entrance will be near Brigue, 680 metres above sea level, and the Italian entrance near Iselle, 750 metres above sea level. There is to be a separate tunnel for ventilation connected with the main tunnel at intervals by cross headings. The work is being executed at the expense of the Swiss and Italian Govern-

ments at an estimated cost of £3,000,000, and is expected to be completed in five and a half years. The hydraulic power for conversion into electric energy will be taken from the Rhone on the Swiss side, which will supply from 1,180 to 2,360 horse power, and from the Cairasca River on the Italian side, which will give about 2,260 horse power. The hygienic precautions for the health of the large number of workmen to be employed are very elaborate.

CHAPTER VIII.

SURVEYS FOR WATER SUPPLY WORKS.

General Features of Water Supply Surveys.—Similarly to railway surveys, surveys for water supply works may be divided into two classes, the preliminary or Parliamentary survey and the working survey. The essential features of any scheme of water supply are the storage reservoir and the conduit or lines of pipes which lead from it either direct to the population to be supplied or first to the service reservoir and filters and then to the points of consumption. The gauging of streams, estimates of rainfall available from catchment area, proper amount of water necessary for wants of population, and various other considerations which lead to a decision of the question of the source of supply and site of reservoir, service reservoirs and filters, are matters which properly come under the head of hydraulics and waterworks construction, and can scarcely be included in a work on surveying. For these see treatises on hydraulics and water supply.

Parliamentary Survey.—It being therefore supposed that the source of supply and the site of the reservoir, service reservoir, filters, &c., have been determined on, the surveying operations necessary for the preparation of the Parliamentary plans, sections, and estimates will now be discussed.

Survey of Reservoir and Cross Sections.—The first thing to do is to range out the centre line of the proposed embankment or dam. Next the lines on which the cross sections of the reservoir are to be taken should be ranged out. These cross sections should invariably be parallel to the centre line of the embankment or dam, and on a working survey are generally taken 1 chain apart, and at such intermediate points as are rendered necessary by the configuration of the ground. In a Parliamentary survey it will be sufficient to take the cross sections only where the ground changes its slope materially in degree or direction.

The best method of procedure is to lay out two lines at right

angles to the centre line of the embankment at points near each end of it. We thus get a line along each side of the reservoir site approximating with the extremities of the cross sections as shown in Fig. 204, where AB is the centre line of the embankment and BC and AD are lines perpendicular to AB and on each side of the site of the reservoir. Having laid out these lines, it remains now to decide upon the points where the cross sections are to be taken, as at 1.1, 2.2, 3.3, &c. Ranging rods should be placed at these points, *i.e.*, at 1, 2, 3, 4, &c., on the lines AD, BC, and the distances A.1, 1.2, 2.3, &c., should be carefully measured and booked. This should be done first along one of the lines, say AD, then as the cross sections are to be parallel to AB, the distances B.1, 1.2, 2.3,

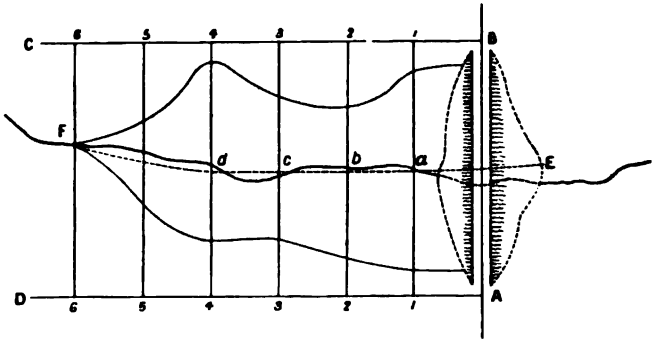

Fig. 204.—Method of Surveying Reservoir.

&c., along BC should of course be made equal to the corresponding distances A.1, 1.2, 2.3, &c., along AD.

By ranging out one line at right angles to AB near the middle of the valley we would get a line running approximately up the centre of the reservoir, along which the points at which the cross sections were to be taken might be fixed. This, however, would entail each cross section being separately "squared off," which, as the cross sections are generally of considerable length, must be done with the theodolite to get anything like accuracy.

By ranging out a line on each side of the valley, as shown in Fig. 204, the parallelism of the cross sections is ensured by the simple measurement of the distances A.1, 1.2, 2.3, &c., and B.1, 1.2, 2.3, &c. Also having once accurately measured the distance AB between the lines AD, BC, we have a good check on the chaining

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of the cross sections; as in taking the cross section 2.2, for instance, starting from the point 2 on line AD, on reaching the point 2 on line BC the measurement should be equal to AB, and so with each cross section.

Having definitely ranged out the lines of the cross sections the rest of the work is simple, as it is merely a matter of taking the levels along each cross section, as described in Chapter III.

Cross and Longitudinal Sections of Embankment.—In addition, cross sections have to be taken along the site of the embankment and a longitudinal section of the embankment. These are taken in the usual way as described in Chapter III.

Extent of Cross Sections and "Top Water Level."—Knowing the quantity of water proposed to be impounded, by plotting one or two of the first cross sections taken, and trying lines on them to represent the surface of the impounded water or "top water level," and by mental estimation of the contours of the valley, the surveyor will be able to arrive at a fair approximate idea of the probable level of the surface of the impounded water, and he will extend his cross sections up each side of the valley as far as this level and for a certain distance beyond it to allow for contingencies or alterations. All existing details, such as fences, roads, buildings, &c., within the proposed limits of deviation must be carefully surveyed.

Having plotted the cross sections, a level is assumed for "top water level," and drawn on each cross section. The quantity of water impounded up to this level is then calculated from the areas of the cross sections, and the "top water level" is raised or lowered until the desired quantity of water is arrived at, and the "top water level" definitely fixed.

Plan of Reservoir.—The plan of the reservoir is now easily made simply by drawing the centre line of the embankment and the base lines AD, BC, then laying down on the lines 1.1, 2.2, 3.3, &c., the points to which the edge of the water extends, as taken from the cross sections on which the "top water level" has been drawn. By joining these points the desired plan of the reservoir is obtained.

Plan of Embankment.—The top width, general form of cross section, and level of the top of the embankment having been

determined on, the cross section of the embankment is plotted on each of the cross sections taken along its site, and from these again the plan of the embankment is easily laid down.

Longitudinal Section of Reservoir.—A longitudinal section on a line up the approximate centre of the reservoir, as along EF, Fig. 204, must also be prepared for Parliamentary work. This again is easily plotted from the cross sections 1.1, 2.2, 3.3, &c., by simply laying down the points *a*, *b*, *c*, &c., along a datum line, and plotting up the levels at these points, as taken from the cross sections 1.1, 2.2, 3.3, &c.

Fence Line and other Details.—The fence line and the limits of deviation are now to be drawn on round the reservoir, which, with the addition of any details, such as the overflow weir, byewash, upstand and bridge, &c., completes the plan.

Byewash.—In cases where a byewash of considerable length is required owing to the lie of the ground, special local circumstances, &c., a special survey must be made and a section of it taken. In this case the byewash will be located precisely on the principles of locating a railway as described in Chapter V. (see page 225).

Line of Pipes.—As regards the line of pipes from the storage reservoir to the service reservoir and filters or to the town, as a rule it may be said that whenever possible these are laid along one side of the public road. This is done to avoid passing through private property where wayleave must be obtained, and where compensation for damage during laying of pipes would have to be paid. Also the pipes are more easily got at for repairs when laid in the public road, and no damage is done to growing crops, &c., when they have to be taken up for repair, &c.

As the water in the pipes is under a considerable head of pressure, the pipes may follow the undulations of the ground, and they are usually laid so as to have a minimum cover of 2 ft. 6 in. to 3 ft. The chief point to avoid is "summits" in the line of pipes, and where these are unavoidable, air valves must be provided. In laying down the gradients, therefore, the surveyor will as far as possible avoid introducing "summits," by which is meant two gradients rising to a point as $\wedge$. For the rest, as the pipes may follow the undulations of the ground, and they will for

the most part be laid along the public roads, the work of making the plan and section is simple, and amounts to little more than taking levels along the route wherever the ground changes. In open country, however, the leveller may exercise a little skill in following the contour of the ground so as to avoid deep cuttings, &c. In this country the 6 in. and 25 in. Ordnance maps are of course always used in getting out the plan and section of the line of pipes, as also the plan of the reservoir. The plan of the line of pipes must show the proposed centre line of the line of pipes, the limits of deviation, and all existing details of the ground within the proposed limits of deviation.

The section must show the level of the invert of the pipes, the level at each change of gradient being marked in figures, the gradients, and the surface line. The chainage must be marked along the datum line, which must be referred to a fixed point, usually an Ordnance bench mark near one end, and the total length of the line of pipes or conduit must also be given.

Aqueduct or Open Conduit.—When tunnel is substituted for pipes, the surveyor will have no special difficulty in getting out the plan and section, but when aqueduct or open conduit is substituted, he must exercise some skill in locating the line on the given gradient, so as to avoid unnecessary cutting or bank, in the same manner as in laying out the centre line of a railway (see Chapter V., page 225).

Service Reservoir and Filters.—When service reservoirs and filters are included in the scheme, plans and sections of these must of course be provided. There is, however, nothing calling for special remark in getting out these, which rather come within the province of the engineer than of the surveyor.

Working Survey and Pegging out.—As regards the detailed or working survey, it need only be said that the operations are essentially the same as for the Parliamentary survey, only more care is exercised so as to get accurate data for setting out the work, quantities, &c. As already stated, the cross sections of the reservoir in this case will be taken at a minimum distance of 1 chain apart. Pegs should be driven in at every chain to one side of the centre line along the route of the conduit or line of pipes, and levels should be taken on the centre

line and on each peg, from which the depths for the excavation and laying of the pipes may be given.

Example of Survey of Reservoir.—Fig. 205, Plate XIII., is an example of the Parliamentary plan of a reservoir containing about 170 million gallons, executed as described in this chapter, and taken from actual practice.

CHAPTER IX.

HYDROGRAPHICAL OR MARINE SURVEYING.

Instruments: Sextant.—For description of sextant and its adjustments and use, see Chapter X.

Determination of the Shore Line and High and Low Water Marks: Survey of Coast Line.—An extensive survey of a coast may be made either from small boats or from the ship, or by traversing round the shore on foot. By the latter method a more particular survey of the creeks and indentations may be made, as much detail is apt to be overlooked from a boat, and from the ship itself only the general outline can be surveyed. In surveying a coast on foot, first the main stations are fixed by triangulating or traversing round the coast. The intervening work is then put in by sextant angles or bearings with prismatic compass, and distances telemetrically measured. A system adopted in the Navy is as follows:—By one surveyor a 500 ft. lead line is run out at right angles to the line to be measured and at its commencement. The second surveyor proceeds to its termination and measures the angle θ subtended by this 500 ft., whence the length of the line is 500 cot θ. The intervening work is then put in with compass bearings and telemetric distances measured with a sextant and 10 ft. pole. When three prominent points can be fixed the extremities of the traverse lines may be determined by the two angles subtended by these three points (the "three point problem," see page 320).

"Running Survey" from the Ship.—"Running survey" from the ship is executed as follows:—First the ship's position relative to the shore is to be carefully determined. When the position of three prominent points on the shore within view of the ship is accurately known, the best way to find the ship's position is to measure the two angles subtended by the three points and fix the position by the three point problem. When this is not available, a base is measured by sound signals. A party is landed

on shore and guns are fired by them and also on board ship. The interval between the flash and the report is measured by counting the beats of a chronometer or watch, and from this time interval and the temperature of the air at the time of observation the distance between the ship and the station at which the shore party are situated is deduced from the velocity at which sound travels. The reason for firing and observing both on shore and on board ship is to make allowance for the effect of wind on the time interval, there being obviously a retardation in one case and an acceleration in the other when the wind is blowing As, however, in the case of the retardation the disturbing cause has acted for a longer time than in the case of the acceleration, the arithmetical mean of the time intervals as determined on shore and on board ship is not to be adopted. The following formula is used—

$$T = \frac{2tt'}{t+t'}$$

where T is the mean time interval required and t and t' are the time intervals found on shore and on board ship. The mean time interval multiplied by the velocity of sound for the temperature at the time of observation gives the required distance. Sound travels with a velocity of 1,090 ft. per second at a temperature of 32° F., and for higher temperatures 1$\frac{1}{7}$ ft. more per degree of temperature; for lower temperatures, 1$\frac{1}{7}$ ft. less. The distance having been found and the bearing observed, the ship's position relative to the coast is now accurately laid down on the chart. The ship then moves slowly forward at 3 or 4 miles an hour, towing the patent log, so that, its path being correctly laid down on the chart, its position at any given time is known. During the ship's course soundings are taken from time to time and angles and bearings are taken to all the main points along the coast, the outline of which is sketched in between these points. In this manner 30 or 40 miles of coast may be surveyed per day.

Surveys for Docks and Harbours.—In the case of smaller surveys for dock and harbour works, the high and low water marks are to be surveyed by ordinary traversing with the theodolite along lines approximately coinciding with the high and low water marks. These lines are chained and offsets are taken to the water mark in the usual manner. Details of the coast line, promontories,

cliffs, &c., are surveyed similarly, or by any of the methods described in Chapters I. and II.

As the low water line is bare for only a short time, it is best surveyed with the sextant by the trilinear method or the three point problem (page 320), observing the angles from each salient point of the low water line to three prominent points already fixed along the shore and sketching in between the salient points.

When the shore is inaccessible a base line or lines must be measured on the water, usually fixing the ends by the trilinear method. The shore and high and low water marks are then surveyed by sextant angles from each end of this base.

Soundings: Sounding Lines and Rods.—The taking of soundings forms one of the most important parts of a survey for dock, harbour, or river works. For nautical purposes the lead line is most generally used, being a strong cord divided into fathoms, and having at its extremity a conical-shaped lead weight. For engineering purposes a chain is generally used when the depth is considerable, and in shallow water a rod divided into feet and tenths and weighted at its lower end. When it is desired to get an idea of the material of the bottom, a piece of tallow is inserted in a hollow in the lower end of the lead weight, to which loose specimens of the bottom adhere and may be brought up. Specimens of the bottom may also be brought up by using a jagged plunger, otherwise the nature of the bottom may be found by boring, dredging, or diving.

Taking and Fixing Position of Soundings.—Soundings on a coast are usually taken along straight lines, approximately at right angles to the contours of the shore. The following are methods of determining the positions of the points at which the soundings are taken. In Fig. 206 let A and B be two fixed points on shore in the line of which the soundings are to be taken at the points 1, 2, 3, &c. The boat being kept in line with A and B, at each of the points 1, 2, 3, &c., observe with a sextant the angles B1D, B2D, &c., to a third fixed point D. To plot the points 1, 2, 3, &c., all we have to do is to draw the line ABC through the points A, B, and through D the line DE parallel to AB, then lay off at D the angles ED1, ED2, ED3, &c., equal to B1D, B2D, B3D, &c., respectively. The points where the line ABC is intersected are then the

required points 1, 2, 3, &c. The point D should be selected so that none of the angles are less than 30°.

Another method is as follows:—In Fig. 207 let it be required to take soundings in line with AB. By means of two angles to

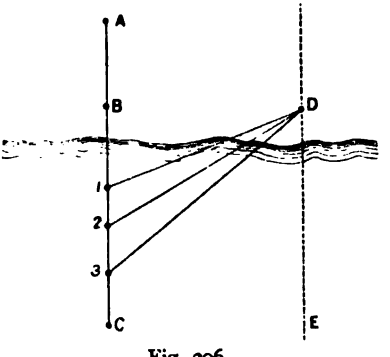

Fig. 206.

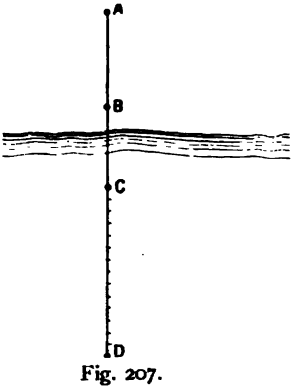

Fig. 207.

Soundings.

three known points fix the commencing point C. Now keeping the boat in line with A and B, let it be rowed at a speed as uniform as possible until the last sounding has been taken at D. Again determine the position of D by two angles to the three known points. The soundings between C and D having been taken at equal intervals of time, if the points C and D are plotted from the angles measured, the soundings may then be plotted at equal intervals along CD.

If greater accuracy is required, in place of taking the soundings at equal intervals of time between C and D, a line may be fixed at B and paid out, the distances being noted from it.

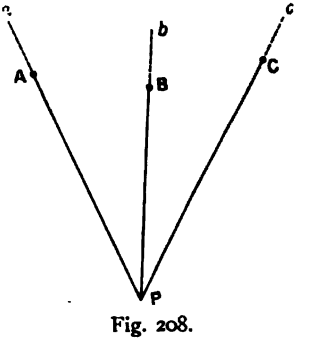

Fig. 208.
The Three Point Problem.

The "Three Point Problem."—The trilinear method of fixing a point, or the three point problem which has several times been referred to, will now be described. In Fig. 208 let

A, B, C be three known points, and let P be the point whose position is required. Let the angles subtended at the point P, viz., APB, BPC, be measured. If now these angles be plotted on a piece of tracing paper as *aPb*, *bPc*, Fig. 208, and this piece of tracing paper be moved about on the plot until the lines *a*P, *b*P, *c*P pass through the points A, B, C respectively, as shown in Fig. 208, the required point P may then be pricked through. The three points A, B, C should be well-defined objects, or signals, and the observer being in the boat at P, measures the angles APB, BPC as rapidly as possible with the sextant. The total angle APC should also be measured as a check, and when there is only one observer, three sextants should if possible be carried, one for each of the three angles, which may afterwards be read off at leisure. Any error due to the boat shifting its position during the observations is thus as far as possible avoided.

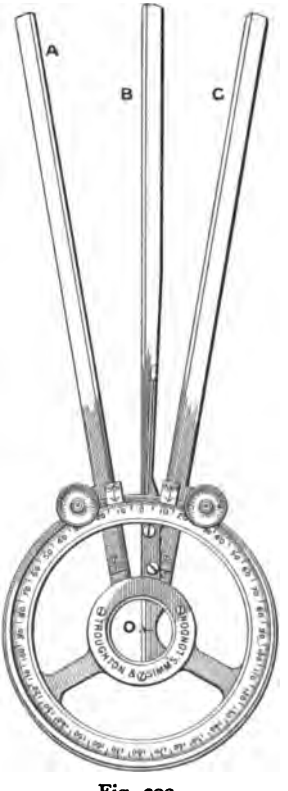

Fig. 209.
The Station Pointer.

The Station Pointer.—To save trouble in plotting, the station pointer has been devised. Fig. 209 shows the instrument as made by Troughton & Simms. It consists of a circle divided from zero both ways to 180°. The arm B is fixed and radial, and its edge B coincides with the zero of the graduations, while the mark at O is at the centre of the graduated circle. The other arms A and C are movable, and are set by means of the indicator arrows and clamp screws shown, their edges also radiating from the centre of the circle O. The arms A and C being set to the observed angles, the instrument is moved over the paper until the edges of A, B, C pass through the three known points. The position

of the required point is then pricked through at O. The instrument is in fact merely a substitute for the piece of tracing paper above described. When a great many points are to be plotted the station pointer saves the time which would have to be spent in protracting the angles on the tracing paper. The usual size of the instrument is a 6 in. circle divided to 30′ with arms 13 in. long, but it may be had with verniers reading to 1 minute, tangent screw motion, and arms to lengthen up to 30 in.

Geometrical and Analytical Solutions of the Three Point Problem.—The following are the geometrical and analytical solutions of the three point problem.

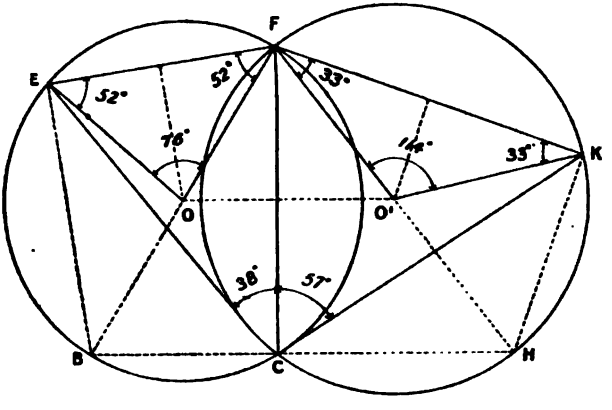

Fig. 210.—Solution of the Three Point Problem.

Geometrical Solution. — Let E, F, K be the known points observed from C, Fig. 210, the angles ECF, FCK being measured. To fix C geometrically, draw lines from E and F making angles with EF each equal to 90° − ECF. These lines will intersect at O, the centre of the circle passing through E and F, and in the circumference of which C is also situated. This may be proved as follows:—Because EF subtends an angle EOF = 180° − 2 (90° − ECF) = 2 ECF at the centre, it subtends therefore an angle = ECF at the circumference; C is therefore on the circumference of the circle whose centre is O. Describe the circle. Draw lines from F and K making angles with FK each = 90° − FCK.

Similarly the intersection of these lines o′ will be the centre of a circle passing through F and K and in whose circumference C will lie. The point C is therefore the intersection of the two circles. In Fig. 210 the observed angles ECF and FCK are 38° and 57°, we therefore draw EO and FO at 52° to EF, also FO′ and KO′ making 33° with FK. The intersection of circles of radii OE and O′F and centres O and O′ is therefore the required point C.

When one of the angles is obtuse, subtract it from 90° and protract that angle on the opposite side of the line joining the two known points to that on which the required point C is.

When the angle EFK is equal to the supplement of the sum of the observed angles, the position of C is indeterminate, as the two centres O and O′ will coincide, and the circle with this common centre will pass through the three known points, and C may be anywhere on its circumference.

The following tests may be used:—

1. The centres of the circles O and O′ are in the perpendiculars from the middle points of EF and FK.

2. OO′ is at right angles to and bisects FC.

3. Draw EB and KH perpendicular to EF and FK. Then FO and FO′ produced should pass through B and H, and BCH should be a straight line, *i.e.*, BH should pass through C.

Analytical Solution.—The distances of C from each of the known points E, F, K may be calculated as follows:—

Let $EF = b$, $FK = d$, $EFK = A$, $ECF = C$, $FCK = C'$

and let $E = 360° - C - C' - A$, $FEC = F$, $FKC = G$

Then we have—

$$\text{Cot } F = \cot E \left(\frac{b \sin C'}{b \sin C \cos E} + 1 \right)$$

$$G = E - F$$

$$CF = \frac{b \sin F}{\sin C} = \frac{d \sin G}{\sin C'}$$

$$CE = \frac{b \sin EFC}{\sin C}$$

$$CK = \frac{d \sin KFC}{\sin C'}$$

Attention should be paid to the algebraic signs of the trigonometrical functions.

Check Angles.—In the case of important stations a third angle to a fourth known point should be taken as a check, although the two angles subtended by three known points are all that is absolutely necessary to fix the position of any station.

Location from Compass Bearings to two Points only.—The position of the boat may also be found by *two* compass bearings to two known points only. The intersection of lines having opposite bearings and drawn through the two known points will be the required position. This method is quick but not accurate.

Location of Point by Simultaneous Observations with two Theodolites.—Simultaneous observations with two theodolites by two observers at two known points on shore is in theory an accurate method of fixing a boat's position, the angles being observed at a given signal or at prearranged times. In practice, however, this method is not accurate, as simultaneous observations are impossible and confusion arises.

Soundings in Rivers or Narrow Waters.—In taking soundings in a river or narrow water (Fig. 211), they may be taken in lines as in Fig. 211 and at equal intervals of time. Steam launches should be used whenever obtainable, and are to be provided with a prismatic compass on a tripod in the stern, the patent log being towed astern with its counter fixed to the gunwale. The velocity of the current may be measured occasionally by anchoring in midstream. A current meter should be used for this purpose, being better than the patent log for slow currents. See page 329.

River Surveys.—When a careful survey of a river has to be made, the following method may be adopted.* At the mouth of the river a base line AB, Fig. 212, as long as possible, is measured, the points A and B being fixed with as much accuracy as possible. One boat party proceeds from A to C taking soundings and surveying the shore line by compass bearings and distances by patent log. Similarly another boat proceeds from B to D sounding and

* Gribble, Preliminary Survey.

surveying the coast line. Having arrived at C and D, these boats anchor there. Two other boat parties now having stationed themselves at A and B, take with the sextant the angles CAB and CBA, also the angles DAB and DBA. These fix the points C and D, and at the same time the angles ACB and ADB are measured by the other boat parties as checks. The boats at A and B now proceed to D and C along the lines AD and BC, taking soundings and noting distances by the patent log by the way. On arriving at D and C they then proceed along the shore lines CE and DF, sounding and surveying as before. The boats at C and D then come up along the diagonal lines CF and DE, and so on until the whole river has been surveyed. Any discrepancy between the total distances as plotted from the sextant angles and those given by the patent logs

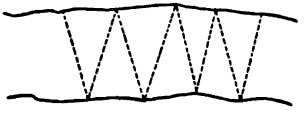

Fig. 211.
Soundings in Narrow Waters.

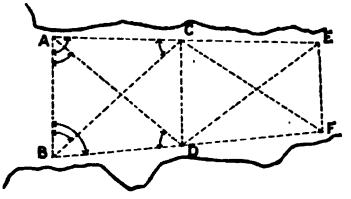

Fig. 212.
River Survey.

are adjusted by distributing the errors proportionally throughout the log distances.

When the base at the mouth of the river is short compared with the total length of the river to be surveyed, the magnetic bearings of the coast lines AC, CE, BD, DF, &c., should be observed. The course of the river as plotted from the measured base and the sextant angles may then be checked by the compass bearings, the error of which is not cumulative, as is the error of the triangulation.

Running Survey of 200 miles of River Nile.—In this connection may be noticed a survey by Mr G. Kilgour, M.I.C.E., of 200 miles of the river Nile* between the First and Second Cataracts, for the Soudan Railway, surveyed from a steam launch,

* Gribble, Preliminary Survey.

and executed in five days, or about 40 miles per day. Mr Kilgour used time measurement of speed, for which there were three scales, full speed, half speed, and dead slow. The value of the scales was found by an accurately measured base on the centre line of the vessel, at the two ends of which, near the bow and stern, two lines of sight were fixed square with the centre line, by which the time was noted at which the vessel passed some well-defined object on the bank at the three rates of speed. The survey was executed from the deck with a plane table adjusted constantly in the meridian by the compass, and astronomical observations were observed at night.

Datum.—For nautical surveys the datum to which soundings are referred is the mean low-water level of ordinary spring tides. For engineering surveys this datum will also serve when it is sensibly horizontal, but on an extensive survey of the sea coast or in tidal waters, such as large rivers and estuaries, the low-water level of spring tides varies so considerably that a fixed datum must be adopted. This may be arbitrarily chosen.

Bench Marks.—On a sandy beach where no good mark is available, a cast-iron water pipe let into the sand vertically and filled with concrete makes a good bench mark. In this country the levels and soundings are referred to the nearest Ordnance bench marks, Ordnance datum being the mean sea level at Liverpool.

Tide Gauges.—As soundings have to be taken at all states of the tide, it is important that the exact level of the surface of the water should be known during the whole of the time that the soundings are being taken. To effect this tide gauges are erected at suitable points. These in their simplest form consist of a post fixed upright in the water, and graduated in feet and tenths of a foot, from the bottom upwards, the zero of the graduations being referred by careful levelling to the datum of the survey.

In some cases it will be possible to record the whole of the rise and fall of the tide by means of one gauge, but in general when the beach slopes with a very flat gradient it will be necessary to have a succession of gauges, extending from high-water mark to low-water mark, the zero of the lowest gauge being referred to

the datum by levelling, and the uppermost mark on this gauge being made to correspond with the lowest mark on the next higher gauge, and so on, thus forming a series of steps, each gauge being carefully fixed as regards its graduations by levelling to it from the datum.

In any case, for the sake of corroboration, at least two gauges should be fixed, and independent observations made at each.

In general the number of gauges required will depend on the variations and fluctuations of the tide and the disturbing influences which act on the tide-waters. The engineer will have to decide from these considerations the number and position of the tide gauges.

For the purpose of reducing soundings it will be sufficient to observe and note the levels on the gauges every fifteen minutes, although it is often done every five minutes.

For general observations made to determine the laws of the tides in any given place, the gauges are read every ten minutes during the hour before and the hour after high and low water. During the remainder of the twenty-four hours every half hour.

In rough waters a tube perforated by a few small holes and having inside it a vertical graduated float may be used. A self-registering tide gauge is made by connecting a float of this description with a chain or cord acting on a train of mechanism which moves a pencil up or down as the float rises or falls. The pencil being pressed against a piece of paper placed on a cylinder which is revolved by clockwork, leaves a self-recorded diagram of the fluctuations of the tide.

Reduction of Soundings.—The time at which each sounding is taken is to be noted, and the soundings are to be reduced to the datum by means of the observations taken at the nearest tide gauge, or by the mean of the observations at the two gauges between which they may have been taken. This is done by ascertaining from the observations at the gauge or gauges the level of the surface of the water at the time the soundings were taken; by subtracting the sounding the reduced level of the bottom is arrived at.

The observations at the gauges being taken only at regular

stated intervals, while the soundings are taken at the most convenient times, which may often be at irregular intervals, the exact reading of the gauge at the time of each sounding is to be found by interpolating between the gauge readings.

When no regular gauge readings have been taken it is possible to reduce the soundings by the following formula :—

Let x = height of mean sea level above datum.

y = rise of the tide above mean sea level.

z = the time between high and low water.

a = interval of time between time of high water and time of sounding.

s = height of sea level above the datum at the time the given sounding is taken.

Then $s = x + y \cos 180° \frac{a}{z}$

In using this formula, note that the cosines of obtuse angles are negative.

Currents.—The direction and velocity of currents should be found and marked on the plan in miles per hour. The direction of the current may be found by observing the bearing, or the angle with some line whose direction is known, of the course of a float placed in the current.

The velocity of a current may also be found from a float by observing the exact times at which the float passes two lines at a known distance apart, and ranged out by signals or buoys in a direction approximately at right angles to the direction of the current. Or the position of the float at stated intervals may be fixed by two angles simultaneously taken to it by observers stationed at two known points.

The mean velocity at any vertical in the cross section of a channel may be taken at about 0.9 of the surface velocity, as the velocity varies considerably at different depths. For full information on this subject the reader must be referred to a treatise on hydraulics.

Current Meter.—The best instrument for measuring the velocity of a current is the current meter. The boat being

moored in the current, the current meter is attached to a pole and lowered to any desired depth. It is a brass instrument having a fan or screw the revolutions of which are registered by a mechanism somewhat similar to that of the patent log. The velocity of the current is indicated by the number of revolutions. By means of wires communicating with the meter the revolutions and velocity of current may be electrically registered in the boat.

Currents out at sea are found by comparing the day's run as given by the log with that found by astronomical observation, the difference, after allowing for instrumental error of log, being due to current.

Signals, Beacons, Buoys.—Signals and beacons are prominent objects with signals on them, &c., usually marking shoals or dangerous rocks, &c. Buoys are floating marks such as barrels or hollow iron or steel spherical or cylindrical floats. They are fixed by a chain which is fastened at one end to an anchor. The chain passes through a ring in the lower side of the buoy, and has a weight fastened to its other end to prevent the buoy from deviating to any considerable extent from its proper position. Buoys are usually placed at regular intervals on each side of a long channel, as at the entrance to a river, &c., and indicate the proper course for ships to take. They are also used in docks and harbours for mooring purposes, being for this purpose often fastened down by chains to rails firmly bedded in a large mass of concrete in the bottom. They are usually painted red, white, or black, or striped, and numbered. Stations afloat may be conveniently fixed by buoys carrying poles or flags. On dangerous coasts self-ringing bells are often attached to buoys.

Additional Details to be shown on Plans.—The nature of the bottom may be indicated on the plan by letters.

All borings, dredgings, diving, &c., should be marked on the plan, as also lighthouses, beacons, buoys, signals, and moorings.

The usual scales are the 6 in. Ordnance scale and the 25 in. Ordnance scale, *i.e.*, 6 in. to 1 mile or $\frac{1}{10560}$, and 25.344 in. to 1 mile or $\frac{1}{2500}$.

Soundings are usually noted in fathoms. The reduced level

of the bottom at each sounding may be computed and marked on the plan, and from these levels contours may be interpolated in by method given on pages 154, 155 (Graphic Interpolation of Contours). Sometimes these contours are drawn for every fathom of depth, *i.e.*, 1 fathom contour, 2 fathom contour, &c., but for engineering purposes it is more useful to have the contours drawn at given reduced levels referred to the datum of the survey.

CHAPTER X.

*ASTRONOMICAL OBSERVATIONS USED IN SURVEYING.**

Instruments: The Sextant.—The only other instrument, in addition to those already described, which the surveyor may

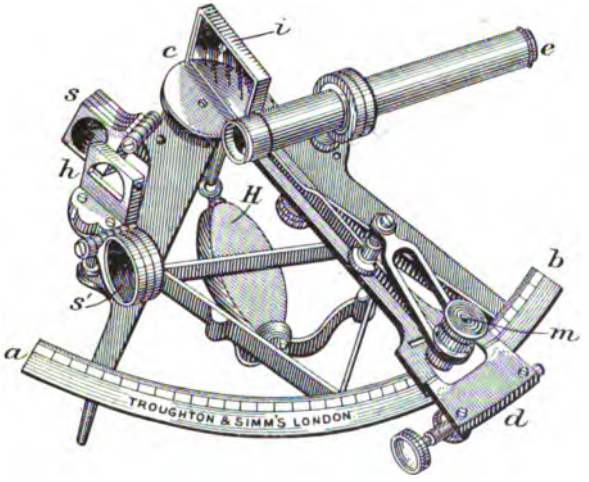

Fig. 213.—The Sextant.

require to use for astronomical observations, is the sextant. The theodolite, however, is the most suitable instrument for all astronomical observations, with the exception of lunar distance observations for longitude, which can only be taken with the sextant (see page 383). As the sextant is a much more portable instrument, and is useful for taking detached observations for latitude, longitude, time, meridian, &c., in rough explorations or reconnaissances of routes, while it answers all the purposes of a theodolite, it is here described in detail. Its only disadvantage is

* For explanation of astronomical terms, see Appendix at end of chapter.

that when the observed objects are at different altitudes it measures the oblique angle between them, while the theodolite measures the horizontal angle.

The instrument, as made by Messrs Troughton & Simms, is shown in Fig. 213. It is held by the handle H, which is usually of wood. *ab* is the graduated arc, *cd* the index arm which moves about a pivot placed at the centre of the graduated arc, *m* a microscope for reading the vernier, *i* is the index glass which is a small mirror attached to the index arm and perpendicular to the plane of the graduated arc. The horizon glass *h* is also perpendicular to the plane of the instrument, and is parallel to the index glass *i* when the index is at zero of the graduated arc. The lower half only of the horizon glass *h* is silvered. In Fig. 213 there is an opening in the upper part of the horizon glass. The horizon at sea or the reflection of the observed body in the "artificial horizon" (see page 335) is viewed directly through the upper part of the horizon glass *h*, while the reflection of the body as reflected from the index glass *i* is viewed in the lower silvered half. *e* is the telescope (in some instruments merely an eyehole or a plain tube is provided in place of the telescope), *s*, *s'* coloured glasses which may be interposed between the sun or its reflection and the observer's eye. The graduated arc *ab* is usually one-sixth of a circle, and measures angles up to 120°, the divisions of the graduated arc being marked with twice their actual value, so that the actual angle subtended by two objects is read on the vernier. The arc is usually divided to 10 minutes, and read by a vernier to 10 seconds. The radius of the graduated arc is usually 7 or 8 in. The principle of the instrument has been already described (see box sextant, Chapter II.).

Adjustments of the Sextant.

1. *To make the Index Glass at Right Angles to the Plane of the Arc.*— Put the index glass near to the centre of the arc, and bring the eye near the index glass and nearly in the plane of the arc. Note if the part of the arc reflected in the mirror appears to be in coincidence with the part seen directly. If so, the glass is at right angles to the plane of the arc; if not, make it so by the screws behind it.

2. *To make the Horizon Glass at Right Angles to the Plane of the Arc.*—Having first adjusted the index glass, look at some

well-defined distant point, such as a star, and if on shifting the index arm one image seems to part from the other, then the horizon glass is not at right angles to the plane of the arc, and must be adjusted by the screws behind it.

Another adjustment is as follows:—Hold the instrument plumb, and make the direct and reflected images of a smooth portion of the distant horizon coincide. Now rotate the instrument until it makes an angle with the vertical. If the two images do not separate the glasses are parallel, and if the index glass has been first adjusted the horizon glass will also be in adjustment.

3. *To make the Line of Sight of the Telescope Parallel to the Plane of the Arc.*—Bring the images of two points into contact at the cross hair nearest the instrument, then by moving the sextant bring them to the other cross hair. If they still coincide, the line of sight is parallel to the plane of the instrument; if not, the adjustment of the cross hairs is to be made by the screws in the diaphragm of the telescope.

4. *To observe if the two Mirrors are Parallel when the Index is at Zero.*—Make the direct and reflected images of a star coincide. If the index is at zero, no correction is required; if not, the difference is the *index error*, which is to be added to or subtracted from all observed angles. The index error may be adjusted by moving the horizon glass by means of the screws provided until the images *do* coincide when the index is at zero, and in that case the perpendicularity of the horizon glass to the plane of the arc (adjustment 2) must be again tested and rectified if necessary until both adjustments are correct. It is, however, more usual to note the index error and use it as a correction to the observed angles.

Practical Hints in observing with the Sextant.—Hold the sextant so that its plane is in the plane of the two points to be observed, and hold it *loosely*. Observe the *left* hand or *lower* object *directly* through the *unsilvered* part of the horizon glass. Alter the index arm until the reflection of the other object is seen in the *silvered* part of the horizon glass, and the two are made to apparently coincide. The reading of the vernier is then the required angle between the two objects.

Always look at the *brighter* of the two objects by reflection. If necessary to effect this, turn the instrument upside down.

If the angle between the two objects is more than the capacity of the instrument (about 120°), observe from one object to some intermediate point and note the angle, then from that to the other object. The required angle is then the sum of the two observed angles.

In observing the sun's altitude at sea, always bring sun's image *down* to horizon.

Parallax of the Sextant (see Fig. 214).—The angle measured with the sextant is the angle between the rays BI coming from the reflected point to the index glass, thence to the horizon glass and thence to the eye, and the rays HE

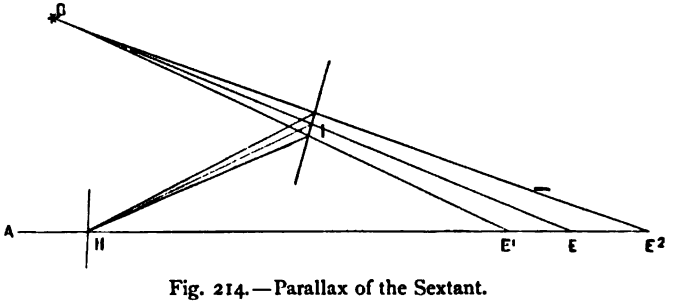

Fig. 214.—Parallax of the Sextant.

coming from the other point directly to the eye, *i.e.*, the angle BEA. But the eye may be at E^1 or E^2 on either side of E. The error in the observed angle due to the eye not being exactly at the point where BE intersects AE is called the *parallax* of the sextant. A difference of 1 in. in the position of the eye makes a parallax of 3 seconds when the reflected object is 1 mile distant, and 3 minutes when 100 ft. distant.

In order to eliminate the effect of parallax as much as possible, view the more distant object by reflection. When both objects are near, take some distant point in line with one of them and observe this point by reflection and the other object directly. For distant objects such as a star the parallax is very small.

The parallax is the angle subtended at the reflected object by the distance EE^1 or EE^2.

Artificial Horizon (Figs. 215, 216).—At sea the altitude of celestial bodies above the visible sea horizon is measured and a correction for *dip* of horizon is applied to the observed altitude. On land, however, in order to observe altitudes with the sextant an "artificial horizon" is required. An artificial horizon simply consists of a shallow trough filled with mercury. Water, oil, or molasses will also do. The reflection of the object in the artificial horizon is viewed directly, and the object itself by reflection from the index glass of the sextant. Sometimes a glass roof is provided, and when used, in order to eliminate irregularities of the glass plates each side of the roof should be marked with a distinctive mark, and it should be reversed at each set of altitudes, except in observing *equal* altitudes of celestial bodies, in which case the observations are to be taken with the *same* side of the roof towards the observer.

In observing the altitude of the sun it is difficult to tell which limb has been observed in the mercury, *i.e.*, whether the upper or lower limb. The following rule will indicate this:—

In the forenoon, or when the sun is rising, if the *lower* limb is observed the images will be continually separating; if the *upper* limb is observed they will be continually overlapping; and *vice versâ* in the afternoon or when the sun is falling, with an inverting telescope.

To observe very small Angles with the Artificial Horizon (Fig. 215).—Fix a string over the artificial horizon. Place your eye so that the string appears to cover its image in the mercury. Then eye and string are in the same vertical plane. Now keeping the eye in this position observe with the sextant the angle between the string as seen directly and the reflection of the object as reflected from the index glass of the sextant. This angle is then $90° +$ altitude of object and required altitude = observed angle $- 90°$. In the case of a depression the required depression $= 90° -$ observed angle (see Fig. 215).

Another method is to fasten a piece of white paper with a small hole in it and having a line on it behind the horizon glass so that the line is perpendicular to the plane of the arc. Now observe the image of the line in the mercury. The line of sight being then vertical, the angle is measured as in the previous method.

Observing Angles with Artificial Horizon. — When using an artificial horizon the observed angle is *twice* the required altitude. Thus in Fig. 216, let the observer's eye be at A and the observed body at S, and let it be reflected in the artificial horizon at B. Then the observed angle is BAS. Draw AE horizontal.

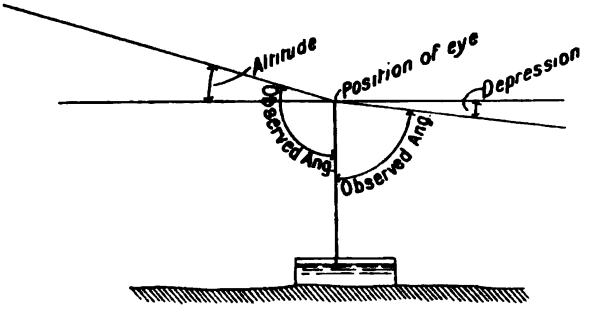

Fig. 215.—Small Altitudes and Depressions with Artificial Horizon.

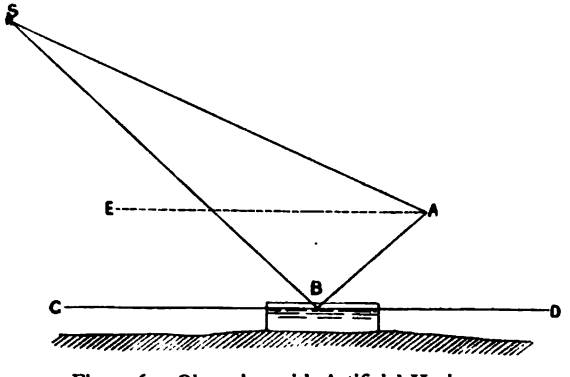

Fig. 216.—Observing with Artificial Horizon.

Then the angle of incidence CBS = angle of reflection ABD = altitude of observed body S.

Also because the distances BS and AS are in the case of celestial bodies infinite compared with AB, AS is parallel to BS.

Therefore CBS = EAS and DBA = BAE, that is, BAS = 2 CBS = twice altitude.

Oblique Angles measured with Sextant.—When the two objects between which the angle is observed differ much in altitude from the altitude of the observer, the angle as measured in the plane of the three points will differ considerably from its horizontal projection. The horizontal projection of the observed angle may be larger or smaller than the observed angle itself. The actual horizontal angle may be observed with the sextant in several ways:—

1. Observe the angle between each object and some third object very distant to the right or left of both. Subtract these

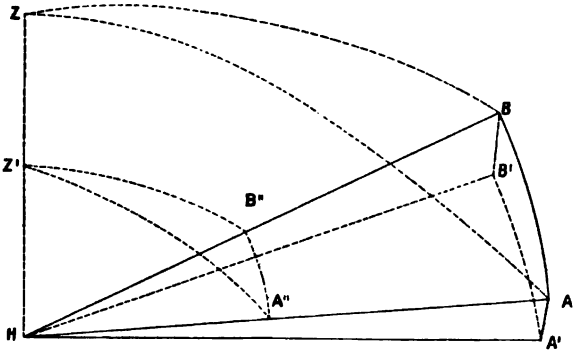

Fig. 217.—Oblique Angles.

angles, and the result will be very nearly equal to the required horizontal angle.

2. Take some point above or below one of the objects, and in line with it and at the same altitude as the other, and observe the angle between it and the other object.

3. Hang two plumb lines in line with the two objects. The horizontal angle between the two plumb lines may then be measured.

To reduce an Oblique Angle to its Horizontal Projection.—In order to do this the altitudes of each of the objects between which the angle is required must also be observed.

In Fig. 217 let AHB be the observed oblique angle, and let A'HB' be its horizontal projection. Let Z be the observer's zenith, then ZA and ZB are the zenith distances of A and B = 90° – respec-

tive altitudes of A and B. The spherical angle AZB is equal to the required horizontal projection A'HB'. Now in the spherical triangle AZB we have—

$$ZA = 90° - \text{altitude of A}$$
$$ZB = 90° - \text{altitude of B}$$
$$AB = \text{observed oblique angle}$$

Having thus the three sides of the spherical triangle AZB, we can calculate the angle AZB which is equal to the required horizontal projection A'HB'. The formula is as follows :—

$$\text{Sin } \tfrac{1}{2} \text{ A'HB'} = \text{sin } \tfrac{1}{2} \text{ AZB} = \sqrt{\frac{\sin (S - ZA) \sin (S - ZB)}{\sin ZA \times \sin ZB}}$$

where $S = \frac{1}{2}$ sum of sides of triangle AZB.

When the altitudes of A and B are not more than 2° or 3° an approximate correction is :—

$$\{90° - \tfrac{1}{2} (ZA + ZB)\}^2 \tan \tfrac{1}{2} \text{ observed angle} \times \sin 1'' - \tfrac{1}{4} (ZA - ZB)^2 \cot \tfrac{1}{2} \text{ observed angle} \times \sin 1''$$

The quantities in brackets are to be expressed in seconds. In Fig. 217 the objects are drawn at equal distances from the observer, *i.e.*, AH = BH. If the objects are at different distances exactly the same reasoning and formulæ apply if we consider a spherical triangle Z'A''B'' formed by the section of a small sphere of radius HZ', as shown in Fig. 217.

The Advantages of the Sextant are—(1) It does not require to be set up on the ground, but can be used while travelling, or on a moving body as on board ship; (2) it can take simultaneous sights to two moving bodies, as to moon and sun or moon and star. Its disadvantage is that it measures oblique angles instead of their horizontal projection.

Necessity for Astronomical Observations.—Unless the bearings of a survey are referred to astronomical or true north it would be difficult if not impossible to retrace the lines of the survey exactly. For surveying in jungle or dense forest the mining dial is excellent, and has been recommended. Unless, however, the actual construction of the road, railway, or other line of communication is to follow immediately upon the location of the lines, the whole line would have to be relocated, as it would be impossible to exactly retrace a line located merely

by compass bearings, even if the line had been staked out; as in tropical countries the pegs rot and are eaten up by ants. If some years have elapsed between the pegging out of the line and its actual construction the whole line would require to be relocated unless the bearings had been referred to true north. In such countries effort is made to preserve the pegging out of the lines by driving the pegs altogether into the ground, and by covering the top of each tenth peg with tin nailed on and stamped with the number of the station, and so on. If the centre line is "nicked out" or "dogbelled," it can be retraced after several years without instruments at all under almost any conditions. In these cases where the lines can be permanently marked with a reasonable certainty of being able to find the marks after the lapse of a considerable time, the necessity for astronomical observations is not so great. As, however, the staking out and permanent marking of a line is a work of considerable expense, and in most projected schemes cannot be undertaken until a considerable time after the location, the importance of astronomical observations must not be underestimated. A line whose bearings have been accurately referred to true north can be retraced with an angular divergence from the original lines of not more than 1 minute. This represents a divergence of about 18 in. in a length of 1 mile, and as the bearings are referred to true north about every mile, the errors are not multiplied, the total error being equal to the sum of the errors between observations. Also it is most improbable that the errors will all be in one direction, and they will in general neutralise each other to a greater or less extent, the result being that a survey which has been accurately referred to true north and carefully chained can be retraced to within a few feet. For instance, take a survey of 20 miles, and suppose that the lines average 1 mile long each, and that there is a difference of 1 minute between the bearing of each line as relaid down on the ground and its original bearing, and the most unfavourable and improbable case that the errors are all in the same direction. If the bearings have been corrected to 1 minute by an observation for meridian every mile the total divergence in a length of 20 miles would then be 20×18 in. $= 30$ ft. In practice, owing to the errors counterbalancing, the actual error would probably be only a few feet. If a line were thus retraced, it would be unnecessary even to relevel the longitudinal section, if it had been carefully levelled

in the first instance, as it would be left to the sectional or sub-divisional engineers to take levels at and on each peg, and fix bench marks, for the purpose of final quantities, giving depth of cuttings, heights of bank, levels for bridges, &c., the actual reduced values of the levels and bench marks of any one section being computed after the engineer immediately behind had brought his levels through his own section to join that section, each engineer assuming a temporary datum for his levels until the levels were connected between each section. When it is considered that in locating a line through thick jungle or dense forest every yard has to be cut, and every piece of information regarding the contour of the ground is also obtained at the expense of heavy cutting, the importance and value of a survey that can be accurately retraced is not to be compared with that of a survey merely referred to magnetic bearings, even if attempts have been made to leave permanent marks. Any one who has had experience of trying to find marks left some years previously on trees in dense forest or jungle will appreciate the force of this. Practically nothing short of a good deep "nicking out" or "dogbell" of the line leaves a permanent trace under such circumstances. As regards retracing survey lines in bare open country, the remarks as to the necessity for astronomical observations apply equally to a line which has only been located. A pegged-out line in open country can, however, usually be found without much difficulty. A long traverse of 20 or 30 miles whose bearings had not been referred to true north but conducted simply by measuring the angles between the lines would probably not be so accurate as a compass survey, the errors being multiplied throughout. Even a very long line, say 200 or 300 miles of railway, if properly surveyed, should be retraced to within a few feet throughout its whole length, as it will seldom happen that villages or pieces of open country at least will not occur at intervals of 20 or 30 miles, where the original line can be actually found by measurement from houses or other permanent marks. The value of compass work such as may be done with the miner's dial is not here underrated if confined to its proper place as a subsidiary. It is possible to run a compass survey with wonderful accuracy and speed, and with it lines have been run between two points 20 miles apart which came in to 1 chain. The exact retracing of such a line, even allowing for change of magnetic variation,

would, however, be quite a different matter, and 1 chain might make all the difference to a location in a hilly country. Also in the absence of permanent marks on the original line the only reliable method of determining the magnetic variation of the compass would be by astronomical observations for true north, taken during the original survey and during the retracing survey.

These remarks apply only to cases where there has been a careful location of the line made, and where it is consequently of great importance that the original located line should be capable of being retraced with accuracy. For rough preliminary route surveys, in which no attempt is made at a careful location, in jungle or dense forest, a compass survey is all that can be desired, the sights being as a rule short. In open country, however, where long sights can be got, a theodolite will generally be used even for rough work. As regards the necessity for observations for latitude, time, and longitude, these are required to get the data for extra meridian observations for true north, &c. Also for exploration and reconnaissance purposes, sextant observations for meridian, latitude, longitude, and time are the only means as a rule by which a rough map may be made.

Observations for Meridian.—The first and most important observation for the surveyor is that for meridian or true north. Its primary object is to determine the *true* bearing of the survey lines and the magnetic variation of the compass. It is also useful for succeeding observations for latitude, longitude, time, &c.

Below are given the principal methods of determining the meridian. They may be classed as follows:—

(1), (2), (3). *Circumpolar stars or sun at equal altitudes.*
(4), (5). *Circumpolar stars at elongation.*
(6), (7), (8). *Circumpolar stars in same vertical.*
(9). *Extra meridian observation of sun or star.*

OBSERVATIONS FOR MERIDIAN.

(1.) *Two greatest elongations of circumpolar star.*
(2.) *Equal altitudes of circumpolar star.*
(3). *Equal altitudes of sun.*
(4.) *Two stars at elongation.*
(5.) *Circumpolar star at elongation.*

(6.) *Pole-star and Alioth in same vertical (approximate).*
(7.) *Circumpolar stars in same vertical.*
(8.) *Test of meridian by time interval between transits of circumpolar stars.*
(9.) *Extra meridian observation of sun or star.*

Before describing in detail the above methods, the following general remarks on them may be noted:—

(1.) This is the most accurate method, but as usually one of the observations must be made in daylight, it is seldom possible with an ordinary theodolite.

(2.) This method admits of considerable accuracy when several pairs of observations with both faces of the instrument (see Chapter II.) are taken, but it is inconvenient owing to the considerable interval of time which must elapse between the observations.

(3.) This is a simple method of finding meridian, but requires a knowledge of the latitude to correct for change of sun's declination, also a considerable interval of time must elapse between the observations.

In addition to the interval of time between the observations, methods (1), (2), and (3) are also open to the objection that the sky may be overcast and the refraction changed by the time that the second observation should be taken. In the case of the sky being overcast the observation would not be possible, while the change of refraction might cause considerable error in the angle of the bodies' elevation. In the case of methods (2) and (3) the celestial body is moving rapidly in azimuth and passes quickly across the cross hairs so that the sights cannot be checked.

(4.) In this observation the disadvantages of the previous methods are obviated, as by selecting suitable stars which are nearly in conjunction or opposition, and which thus elongate at nearly the same time, the observations may be rapidly completed. As the star's motion is sensibly vertical for a considerable time near elongation, both faces of the instrument may be used, and there is time to take the sights carefully. This method also possesses the advantage of methods (1) and (2), that no knowledge of the latitude is required.

(5.) In this method a knowledge of the latitude is required. It is, however, a very convenient method, and a very approximate idea of the latitude suffices. To a star near the pole several pairs of observations with both faces of the instrument may be

taken, as the star's motion is sensibly vertical for about 15 minutes near elongation.

(9.) In this method a knowledge of latitude, approximate longitude, and approximate local mean time is required. Two observations with both faces of the instrument are taken, and the operation is rapidly performed. The sun observation is on the whole the surveyor's best method of finding his meridian, its great advantage being that it is taken in the daytime.

It may be as well to remark that no observation which does not permit of both faces of the instrument being used can be considered satisfactory.

(1.) **Meridian by two greatest Elongations of a Circumpolar Star.**—Observe the angles between the line whose bearing is required and a circumpolar star at its eastern and western elongations. Then the mean of these two angles is the angle between the given line and the meridian. This is a very accurate method, and the star's motion being sensibly vertical for some time at elongation, both faces of the instrument are used, and there is time to take careful sights. For the reason already stated on page 342, however, this observation is seldom possible with an ordinary theodolite.

(2.) **Meridian by Equal Altitudes of a Circumpolar Star.**—This is perhaps the simplest method of finding true north, and requires no calculation.

In Fig. 218 let P represent the north pole and S_1 a circumpolar star, say one of the stars in the Great Bear or Cassiopeiæ. As the stars appear to circle round the pole, after a certain interval of time the star S_1 following the path of the circle ENWL will appear to be at S_4 *at the same altitude* but on the opposite side of the pole P. Obviously the two positions S_1 and S_4 are equidistant from the meridian NPL, *i.e.*, $S_1x_1 = S_4x_1$. Similarly if S_2 and S_3 be the star's apparent positions when at the altitude Px_2 above the pole, $S_2x_2 = S_3x_2$. We have therefore the following simple method of finding the true bearing of any line on the earth's surface. Set up the theodolite over any station on the given line, and measure the angle between it and the star when it is say at S_1. Note this angle and also the vertical angle, *i.e.*, the star's altitude. Change face quickly and again note the horizontal and vertical angles when the star is say at S_2. Now unclamp the upper plate and

wait until the star approaches nearly to the position S_3. The course of the star should then be followed by rotating the upper part of the instrument until the star is seen in the field of the telescope. The vertical arc should be kept clamped at the same vertical angle as observed at S_2, or if it is by oversight unclamped after observing the star at S_2, it must be again set to the same angle. Having got the star into the field of the telescope the upper plate may be again clamped. The star is now kept in the centre of the field by means of the tangent screw of the horizontal plates until it cuts the axis or intersection of the cross hairs. The star's position is then obviously at S_3, being at the same altitude as when observed at S_2. Having read the horizontal angle to the star at S_3, change face quickly and set the vertical arc to the vertical angle observed at S_1, and again follow the star as already described until it coincides with the intersection of the cross hairs. The star's position will then be at S_4, and the horizontal angle may be read off. Take the mean of the two horizontal angles observed to S_1 and S_4. Also take the mean of the two horizontal angles observed to S_2 and S_3. These two means will differ slightly, chiefly owing to errors of adjustment of the instrument, and partly owing to errors of observation. The mean of the two results is to be taken, and will be practically free from errors of adjustment of the instrument, as both faces have been used, and the altitude of the star when at S_2 and S_3 is only slightly different from its altitude when at S_1 and S_4. More briefly, take the mean of the four horizontal angles observed to the star when at S_1, S_2, S_3, and S_4. This angle is then the angle between the meridian and the line whose true bearing is required.

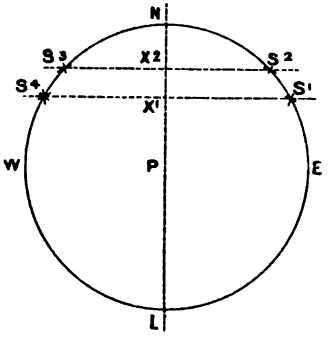

Fig. 218.
Meridian by Equal Altitudes of Circumpolar Star.

Thus in Fig. 219 let AB_1 be the given line whose bearing is required, and let AP be the meridian of A passing through the pole P; S_1 and S_4 the positions of the star when at the same altitude but on opposite sides of the pole P. Having set up the

theodolite at A, and measured the angles B_1AS_1, B_1AS_4, the mean of these is the required angle B_1AP. If, however, the line should lie *between* the two observed positions of the star as AB, then the angle BAP is equal to half the *difference* of the observed angles BAS_1, BAS_4.

When the line lies to the *east* of the meridian AP as AB_1 the true bearing is the angle B_1AP, but when the line lies to the *west* of the meridian AP as AB_2 its true bearing is $360° - B_2AP$.

The star may be observed either when on its upper path or on its lower path.

Referring to Fig. 218, the star appears to move horizontally for some time when near N or L, *i.e.*, at upper or lower culmination,

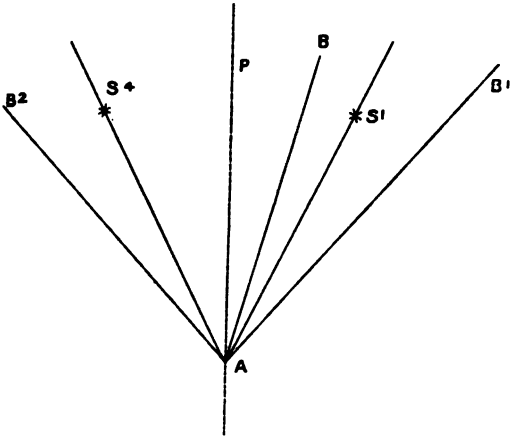

Fig. 219.—Meridian by Equal Altitudes of Circumpolar Star.

while when near E or W, *i.e.*, at eastern or western elongation, it appears to move vertically for some time. The star's position, therefore, should not be too near culmination when observed, otherwise it will appear to be at the same altitude throughout a considerable range of azimuth, and the observed horizontal angle will be liable to a considerable amount of error. On the other hand, if observed when very near elongation, nearly twelve hours would elapse before the star again reached the same altitude.

On account of the considerable interval of time which must elapse between the observations, this method is inconvenient, although the instrument may be left and other work done during the interval. In common with all other star observations, it has

also the disadvantage of keeping the surveyor up at night after probably having had a long day in the field. There is also the risk of the sky becoming overcast during the interval between the two sets of observations. The refraction may also change considerably owing to atmospheric changes in the interval, and thus affect the vertical angles by an unknown amount. On the other hand, the observation is independent of latitude, longitude, or local mean time, and no calculation beyond simple addition or subtraction of the observed horizontal angles is required.

In taking star observations it is necessary to illuminate the cross hairs, otherwise they would be invisible. In many theodolites there is a special lamp provided which can be fixed near the end of the horizontal axis of the telescope. The axis is made hollow, and has a piece of glass at its extremity to admit the light from the lamp. The writer's experience is that this is not a satisfactory arrangement. Too much light will illuminate the cross hairs well, but will make so much glare that the star is invisible. Too little light, on the other hand, will not render the cross hairs distinctly visible. In any case a hand lamp or candle is required to read off the angles, and the best way to get a satisfactory sight on a star is to hold the lamp or candle a little in front of the object glass and slightly to one side. By moving the light it may be placed so that both cross hairs and star are distinctly visible. If three nails are driven into a small piece of board, a candle may be stuck between them, and the arrangement may be readily held in the hand. It is possible for an observer to hold his light and take the observations by himself, but it is better to have an assistant to hold the light.

This observation has been described in some detail, as the remarks will apply to a great extent to all star observations.

Example of finding Meridian by Equal Altitudes of Circumpolar Star (see Figs. 218, 219).—Let the four observed angles be—

$$\begin{aligned} B_1AS_1 &= 20^\circ\ 15'\ 20'' \\ B_1AS_2 &= 20^\circ\ 16'\ 30'' \\ B_1AS_3 &= 61^\circ\ 48'\ 0'' \\ B_1AS_4 &= 61^\circ\ 48'\ 10'' \end{aligned}$$

$$4 \mid 164^\circ\ 8'\ 0''$$

Angle $B_1AP = 41^\circ\ 2'\ 0''$ = true bearing of AB_1

Fig. 220 shows a handy form of sight for night work, consisting of a stake formed from a flat board either with a very small hole in it or a larger hole having two wires stretched across it. In the daytime this stake may be driven in on the line whose bearing is required, and the centre of the small hole or the intersection of the cross wires accurately adjusted to the line by the theodolite. On placing a light behind the hole we get a good point to sight on when taking the star observation at night.

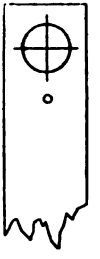

Fig. 220. Night Sight.

(3.) **Meridian by Equal Altitudes of the Sun.**—Exactly the same method as the last may be used for the sun, taking the sun's west limb in the forenoon and east limb in the afternoon, or *vice versâ*. The sun's image should be focussed on to the cross hairs as ⊙| or |⊙

Owing to the change in the sun's declination during the interval between the observations, a correction must, however, be applied. When the sun's declination is altering towards the $\left\{\begin{matrix}\text{north}\\\text{south}\end{matrix}\right\}$ the approximate direction of the meridian, as given by the mean of the sun's observed positions, is too far to the $\left\{\begin{matrix}\text{right.}\\\text{left.}\end{matrix}\right\}$ The correction to be applied is equal to ½ change of sun's declination in interval between observations × sec latitude × cosec ½ angle between two observed positions of sun.

The change of declination for the given interval of time may be found from the Nautical Almanac.

The latitude, if not known from observation and account, may often be had near enough by scaling from a map, if there is one available, otherwise the latitude must be found from a previous observation (see observations for latitude).

In observing to the sun it is necessary to interpose a piece of coloured glass between the object glass and the sun.

Latitude and Longitude by Account.—By latitude or longitude by account is meant latitude or longitude as deduced from a previous observation at one of the stations of the survey ± the latitude and departure of the intervening survey lines.

TABLE FOR THE CONVERSION OF DEPARTURE INTO DIFFERENCE OF LONGITUDE.*

Degree of Latitude.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
0°	...	1.0001	1.0006	1.0013	1.0026	1.0038	1.0055	1.0075	1.0098	1.0125
10°	1.0154	1.0187	1.0224	1.0261	1.0306	1.0353	1.0403	1.0457	1.0514	1.0578
20°	1.0642	1.0711	1.0785	1.0864	1.0946	1.1034	1.1126	1.1224	1.1326	1.1434
30°	1.1547	1.1666	1.1792	1.1924	1.2062	1.2208	1.2361	1.2521	1.2690	2.2868
40°	1.3054	1.3250	1.3456	1.3673	1.3902	1.4142	1.4395	1.4663	1.4945	1.5242
50°	1.5557	1.5890	1.6242	1.6616	1.7013	1.7435	1.7883	1.8361	1.8871	1.9416
60°	2.0000	2.0626	2.1301	2.2027	2.2812	2.3662	2.4586	2.5593	2.6695	2.7904
70°	2.9238	3.0716	3.2361	3.4204	3.6280	3.8637	4.1337	4.4454	4.8097	5.2406
80°	5.7587	6.3925	7.1856	8.2057	9.5664	11.475	14.334	19.108	28.653	57.307

* From "Hints to Travellers."

Rule.—Multiply the departure measured at any parallel of latitude by the number in the table corresponding to the latitude. The result is difference of longitude on the equator, which must be converted into minutes on the basis that 1 minute of longitude = 1.15287 miles at the equator.

Thus the departure measured on parallel of latitude 60° was 34.67 miles.

The number in table corresponding to latitude 60° is 2.0000.

$34.67 \times 2 = 69.34$ miles difference of longitude on equator.

$\frac{69.34 \text{ miles}}{1.15287 \text{ miles}} = 60.14$ minutes difference of longitude.

The following table will also be useful :—

LENGTH OF A DEGREE OF LONGITUDE AT MEAN SEA LEVEL AT DIFFERENT LATITUDES.

Degree of Latitude.	Statute Miles.	Degree of Latitude.	Statute Miles.	Degree of Latitude.	Statute Miles.
0	69.16	28	61.11	56	38.76
2	69.12	30	59.94	58	36.74
4	68.99	32	58.70	60	34.67
6	68.78	34	57.39	62	32.55
8	68.49	36	56.01	64	30.40
10	68.12	38	54.56	66	28.21
12	67.66	40	53.05	68	25.98
14	67.12	42	51.47	70	23.72
16	66.50	44	49.83	72	21.43
18	65.80	46	48.12	74	19.12
20	65.02	48	46.36	76	16.78
22	64.15	50	44.54	78	14.42
24	63.21	52	42.67	80	12.05
26	62.20	54	40.74	82	9.66

	Metres.	Feet.	Statute Miles.
* 1 minute of latitude at the equator =	1842.787 =	6045.95 =	1.14507
1 minute of latitude at latitude 45° =	1852.181 =	6076.76 =	1.15090
1 minute of latitude at the pole =	1861.655 =	6107.85 =	1.15679
1 minute of longitude at the equator =	1855.345 =	6087.15 =	1.15287

In converting small differences of latitude of about 5 miles into arc it is usually near enough to take 1 minute of arc as equal to 1.15 mile. Similarly in converting small departures into longitude it is usually near enough to take 1 minute of longitude as equal to 1.15 mile on the equator. As the length of a minute of longitude varies from 1.15287 mile on the equator to 0 at the pole, the departure must always first be converted into equivalent distance on the equator by means of the table on page 348, before being converted into arc.

(4.) **Meridian by two Stars at Elongation.**—This method possesses the advantage of (2), that a knowledge of the latitude is not necessary, while at the same time the long interval of time between the observations is dispensed with. Select two stars which are nearly in conjunction or opposition so that they elongate at nearly the same time. Now measure the angles between the line whose bearing is required and each of the stars at elongation. The sum or difference of these angles is the angle between the stars at elongation according as the line whose bearing is required lies between the stars or to one side, and whether the stars are at opposite elongations or at the same elongation. This angle is obviously also the sum or difference of the star's azimuths according as they are at opposite or at the same elongations.

Let the observed angle between the stars be denoted by $\alpha = A_1 \pm A_2$.

Let the declinations of the stars be δ_1 and δ_2.

Let the azimuths of the stars be A_1 and A_2.

The star's declinations are to be found in the Nautical Almanac.

---

* These are the values published by U.S. Coast and Geodetic Survey, Appendix 12, Report for 1881, and are calculated from Clarke's spheroid, which is now the standard of that Survey.

Then we have—

$$\text{Cot } A_1 = \cot a \pm \frac{\cos \delta_2}{\cos \delta_1} \text{ cosec } a$$

$$\text{Cot } A_2 = \cot a \pm \frac{\cos \delta_1}{\cos \delta_2} \text{ cosec } a$$

The plus or minus sign being used according as $a = A_1 + A_2$ or $A_1 - A_2$. The azimuth is the angle between the star and the meridian (see Appendix at end of this chapter). Having therefore either of the azimuths A_1 or A_2, the star's true bearing and consequently the line's true bearing are readily deduced as already described for method (2).

Both faces of the instrument should be used to observe each star, their motion at elongation being sensibly vertical for some time both faces may be used; and they should be selected so that the interval of time between their elongations is sufficient to permit of the observations being carefully taken and without undue haste. To calculate time of star's culmination or elongation see page 353.

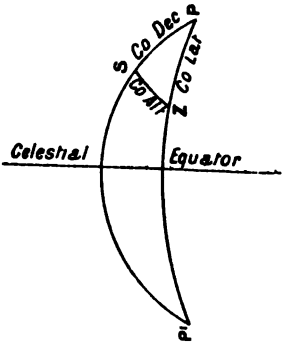

Fig. 221.
Meridian by one Circumpolar Star at Elongation.

(5.) Meridian by Observation of one Circumpolar Star at Elongation. — This method is similar to the last, but only one star is observed, and a knowledge of the latitude is required. In the northern hemisphere, Polaris, the pole-star, is a convenient star.

Measure the angle between the line whose bearing is required and the star at elongation, using both faces of the instrument. As the star's motion is sensibly vertical for a considerable time at elongation there is ample time to take careful observations with both faces of the instrument.

In Fig. 221 let P and P_1 represent the celestial poles
PZP_1 the observer's meridian
Z the observer's zenith
S the star

Then in the spherical triangle SPZ we have—

PZ = colatitude = 90° – latitude

PS = star's codeclination = 90° – declination

and angle PZS = star's azimuth

$$\text{Sin PZS} = \frac{\sin \text{PS}}{\sin \text{PZ}}$$

$$\textit{i.e.}, \text{ sin star's azimuth} = \frac{\sin \text{ codeclination}}{\sin \text{ colatitude}}$$

or log sin star's azimuth = log sin codeclination – log sin colatitude + 10. The star's declination is to be found in the Nautical Almanac, and as a star's declination is constant within a few seconds throughout any given year, it need only be corrected for the day of the year, *i.e.*, only the date of the observation is required to be known, not local mean time.

The latitude, if it cannot be had near enough from a map, must be found by observing the altitude of a circumpolar star at culmination (see Observations for Latitude).

If the latitude has been observed at a previous station on the survey, the latitude may be had near enough from account. An error of 1 minute in latitude which is over a mile will give an error of only about 1 or 2 seconds in the calculated azimuth, while the declination will be had correctly to about 1 second from the date and Nautical Almanac. Altogether this is a very satisfactory observation, and with a 6 in. theodolite the true bearing should be found to within about ¼ minute.

When the star is observed at western elongation as shown in Fig. 221, then its true bearing = 360° – calculated azimuth PZS.

If observed at eastern elongation its true bearing = calculated azimuth.

The true bearing of the line is then simply deduced from the star's true bearing by addition or subtraction of the observed angle between the line and the star.

Example of finding Meridian by Observation of one Circumpolar Star at Elongation.—In latitude 25° 58′ N. observed Polaris at eastern elongation on 25th February 1893. Observed angles between line and star being—

10° 21′ 30″ face right
10° 22′ 20″ face left

2 | 20° 43′ 50″

Mean = 10° 21′ 55″

The line whose bearing was required being to east of star.

From the Nautical Almanac—

Codeclination Polaris for date = 1° 15′ 40″
Colatitude = 90° − 25° 58′ = 64° 2′ 0″

Therefore sin star's azimuth $= \frac{\sin 1° 15' 40''}{\sin 64° 2' 0''}$

Log sin 1° 15′ 40″ + 10 = 18.342587
Log sin 64° 2′ 0″ = 9.953783

Log sin star's azimuth ÷ 8.388804 = log sin 1° 24′ 10″

The true bearing of the line is therefore—

1° 24′ 10″ + 10° 21′ 55″ = 11° 46′ 5″

To Calculate the Time of a Star's Culmination or Elongation.*—In taking star observations for meridian it is convenient to know the approximate time of elongation so as to be able to select a suitable star and know when to begin to observe. The calculation of the time of culmination is as follows:—Take from the Nautical Almanac the star's right ascension, and when this is less than the sidereal time at mean noon increase it by 24 hours. Subtract from the right ascension the sidereal time at mean noon which is also given in the Nautical Almanac. This gives the interval in sidereal time between mean noon and the star's culmination. Reduce this sidereal time interval to mean time by applying the correction given in the Nautical Almanac Tables, and the result is the interval in mean time between mean noon and the star's culmination, or in other words the approximate mean time of star's culmination. To this is to be applied a correction for longitude, which is usually so small as to be unnecessary for the present purpose. When the longitude is east, add this correction; when west, subtract it.

Example of Calculation of Time of Star's Culmination.—The following example is taken from "Hints to Travellers." At a place in longitude 30° E. required the mean time of the meridian passage of Aldebaran on 29th November 1881.

* See also sidereal time, Appendix at end of chapter.

	H.	M.	S.
Right ascension of Aldebaran + 24 hours - - =	28	29	11
Sidereal time at mean noon - - - =	16	34	2
Interval in sidereal time between mean noon and star's culmination - - - - =	11	55	9
Conversion of this sidereal time interval to mean time { 11h. retardation = 1m. 48.13s. ; 55m. „ = 9.01s. } =		1	57
Approximate mean time of culmination - =	11	53	12
30° E. long. = 2h. ; 2h. × 9.86s. = 20s. acceleration =			20
Mean time of culmination - - - =	11	53	32

The time of elongation varies according to the latitude from 5 hours 49 minutes to 5 hours 54 minutes from the time of culmination. For the present purpose it is near enough to add 5 hours 51 minutes to the time of culmination, as calculated above, in the case of a western elongation, and subtract in the case of an eastern elongation. We then know within about 3 minutes when to begin to observe.

The exact time of elongation may, however, be calculated as follows:—

In Fig. 221 the angle SPZ = the hour angle, and

$$\text{Cos SPZ} = \frac{\text{cot declination}}{\text{cot latitude}}$$

This is the interval of sidereal time between culmination and elongation in degrees, and is to be reduced to sidereal hours and then to mean hours (see page 377). We then have the exact interval of mean time between culmination and elongation, whence exact time of elongation may be deduced.

Calculation of Star's Altitude at Elongation.—It is also useful to know the star's altitude at elongation to assist in finding the star. This is given by—

$$\text{Sin altitude} = \frac{\text{sin latitude}}{\text{sin declination of star}}$$

(6.) **Meridian by observing the Pole-star** (α, Ursæ Minoris) **and Alioth** (ε, Ursæ Majoris) **in same Vertical** (approximate).—A vertical through these stars passes very near

the pole. By observing to them with a plummet string, or the theodolite, when they are in the same vertical, therefore we can fix an approximate meridian.

(7.) **Meridian by observing Circumpolar Stars in same Vertical.**—In Fig. 222 let Z be the observer's zenith

P the pole

AZPB the observer's meridian

S_1 and S_2 the stars in the same vertical ZS_2S_1C

We have—

Angle PZS_1 = azimuth of stars when in same vertical

ZP = colatitude and sin ZP = cos latitude

In the spherical triangle PS_1S_2 calculate the angle $PS_1S_2 = \alpha$.

Then in the spherical triangle PZS_1 we have—

$$\frac{\text{Sin } PZS_1}{\text{sin } PS_1} = \frac{\text{sin } PS_1Z}{\text{sin } PZ}$$

$$\text{or sin azimuth } PZS_1 = \frac{\text{sin } PS_1 \times \text{sin } \alpha}{\text{cos latitude}}$$

PS_1S_2 is a spherical triangle which alters its form very slowly, as the fixed stars S_1 and S_2 change their declinations only a few seconds throughout each year. Sin $PS_1 \times$ sin α is therefore constant throughout each year within a few seconds. If the constant sin $PS_1 \times$ sin α is computed for various pairs of stars for several years, the computation of the azimuth is then a simple matter. The stars β and ϵ (Ursæ Majoris) and β and δ (Draconis) are two useful pairs.

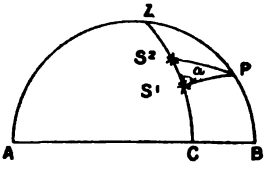

Fig. 222.
Meridian by Circumpolar Stars in same Vertica.

As, however, the accuracy of the observation depends on the instrument being in accurate vertical adjustment, it is not in practice so good as the other methods in which both faces of the instrument are used.

(8.) **Test of Meridian by Time Interval between Transits of Circumpolar Stars.**—Calculate as explained on page 353 the exact time of culmination of two stars which differ nearly 12 hours in right ascension, and which therefore reach their

upper and lower culminations respectively at nearly the same time. In Fig. 223 let S_1PL be the true meridian, S_1 and S_2 the stars. Then when S_1 has culminated the interval of time taken by the star S_2 to reach lower culmination at L is equal to the difference between the times of culmination as calculated.

If, however, the observed meridian be to one side, as shown by the dotted line, this time interval will be less. If the observed meridian be to the other side, the time interval will be greater. We have therefore an accurate means of testing the meridian,

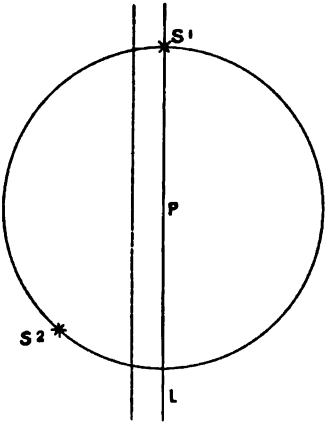

Fig. 223.—Test of Meridian.

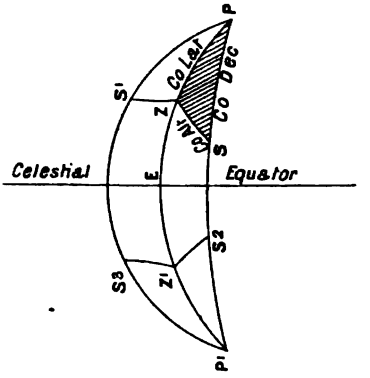

Fig. 224.
Meridian by one extra Meridian Observation.

but it will, however, depend on the instrument being in correct vertical adjustment.

(9.) **Meridian by one extra Meridian Observation of Sun or Star.**—This is usually the surveyor's most convenient method of obtaining the meridian, as only one observation is required, and it may be taken at any time. The sun observation is as a rule more convenient than the star observation, as it may be made in a few minutes either in the forenoon or in the afternoon while the surveyor is actually engaged on his survey lines, and the true bearing of the line being run is then determined by direct observation. As the survey party may often be encamped at some little distance from the survey lines, in that case

it will be somewhat inconvenient to take a *star* observation for true north at one of the stations of the survey.

In Fig. 224 let P and P_1 be the north and south celestial poles
Z the observer's zenith
PZP_1 the observer's meridian
S the observed position of the sun or star

Then PZ = colatitude of observer
PS = codeclination of star or sun
ZS = observed coaltitude or zenith distance of sun or star

If the above three sides of the spherical triangle SPZ are known, the angle PZS, which is the celestial body's azimuth, may be calculated. Of the three sides, the coaltitude is obtained by direct measurement of the altitude with the theodolite, the codeclination is got from the Nautical Almanac, and the latitude, if not to be had near enough from a map, is to be found by observing a star's altitude at culmination (see Observations for Latitude).

The most convenient formula for the calculation of the azimuth PZS is as follows:—

Let $S = \frac{1}{2}$ sum of sides of spherical triangle PZS
$= \frac{1}{2}$ (coalt + colat + codec)

Then—

$$\text{Cos } \tfrac{1}{2} \text{ PZS} = \sqrt{\frac{\sin S \times \sin (S - PS)}{\sin PZ \times \sin ZS}}$$

$$= \sqrt{\sin S \times \sin (S - PS) \times \text{cosec } PZ \times \text{cosec } ZS}$$

$$= \sqrt{\sin S \times \sin (S - \text{codec}) \times \text{cosec colat} \times \text{cosec coalt}}$$

or 2 log cos $\frac{1}{2}$ azimuth PZS = log sin S + log sin (S − codec) + log cosec colat + log cosec coalt − 20

When the observer is situated in the northern hemisphere, the spherical triangle will be as represented by PZS, Fig. 224, if the celestial body is observed when east of the meridian, and as represented by PZS_1 if observed when west of the meridian.

In the former case the true bearing of the celestial body is equal to its azimuth PZS, and in the latter case its true bearing is equal to 360° − the azimuth PZS_1.

If the observer is situated in the southern hemisphere, the spherical triangles will be as represented by the triangles $P_1Z_1S_2$ and $P_1Z_1S_3$, and the true bearings of the celestial body will be

$180° -$ azimuth $P_1Z_1S_2$, or $180° +$ azimuth $P_1Z_1S_3$, according as the observed body is to the east or west of the meridian.

The horizontal angle between the survey line and the sun or star having been measured, the true bearing of the line is deduced from the celestial body's true bearing by a simple addition or subtraction.

The details of the observation are as follows:—The theodolite being set up on one of the stations of the survey line, the horizontal angle between the sun or star and the line and the altitude of the celestial body are measured simultaneously, both faces of the instrument being used, and the mean of the two observations taken as the complete observation. The

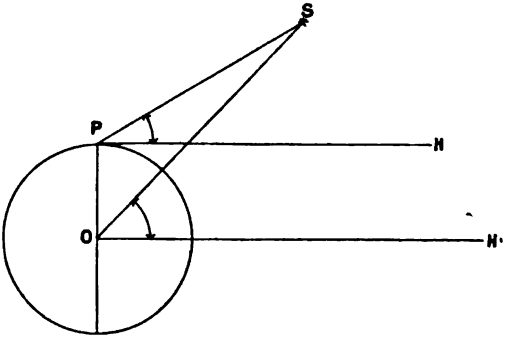

Fig. 225.—Parallax.

operation of observing the angles and changing face must be quickly performed, as the celestial body's apparent motion in the heavens is rapid, and the sun's declination may be changing rapidly. This it may be mentioned is one objection to this observation, as there is not time to take the sights carefully.

Celestial bodies should not be observed when at a less altitude than 15° or 20°, as the refraction is then very uncertain. The sun's upper or lower limb and right or left limb will be observed as $\frac{|}{\odot|}$ or $\frac{|\odot}{|}$ and a correction for sun's semidiameter is required in both the observed vertical and horizontal angles to reduce them to the sun's centre.

The vertical angle must also be corrected for refraction (see

pages 370, 159) and parallax. The parallax is the difference between the altitude as measured at the point of observation on the earth's surface and the altitude if it were measured at the earth's centre. Thus in Fig. 225, if P be the point of observation on the earth's surface and O the earth's centre, then SPH is the observed angle which differs from the angle SOH[1] by an amount which is called the parallax. If a star be observed, the distances of the stars being so great in comparison with the earth's radius, the parallax is inappreciable. In that case also the correction for semidiameter is unnecessary, but otherwise the observations and calculations are the same for a star as for the sun.

In the case of the sun, therefore—

True altitude = mean of observed altitudes – refraction ± semidiameter + parallax.

In the case of a star—

True altitude = mean of observed altitudes – refraction.

The corrections for refraction, semidiameter, and parallax are to be found in the Nautical Almanac.

As regards the other data required, viz. :—

The colatitude = PZ, Fig. 224

The codeclination = PS, Fig. 224

the latitude is often to be had with sufficient accuracy from a map, if any are available, otherwise it must be obtained by observing the altitude of a circumpolar star at culmination (see Observations for Latitude). If the latitude has been previously observed at any station on the survey, the latitude of the point of observation may be found with sufficient accuracy by computing the latitude and departure between the two stations from the length and bearings of the survey lines. The colatitude = 90° – latitude.

As to the declination, this is taken from the Nautical Almanac. The codeclination is 90° ∓ declination according as the celestial body's declination is north or south, and whether the observer is situated in the northern or southern hemisphere.

A star's declination is constant within a few seconds throughout the year, and may be taken from the Nautical Almanac without further trouble; but in the case of the sun it is different, as its declination will vary nearly 1 minute per hour at certain seasons of the year. The sun's declination at noon at Greenwich is given in the Nautical Almanac for every day of the year, and in order to correct the declination the Greenwich date of the

observation is required, *i.e.*, the time at Greenwich at the instant of observation.

In order to get the correct declination, therefore, the time of the observation with each face of the instrument is noted on the watch, the mean of these being taken as the time of the complete observation. If the observation is taken in the forenoon, 12 hours is to be added to get astronomical mean time, as the astronomical day commences and terminates at noon. This time must then be corrected for longitude, *i.e.*, for the difference between the time at Greenwich and that at the place of observation. Having thus arrived at the Greenwich date of the observation, the sun's declination at previous mean noon at Greenwich is taken from the Nautical Almanac, and the change of declination for the time elapsed since is added or subtracted according as the sun is northing or southing, and whether it is north or south of the equator. The rate of change of sun's declination is given in the Nautical Almanac.

It appears, therefore, that in order to arrive at the sun's declination we require—

1. Local mean time of observation by watch.
2. Longitude.

As regards the first, the maximum error of declination due to 1 hour error of watch on local mean time is less than 1 minute of arc as the maximum rate of change of sun's declination is less than 1 minute per hour.

As regards the second, an error of 15° in longitude will give the same error in declination, *i.e.*, 1 minute of arc, as 15° longitude = 1 hour time.

As 15° longitude is over 1,000 miles at the equator, it is seldom that the longitude is not known approximately enough to correct the declination. Also by observing the time of the sun's culmination, or by taking the mean of the times when equal shadows are cast by a needle stuck vertically in a horizontal board (see page 374), the watch may be corrected to within 15 minutes of local mean time, and if the equation of time be applied, to within 1 or 2 minutes of local mean time.

Thus an error of ½ hour in local mean time by watch plus an error of 7½° in longitude = over 500 miles at the equator, will give a total maximum error of declination of only about 1 minute.

Errors in Calculated Azimuth by Sun Observation due to given Errors in Latitude and Declination.—The resulting errors in the calculated azimuth due to errors in latitude and declination may be found from the following equations:—

$$\text{Error in azimuth due to error in latitude} = + \frac{d\text{L}}{\cos \text{L} \tan t}$$

$$\text{Error in azimuth due to error in declination} = - \frac{d\text{D}}{\cos \text{L} \sin t}$$

Where $d\text{L}$ = error in latitude
$d\text{D}$ = error in declination
L = latitude
t = hour angle ZPS, Fig. 224

The following table gives the errors in azimuth, at different hours of the day, due to an error of 1 minute in latitude and 1 minute in declination:—

Hour.	For an Error of 1 minute in Latitude.			For an Error of 1 minute in Declination.		
	Lat. 30°	Lat. 40°	Lat. 50°	Lat. 30°	Lat. 40°	Lat. 50°
	Min.	Min.	Min.	Min.	Min.	Min.
11.30 A.M. and 12.30 P.M.	8.75	9.90	11.81	8.83	10.03	12.93
11 A.M. and 1 P.M. -	4.30	4.84	5.82	4.45	5.02	6.02
10 A.M. and 2 P.M. -	2.02	2.23	2.73	2.32	2.62	3.13
9 A.M. and 3 P.M. - -	1.16	1.32	1.54	1.62	1.83	2.23
8 A.M. and 4 P.M. - -	0.65	0.72	0.93	1.32	1.52	1.82
7 A.M. and 5 P.M. - -	0.33	0.32	0.35	1.23	1.37	1.63
6 A.M. and 6 P.M. - -	0.00	0.00	0.00	1.13	1.33	1.55

These formulæ and tables will be useful in determining the degree of accuracy attained in the calculated azimuth when there is any uncertainty as to the correct values for latitude and declination having been used. As the errors have opposite signs, if the errors in latitude and declination are *both in excess* or *both in defect* the resulting errors in azimuth tend to neutralise each other.

Example of finding Meridian by Extra Meridian Observation of Sun.—On 25th February 1893, in latitude 25°

58′ N., longitude 93° 45′ E. from map, observed in forenoon the sun's left limb |⊙, and altitude of lower limb ⊙, with the theodolite, both faces of the instrument being used.

The observations were as follows:—

Altitude ⊙	45° 38′ 0″	face right
Horizontal angle between line and |⊙	120° 2′ 30″	face right
Time	10h. 20m. 20s.	face right
Altitude ⊙	46° 4′ 30″	face left
Horizontal angle between line and |⊙	119° 15′ 0″	face left
Time	10h. 22m. 40s.	face left

The line whose bearing was required being to right or west of sun.

Coaltitude.

Mean of observed altitudes $\frac{1}{2}(45° 38′ 0″ + 46° 4′ 30″) = 45° 51′ 15″$

Refraction	− 0° 0′ 54″	
Semidiameter	+ 0° 16′ 11″	
Contraction (see Appendix)	...	
Parallax	+ 0° 0′ 6″	+ 0° 15′ 23″
True altitude of sun's centre		= 46° 6′ 38″

Coaltitude = 90° − 46° 6′ 38″ = 43° 53′ 22″

Colatitude.

Colatitude = 90° − latitude
= 90° − 25° 58′ = 64° 2′

Codeclination.

Mean of observed times by watch—
$= \frac{1}{2}$(10h. 20m. 20s. + 10h. 22m. 40s.) = 10h. 21m. 30s.
Error of watch on local mean time = oh. 4m. 30s.

True local mean time of observation = 10h. 26m. 0s.

The longitude 93° 45′ is to be reduced to hours (see page 376).

93° × 4 = 372 minutes = 6h. 12m.
45′ × 4 = 180 seconds = oh. 3m.

Longitude 93° 45′ = 6h. 15m.

Then local mean time of observation = 10h. 26m. Feb. 25
+ 12h. 0m.

Astronomical mean time of observation = 22h. 26m. Feb. 24
Difference of time for 93° 45′ long. as above = − 6h. 15m.

Greenwich date of observation = 16h. 11m. Feb. 24

For sun's declination at time of observation we have—

Sun's declination at mean noon at Greenwich,
Feb. 24th = 8° 53′ 28″ south
Difference of declination for 16h. 11m. = − 0° 15′ 6″

Sun's declination at time of observation 8° 38′ 22″ south

Codeclination = 90° + declination
= 90° + 8° 38′ 22″
= 98° 38′ 22″

The three sides of the spherical triangle SPZ, Fig. 224, are therefore—

Coaltitude = 43° 53′ 22″
Colatitude = 64° 2′ 0″
Codeclination = 98° 38′ 22″

2 | 206° 33′ 44″

½ sum of sides of triangle SPZ = S = 103° 16′ 52″

S − codeclination = 103° 16′ 52″ − 98° 38′ 22″ = 4° 38′ 30″

Log sin S = log sin 103° 16′ 52″ = 9.988227
Log sin (S. − codec) = log sin 4° 38′ 30″ = 8.908075
Log cosec colat = log cosec 64° 2′ 0″ = 10.046217
Log cosec coalt = log cosec 43° 53′ 22″ = 10.159098

39.101617
− 20

2 | 19.101617

Log cos ½ PZS = log cos 69° 10′ 39″ = 9.550808
Therefore angle PZS = 2 × 69° 10′ 39″ = 138° 21′ 18″

In this case, as the observer is situated in the northern hemisphere, and the sun is observed in the forenoon when east of the

meridian, the angle PZS = sun's true bearing, which is therefore 138° 21′ 18″.

Mean of observed horizontal angles between line and sun's left limb

$$= \tfrac{1}{2}(120^\circ\ 2'\ 30'' + 119^\circ\ 15'\ 0'') \quad - \quad = 119^\circ\ 38'\ 45''$$

Deduct sun's semi-diameter - = 0° 16′ 11″

Angle between line and sun's centre = 119° 22′ 34″

As the line was to right or west of sun, its true bearing is therefore true bearing of sun + angle between line and sun, *i.e.*—

$$\text{True bearing of line} = 138^\circ\ 21'\ 18'' + 119^\circ\ 22'\ 34''$$
$$= 257^\circ\ 43'\ 52''$$

The compass variation, if required, is the difference between the magnetic bearing of the line as observed by the compass on the theodolite and its true bearing as above calculated.

Accuracy of Extra Meridian Observation of Sun or Star.—The longitude may always be had within at least 1°, which is about 70 miles on the equator, and this corresponds to $\frac{1}{15}$ of an hour, or an error in declination of only a few seconds; also the error of watch on local mean time need not exceed a few minutes, which also corresponds to only a few seconds of error in declination. The total error in declination need not, therefore, exceed a few seconds. With a 6 in. theodolite the altitude of a circumpolar star at culmination may be measured to less than $\frac{1}{2}$ minute, the maximum error in latitude being say $\frac{1}{4}$ minute. In general the true bearing as found from an extra meridian observation with a 6 in. theodolite will be within 1 minute of the truth.

As the object of the observation is usually the correction of the bearings of the survey lines run since the last observation taken, for most civil engineering purposes a determination of the meridian to within 1 minute will generally be sufficient.

With a 6 in. theodolite the meridian would be determined to within about $\frac{1}{4}$ minute by observation of a circumpolar star at elongation, method (5), as a considerable error in latitude only affects this observation by a few seconds, while the star's declination, which is the only other data required, being constant within a few seconds throughout any given year, is obtained from

the Nautical Almanac to 1 second, if the variation for the day of the year is allowed for, local mean time not being required, only the date. The observation need not occupy longer than the extra meridian observation of the sun, while the calculation of the azimuth is much simpler and more expeditiously performed.

Altogether method (5) is the quickest and most accurate, but the great advantage of the sun observation lies in the fact that it is taken through the day.

Calculation when either Altitude, Latitude, or Declination is not known.—In Fig. 224, if the hour angle SPZ is known, any one of the three sides of the spherical triangle SPZ may be dispensed with as follows:—

1. *Altitude not known.*—In this case the data are—

Colatitude PZ, Fig. 224
Codeclination PS, Fig. 224
Included angle SPZ, Fig. 224

The angle SPZ is the hour angle, and is the difference oetween the local mean time of the observation and the local mean time of the celestial body's culmination, reduced to arc; the local mean time of the observation is therefore required very accurately.

The formulæ are as follows:—

$$\text{Tan } \tfrac{1}{2}(\text{SZP} - \text{ZSP}) = \frac{\sin \tfrac{1}{2}(\text{codec} - \text{colat})}{\sin \tfrac{1}{2}(\text{codec} + \text{colat})} \cot \tfrac{1}{2} \text{SPZ}$$

$$\text{Tan } \tfrac{1}{2}(\text{SZP} + \text{ZSP}) = \frac{\cos \tfrac{1}{2}(\text{codec} - \text{colat})}{\cos \tfrac{1}{2}(\text{codec} + \text{colat})} \cot \tfrac{1}{2} \text{SPZ}$$

Having obtained half the difference and half the sum of the two angles SZP and ZSP as above, the two angles are got by addition and subtraction of the half difference and the half sum, and are located by placing the greater angle opposite the greater side in the spherical triangle SPZ, Fig. 224. The true bearing is then deduced from the azimuth SZP, as previously described.

2. *Latitude not known.*—If the latitude is dispensed with, the data are—

Coaltitude SZ, Fig. 224
Codeclination PS, Fig. 224
Hour angle SPZ, Fig. 224

Then we have—

$$\text{Sin SZP} = \frac{\text{sin codec} \times \text{sin SPZ}}{\text{sin coalt}}$$

and the true bearing is deduced from the azimuth SZP as before. This, however, is a case of two sides of a triangle and the angle opposite one of them being given, and is therefore ambiguous, *i.e.*, there is an uncertainty whether the angle SZP is that obtained from the above formula or its supplement. There will only be a difficulty in deciding this point when SZP is nearly a right angle, and it will generally be found that the larger angle is the one required, especially in the case of a sun observation.

3. *Declination not known.*—In this case the data are—

Colatitude PZ, Fig. 224
Coaltitude SZ, Fig. 224
Hour angle SPZ, Fig. 224

The formulæ are then—

$$\text{Sin PSZ} = \frac{\sin \text{colat} \times \sin \text{SPZ}}{\sin \text{coalt}}$$

$$\text{Cot } \tfrac{1}{2} \text{ PZS} = \frac{\sin \tfrac{1}{2} (\text{coalt} + \text{colat})}{\sin \tfrac{1}{2} (\text{coalt} \sim \text{colat})} \tan \tfrac{1}{2} (\text{SPZ} \sim \text{PSZ})$$

This also is an ambiguous case.

Convergence of Meridians.—As the meridians converge to the pole the bearing of a very long straight line is not the same throughout its length. The straight line run by a theodolite is in reality an arc of a great circle the bearing of which theoretically changes at every point of its length. A line kept always on the same bearing would not be a straight line but a curved line. Such a line is, however, represented by a straight line on a Mercator's chart, which is one of the chief reasons why a Mercator's chart so much facilitates the navigation of a vessel between two points. The vessel being kept on the same bearing throughout pursues a curved path between two points, which curved path is represented on the chart by a straight line. This path is not, however, the shortest distance between the two points, the shortest distance between two points being an arc of a great circle or that traced out with the theodolite. If the line is accurately produced from the two given points, *i.e.*, if it is truly straight, the bearing at any point will differ from its bearing at the starting point by an amount called the convergence. By taking an observation for meridian at any point we therefore have the means of checking the accuracy with which a straight line is being run.

Let c = convergence in seconds.

P = difference of longitude in seconds between the two extremities of any line AB.

ϕ = latitude of the point A.

ϕ' = latitude of the point B.

Then $c = \frac{\text{P} \sin \frac{1}{2}(\phi + \phi')}{\cos \frac{1}{2}(\phi - \phi')}$

When A and B are in the same latitude, *i.e.*, when the line AB runs due east and west, the formula becomes $c = \text{P} \sin \phi$. In this case we have also AB = P $\cos \phi$ in seconds of arc. This multiplied by earth's radius at given latitude gives actual length of line AB. If AB is known, $c = \text{AB} \tan \phi$.

In latitude 40°, in a line 1 mile long, the convergence would be about 44 seconds or $\frac{3}{4}$ minute. In higher latitudes it would be more, and in lower latitudes it would be less. In 50 miles in latitude 40° the convergence is about 40 minutes. Thus in latitude 40° the bearing at one extremity of a truly straight line 1 mile long would differ from its bearing at the other extremity by about $\frac{3}{4}$ minute.

This will emphasise the necessity for the surveyor to take an observation for meridian and correct his bearings about every mile about latitude 40°. At a lower latitude a greater interval between the observations for meridian will suffice. This is of course apart from considerations of instrumental error in ranging the lines and measuring the angles between them, which considerations, when the lines are very short and the number of bearings in a given distance is consequently great, may necessitate the taking of observations for meridian more frequently than above stated.

In the case of long straight lines of 2 or 3 miles, it will only be necessary to take an observation for meridian at each extremity of each line. The true bearing of *each* extremity should then be marked on the plan. For running *very* long straight lines, see page 412.

On a true north and south line there is no convergence; also on a due east and west line at the equator there is no convergence. On lines approximating to these positions the convergence will be small.

Observations for Latitude.—The surveyor's principal methods of obtaining latitude are as follows:—

(1.) *Meridian altitudes of sun or stars.*

(2.) *Zenith pair observations of stars.*

(3.) *Extra meridian observations of sun or stars.*

Of these the first two methods are the best, and are based on the principle that the "elevation of the pole is equal to the latitude of the place." This may be proved as follows:—

In Fig. 226 let E_1ES represent the earth, O its centre, EE_1 the equator, P the celestial pole, S the station of the observer, and Z the zenith of the observer.

Then the angle EOS = latitude of the point S.

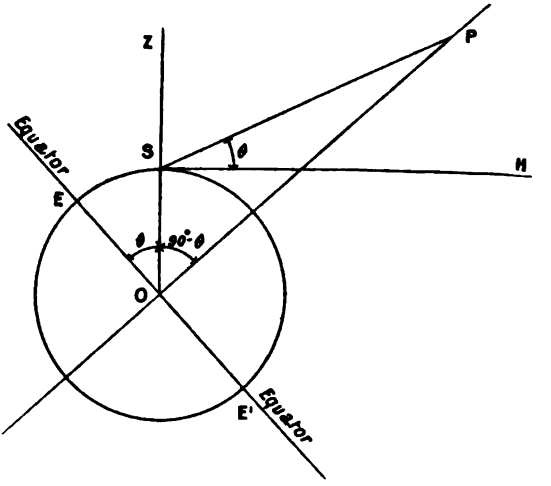

Fig. 226.—Latitude Observations.

Draw SH tangent to the earth's surface at S.

Then HSP = elevation of the pole = θ, and PSZ = $90° - \theta$.

As the distances PS and PO are infinite in comparison with the earth's radius SO, the lines PS and PO are practically parallel.

Therefore POS = PSZ = $90° - \theta$.

But the angle EOS = $90° -$ POS = $90° - (90° - \theta) = \theta$ = elevation of pole.

That is, the latitude EOS = elevation of pole HSP.

(1.) **Latitude by Meridian Altitude of Sun or Star.**—Measure the meridian altitude of sun or star, *i.e.*, its altitude at

culmination. Then zenith distance = 90° – altitude. The sum or difference of the codeclination and the zenith distance is then the colatitude and the latitude = 90° – colatitude.

Thus in Fig. 227 let O be the earth's centre

Z the observer's zenith

EE_1 the equator

P the pole

S the body observed at culmination.

Let OH be perpendicular to OZ. Then HOS = θ = observed altitude, and SOZ = 90° – θ = zenith distance ZS.

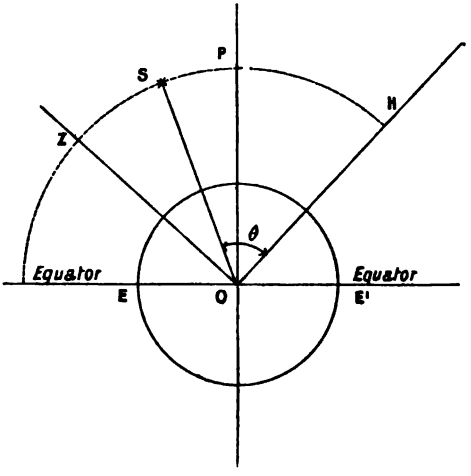

Fig. 227.—Latitude by Meridian Altitude of Sun or Star.

As the body is shown at upper culmination in the figure and SP = codeclination, we have—

$$ZP = ZS + SP$$

or colatitude = zenith distance + codeclination.

Or otherwise—

Altitude of pole = HOP = HOS – POS.

= altitude – codeclination.

i.e., latitude = altitude – codeclination.

The following are the different cases which may arise :—

In the Northern Hemisphere.—When the body culminates to north of observer :—

1. Altitude measured at upper culmination.

Lat = alt − codec.

2. Altitude measured at lower culmination.

Lat = alt + codec.

When the body culminates to south of observer :—

1. When the body's declination is north.

Colat = alt − dec.

2. When the body's declination is south.

Colat = alt + dec.

In the Southern Hemisphere.—When the body culminates to south of observer :—

1. Altitude measured at upper culmination.

Lat = alt − codec.

2. Altitude measured at lower culmination.

Lat = alt + codec.

When the body culminates to north of observer :—

1. When the body's declination is south.

Colat = alt − dec.

2. When the body's declination is north.

Colat = alt + dec.

The observed altitudes are to be corrected for refraction, and as this correction varies with the temperature and pressure of the atmosphere, to be precise the thermometer and barometer readings at time of observation should be taken. As, however, the refraction differs at most only about 3 seconds per 10° F. and 5 seconds per inch of barometer, from the refractions for mean temperature 50° F. and mean pressure 30 inches barometer, for which the tables of refraction are made out, the thermometer and barometer readings will as a rule be unnecessary. For an exceptionally high elevation above sea level combined with a very high temperature the variation in the refraction should, however, be allowed for. Values of refraction are to be found in the Nautical Almanac.

In the case of a star, its distance being infinite compared with the earth's radius, no correction for parallax is necessary. In the case of the sun, however, a correction for parallax and also for

semidiameter is required (see pages 358, 359). The corrections for parallax and semidiameter are to be found in the Nautical Almanac.

The whole operation of taking and reducing a meridian altitude observation for latitude is therefore very simple, and is as follows:—Measure the altitude of the body at culmination, correct this altitude for refraction and for parallax and semidiameter. Find the declination of the body from the Nautical Almanac, and apply one of the rules (page 370) to deduce the latitude.

As the motion of a circumpolar star is sensibly horizontal for about 15 minutes at culmination, there is ample time to take two or three pairs of observations with both faces of the instrument. (It is not necessary that the altitude should be measured *precisely* at culmination.) One pair of observations with a 6 in. theodolite should give the latitude within 20 seconds of the truth.

In the case of the sun, the approximate longitude and approximate local mean time are required to arrive at the declination. This has been already explained on pages 359, 360.

If the altitudes of a circumpolar star be observed at *both* upper and lower culminations, then the mean of these altitudes corrected for refraction is the altitude of the pole, which is equal to the latitude. This entails a long interval between the observations, but the upper and lower culminations may be observed on different nights. In general one of the culminations will be in daylight if successive culminations are observed.

Latitude by Meridian Altitude of Sun.—This is the sailor's chief method of finding his latitude. As he uses a sextant to observe the altitude above the visible horizon, a correction for dip of horizon is required. The operation on land with a theodolite is precisely the same, except that no dip correction is required. The star observation is also exactly similar, except that the declination is easier to get, being simply taken from the Nautical Almanac, and corrected for the day of the year.

Example of finding Latitude by Meridian Altitude of Sun.—August 26, 1882, long. 92° E., observed meridian altitude of sun's lower limb 56° 42′ 10″, sun bearing to N., eye 18 ft. above sea level. Index error of sextant 1′ 32″ to be subtracted.

Sun's declination for date from Nautical Almanac 10° 22′ 15″ N.
Variation declination, 52.25″ per hour from Nautical Almanac
92° long. = 6h. 8m. = 6.1h.
Correction of declination for long = 6.1 × 52.25″ = +5′ 19″

Reduced declination = 10° 27′ 34″ N.

Observed altitude		56° 42′ 10″
Index error		− 1′ 32″
		56° 40′ 38″
Dip	− 4′ 11″	
Refraction	− 33″	
Semidiameter	+ 15′ 52″	+ 11′ 8″
		56° 51′ 46″
		90°
Zenith distance		33° 8′ 14″ S. of sun.
Sun's declination		10° 27′ 34″ N.
Latitude		22° 40′ 40″ S.

If a sextant is used to measure the altitudes of heavenly bodies on land, an artificial horizon is required, and the observed angle is twice the required altitude (see Fig. 216).

(2.) **Latitude by Zenith Pair Observations of Stars.**—By observing the meridian altitudes of two stars, one on each side of the observer's zenith, more accuracy may be attained. Two stars should be selected which culminate at nearly equal altitudes and within about half an hour of each other. The latitude is to be deduced from the observed altitudes exactly as already explained for method (1), and the mean of the two latitudes will be the correct latitude. The observation is in fact a repetition of method (1), with the added advantage that by taking the mean of two stars, one on each side of the zenith, errors of refraction and graduation of the vertical arc are eliminated.

(3.) **Latitude by Extra Meridian Observation of Sun or Star.**—This method is similar to method (9) for meridian, and in fact when the mean time is accurately known, the latitude and meridian can be determined from the same observation.

Referring to Fig. 224, if the meridian is known, then in the spherical triangle SPZ we have as data—

$$\text{Coalt} = \text{SZ}$$
$$\text{Codec} = \text{SP}$$
$$\text{Azimuth} = \text{SZP}$$

Then the sine of the hour angle SPZ = sin coalt $\frac{\sin \text{SZP}}{\sin \text{codec}}$

Having now the four quantities coalt, codec, SZP, and SPZ, we have—

$$\text{Tan } \tfrac{1}{2} \text{ colat} = \frac{\sin \frac{1}{2}(\text{SZP} + \text{SPZ})}{\sin \frac{1}{2}(\text{SZP} - \text{SPZ})} \tan \tfrac{1}{2} (\text{codec} - \text{coalt}) \quad . \quad . \quad . \quad (\alpha)$$

When the meridian and latitude are to be determined from the same observation, the local mean time of the observation must be accurately known and observed with a more closely divided timekeeper than an ordinary watch. In this case from the local mean time we deduce the hour angle SPZ, and from it, the coalt, and codec we calculate the azimuth SZP by the second case (page 365), *i.e.*,

$$\text{Sin SZP} = \frac{\sin \text{codec} \times \sin \text{SPZ}}{\sin \text{coalt}}$$

Having then the four quantities SPZ, SZP, coalt, and codec, tan ½ colat is given by (α) above.

Prime Vertical Observations for Latitude.—The most accurate method of determining the latitude is by observing the times of transit of a star across the prime vertical or east and west line. The times of transit must be measured with great accuracy, as the result is greatly affected by a small difference in time. The accuracy of the observation will also depend on the instrument being in correct vertical adjustment.

It is not therefore suited to the surveyor, as more powerful instruments and more accurate means of measuring the time are required than are as a rule at the disposal of an ordinary survey party; as ½ minute of latitude = over ½ mile, observations for latitude are not of value to the surveyor as a direct check on his survey lines. The use of the observations for latitude is to provide the requisite data for observations for meridian by methods (3), (5), (7), or (9) in which the latitude is required, and for local mean time observations. Stars should not be observed within 25° of the zenith.

Observations for Time. — The principal methods of obtaining local mean time are :—

(1.) By meridian transit of sun or star.

(2.) By extra meridian observation of sun or star.

Approximate Method of finding Time and Meridian. —Before describing these, however, the following simple approximate method may be noted.

Take the mean of the times when the sun casts equal shadows before noon and after noon, as noon on local mean time, and correct the result by the equation of time for date.

In order to do this, take a sheet of drawing paper and mount it on a plane table or any suitable stand which can be adjusted

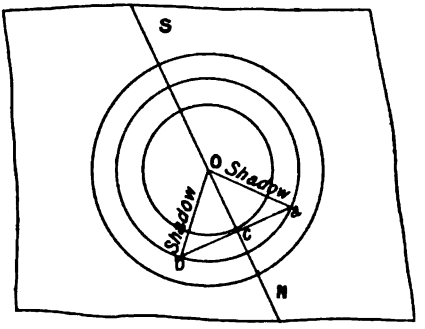

Fig. 228.—Meridian by Shadows.

to a horizontal position. Draw one or two concentric circles as shown in Fig. 228, and fix a needle vertically at the common centre o. Note the time when the extremity of the needle's shadow just touches the circumference of one of the circles as at *a*, about an hour or so before noon. Again note the time when the extremity of the needle's shadow just touches the circumference of the same circle in the afternoon as at *b*. The mean of these times will be the time of apparent noon, which may be converted into mean time by adding or subtracting the equation of time for date as given in the Nautical Almanac. If the chord joining *ab* be bisected as at *c*, then *co* is an approximate meridian or true north and south line. These results are approximate only, because of the change of the sun's declination during the interval, but the time as found in this way is sufficiently near the

truth to be used for finding the sun's declination at the time of taking an extra meridian observation for true north for instance. It would also be near enough to know by watch the approximate time of culmination or elongation of any given star which it was proposed to observe, and of which the mean time of culmination or elongation had been calculated.

If several concentric circles are drawn as shown in Fig. 228, and the times noted when the needle's shadow touches each in the forenoon, one of these circles may be available for observation in the afternoon even if the sky is overcast for some time and no shadow is apparent on the others.

(1.) **Time by Meridian Transit of Sun or Star.**—If the theodolite is set up in the meridian, and the time of the transits of the sun's right and left limbs is noted or the time of transit of a star, the error of watch on local mean time may be deduced, as it will be equal to the difference between the time of transit by watch and the true time of transit as calculated.

In the case of the sun the observed time of transit is equal mean of times of transit of sun's right and left limbs, and the true local mean time of transit is noon ± equation of time for date. The time of star's transit may be calculated as explained on page 353.

As, however, the result depends on the instrument being in exact vertical adjustment, and on the correctness of the surveyor's meridian, and as a very powerful instrument and very exact means of noting the times of the transits are required in order to get an accurate result, this method is not suitable for the surveyor.

The surveyor's only reliable and practical method of obtaining local mean time is by the following method, viz., an extra meridian observation of sun or star.

(2.) **Time by Extra Meridian Observation of Sun or Star.**—Referring again to Fig. 224, having given the three sides of the spherical triangle SPZ, viz., the colat, coalt, and codec, the hour angle SPZ may be calculated, and we have—

$$\text{Sin } \tfrac{1}{2} \text{ SPZ} = \sqrt{\frac{\sin (s - \text{codec}) \times \sin (s - \text{colat})}{\sin \text{codec} \times \sin \text{colat}}}$$

$$\text{or } 2 \log \sin \tfrac{1}{2} \text{ SPZ} = \log \sin (s - \text{codec}) + \log \sin (s - \text{colat}) + \log \operatorname{cosec} \text{codec} + \log \operatorname{cosec} \text{colat}$$

$$\text{where } s = \tfrac{1}{2} \text{ sum of sides} = \tfrac{1}{2} (\text{colat} + \text{codec} + \text{coalt})$$

If the angle SZP is known—

$$\text{Sin SPZ} = \frac{\text{sin SZP} \times \text{sin coalt}}{\text{sin codec}}$$

or log sin SPZ = log sin SZP + log sin coalt + log cosec codec

The hour angle SPZ is the interval of time expressed in degrees between the time of observation and the time of culmination.

In the case of the sun the time of culmination is 12 ± equation of time, and the time of a star's culmination is to be calculated as explained on page 353. Convert the hour angle into hours, minutes, and seconds, and subtract it from the time of culmination when the body is observed to east of meridian, and add it when observed to west of meridian. The result will be the true local mean time of the observation, and the error of watch on local mean time is the difference between the time of observation by watch and the time of observation as calculated.

Two observations with both faces of the instrument are taken, and the mean of the two times is taken as the mean time of the observation.

The most suitable stars to observe for time are those situated easterly or westerly, and about midway between horizon and meridian. Greater accuracy is attained if two stars are observed one to the east and the other to the west at about the same altitude, and the mean of the results taken.

To Convert Arc to Time and vice versa.*—As 360° = 24 hours, we have—

$$15^\circ = 1 \text{ hour, and}$$
$$1^\circ = 4 \text{ minutes.}$$

To convert arc to time, therefore, we have—

$$\left.\begin{array}{l}\text{Degrees arc} \times 4 = \text{minutes time} \\ \text{Minutes arc} \times 4 = \text{seconds time} \\ \text{Seconds arc} \times 4 = \text{thirds time}\end{array}\right\} \quad \ldots\ldots\ldots (1)$$

$$\text{or } \frac{\text{degrees arc}}{15} = \text{hours time} \quad \ldots\ldots\ldots (2)$$

Conversely to convert time to arc, we have—

$$\left.\begin{array}{l}\text{Hours time} \times 15 = \text{degrees arc} \\ \text{Minutes time} \times 15 = \text{minutes arc} \\ \text{Seconds time} \times 15 = \text{seconds arc}\end{array}\right\} \quad \ldots\ldots\ldots (3)$$

* See also Sidereal Time, Mean Time, Appendix at end of chapter.

To Convert Sidereal Time to Mean Time and vice versa.—Strictly speaking these rules apply only to "sidereal time," and before reducing *mean time* to arc it should be converted into sidereal time.

24 hours sidereal time = 23h. 56m. 4.0906s. mean time

or 1 hour sidereal time = 9.83s. less than 1 hour mean time.

To Convert Sidereal Time to Mean Time, therefore, multiply the sidereal time in hours and decimals by 9.83, and the result is the difference between sidereal time and mean time in seconds. This deducted from the sidereal time will then be the equivalent mean time.

To Convert Mean Time to Sidereal Time, multiply the hours and decimals mean time by 9.83, and the result is seconds of time, which added to mean time gives equivalent sidereal time. This may be expressed as follows:—

Sidereal time – (sidereal hours × 9.83) seconds = mean time

Mean time + (mean hours × 9.83) seconds = sidereal time.

Tables for the conversion of sidereal time to mean time are given in the Nautical Almanac.

Equation of Time.—This varies, and is given in the Nautical Almanac for noon at Greenwich, and should therefore be calculated for Greenwich date before being applied. However, the maximum variation of the equation of time is only 1¼ seconds per hour, and 1 hour = 15° long. Therefore, if we know our position within 15° long., we can deduce Greenwich mean time from local mean time to within 1 hour, and can therefore correct the equation of time to within 1¼ seconds (see Appendix, Equation of Time).

Observations for Longitude.—There are two general methods of finding longitude, viz.:—

I. By chronometer.

II. By absolute observations.

I. **Longitude by Chronometer.**—This is the most useful and general method of finding longitude. The local mean time having been found by extra meridian observation of sun or star, the difference between local mean time and Greenwich time as indicated on the chronometer is the longitude in mean time. This converted into degrees is the required longitude.

The surveyor is, however, rarely able to transport chronometers giving Greenwich mean time under the conditions of an ordinary survey, and in cases in which he cannot get even an approximate idea of his longitude he must have recourse to what are called absolute methods. The results of absolute methods are at best only approximate with the instruments at the surveyor's disposal and under the conditions of an ordinary survey, and the calculations very laborious. As the surveyor only requires his longitude very approximately for the computations of extra meridian observations, and as he can dispense with it altogether by selecting suitable star observations to determine meridian, latitude, and time, it is very seldom that he will require to take absolute observations for longitude. Before describing these, a simple, practical, and useful method of determining *difference* of longitude will be explained.

Determination of Difference of Longitude by Watch.—If the error of watch on local mean time is found at one station and the watch transported to another station, and error of watch on local mean time again found at that station, then the difference between the two local mean times is a measure of the difference of longitude. Every watch or chronometer gains or loses so much per day, and this is termed the *rate* of the watch or chronometer. In order to render a watch useful for astronomical observations in surveying it is thus only necessary to determine its *rate*. Knowing the *rate*, any ordinary timekeeper may be made useful for astronomical observations, and the true difference between the local mean times at any two stations may be found by allowing for the *rate*. As the rate is liable to alter, it should be determined at *each* of the two stations the difference of longitude between which is required. The method of finding the rate of the watch is as follows:—Take an extra meridian observation for time on a certain day and find error of watch on local mean time. After an interval of some days again take an observation for local mean time at the same station, and again find the error of watch on local mean time. The difference between the errors of watch on local mean time divided by the number of days between the observations is then the *daily rate* of the watch, either gaining or losing. The watch being transported to the other station, the same observations are repeated there. The average of the two

daily rates is then taken as the mean daily rate. If the daily rate is only found at one station, that rate will have to be used.

Then difference of longitude = difference between the local mean times at the two stations allowing for rate of watch.

Example of finding Difference of Longitude by Watch.—The following is an example taken from "Hints to Travellers":—

	H.	M.	S.	
Error of watch at Mombasa, 8 A.M., 14th July	2	18	32	slow.
„ „ „ 9 A.M., 20th July	2	17	14	„
Therefore for interval of 6.04 days' difference =		1	18	

	M.	S.
6.04	1	18

Daily rate = 12.91 sec. gaining.

	H.	M.	S.	
Error of watch at Taveta, 4 P.M., 25th July	2	8	5	slow.
„ „ „ 8 A.M., 30th July	2	6	48	„
Therefore for interval of 4.67 days difference =		1	17	

	M.	S.
4.67	1	17

Daily rate = 16.5 sec. gaining.
Former daily rate = 12.9 sec. gaining.

2	29.4

Mean daily rate = 14.7 sec. gaining.

	H.	M.	S.	
Error of watch at Mombasa, 20th July, 9 A.M. =	2	17	14	slow.
5.3 days' mean daily rate =		1	18	gaining.
Error of watch at Mombasa, 25th July, 4 P.M. =	2	15	56	slow.
„ „ Taveta „ „ =	2	8	5	slow.
Diff. of long. between Mombasa and Taveta =		7	51 = 1°57′45″	

As the watch is less slow at Taveta than at Mombasa, Taveta is west of Mombasa.

Longitude of Mombasa - -	39° 40′ 0″ E.
Difference of longitude - -	1° 57′ 45″ W.
Longitude of Taveta - -	37° 42′ 15″ E.

Strictly speaking, the above difference of 7′ 51″ mean time should have been converted into sidereal time before conversion into arc. This would make the difference of longitude = 1° 58′ about, instead of 1° 57′ 45″ as above.

II. **Longitude by Absolute Methods.**—The principal absolute methods of determining longitude are:—

(1.) Eclipses of Jupiter's satellites.
(2.) Moon culminating stars.
(3.) Lunar occultations.
(4.) Lunar distances.

With the exception of (1), these are all simply based on the principle that if the moon's right ascension or the moon's distance from a fixed star is found from observation, then the Greenwich mean time of the observation may be deduced from the registered right ascensions and lunar distances in the Nautical Almanac. The difference between Greenwich mean time and local mean time is then the longitude in time which is converted into degrees.

(1.) **Longitude by Eclipses of Jupiter's Satellites.**—The configuration of Jupiter's satellites is given in the Nautical Almanac for every day of the year except for those days on which Jupiter is so close to the sun that his satellites are invisible. The diagrams of the configuration are for north latitude, and are to be reversed when the observer is in south latitude. The eclipses are caused by Jupiter's shadow falling on the satellite, and take place on the east side of the planet if it comes to meridian before midnight, and on the west side if it comes to meridian after midnight. The eclipse of the first satellite is the best to observe, as its motion is the most rapid. The Greenwich mean times of the immersions and emersions are given in the Nautical Almanac, and the observation simply consists in observing the local mean time of either immersion or emersion or both. The longitude in time is then the difference between the local mean time of the eclipse and the Greenwich mean time of the same as given in the Nautical Almanac. This is then converted into degrees. A telescope of 40 power is required, and as the telescope of a 6 in. theodolite is usually not above 20 power, a special telescope is necessary. Any good telescope of sufficient power attached to a post or tree will do. The Greenwich mean time of the eclipse should be converted

into local mean time by applying the approximate longitude so as to know approximately when to begin to observe.

The error of watch on local mean time must of course be determined by a previous observation.

Raper says of this observation: "This method, though easy and convenient, is not very accurate; the eclipse is not instantaneous, and the clearness of the air and the power employed affect considerably the time of the phenomenon. Observers have been found to differ 40 or 50 seconds in the same eclipse." One minute of time is equal to 15 minutes of longitude, which is over 15 miles at the equator.

(2.) **Longitude by Moon Culminating Stars.**—This is perhaps the surveyor's best method. It consists in observing the local mean time of the meridian transit of the moon's bright limb and also the local mean time of the meridian transit of a fixed star near the moon and whose right ascension is known. A list of convenient stars called moon culminating stars is given in the Nautical Almanac. From the time interval between the culminations the right ascension of the moon's bright limb at the instant of observation may be found. In the Nautical Almanac the right ascension of the moon's bright limb is given for every hour for both lower and upper culminations, and the variation in right ascension is also given.

By interpolating between the values of the right ascension given in the Nautical Almanac we can find the Greenwich mean time corresponding to the moon's right ascension at the time of observation, *i.e.*, Greenwich mean time of observation of transit of moon's bright limb.

The longitude in time is then the difference between the local mean time and Greenwich mean time of observation. This converted into degrees gives longitude in degrees.

The accuracy of the observation is affected by any error in the surveyor's meridian and in the vertical adjustment of the instrument. The observation can be taken by a single observer with a stop watch. A disadvantage of this observation is that for about half the lunar month the transits occur at very inconvenient hours, while the lunar distance method, method (4), can be employed almost at any time when the moon is visible. Also most of the moon culminating stars are of small magnitude, and are not easy

to find even with the aid of the altitude at culmination and local mean time of culmination. There is also the chance of either moon or star being overclouded at culmination owing to sky becoming overcast during interval between transits.

The reduction of the observation is, however, simple, compared with either of the other methods, (3) and (4), while for the lunar distance method a sextant must be used, and two observers are required if one altitude is measured, or if the altitudes of both celestial bodies are observed three observers are required. The reduction of the observation is as follows :—

Convert the mean time interval between star's transit and moon's transit into sidereal time. Take from the Nautical Almanac the star's right ascension, and add or deduct the sidereal time interval between the transits. This gives the moon's right ascension at time of observed transit.

The Greenwich mean time corresponding to this right ascension is now to be found by interpolating between the values of moon's right ascension given in the Nautical Almanac.

(3.) **Longitude by Lunar Occultations.**—When a star is occulted by the moon, the right ascensions of the moon's limb and the star are the same. The star's right ascension is to be found in the Nautical Almanac, and the Greenwich mean time corresponding to this observed right ascension of moon's limb is then to be deduced by interpolating between the registered values of moon's right ascension in the Nautical Almanac.

The longitude in time is then deduced from the difference between the local mean time and Greenwich mean time of the observation, and then converted into degrees.

The Greenwich mean times of the occultations of fixed stars by the moon are given in the Nautical Almanac for both immersion and emersion. By applying the approximate longitude in time the approximate local mean time of the occultation may be found, and the observer will know approximately when to begin to observe.

In an ordinary 6 in. theodolite the telescope is not quite powerful enough to get a good observation of the exact time of the occultation, and a more powerful telescope should be used similar to that described for observing the eclipses of Jupiter's satellites (page 380).

This observation is independent of errors of adjustment of the instrument.

(4.) **Longitude by Lunar Distances.**—This method consists in observing the angular distance between the moon and a star or moon and sun. The computation is easier if one or both altitudes of the celestial bodies are also observed. One observer is required if only the lunar distance is measured, in which case the local mean time is accurately required; two observers if one altitude is also taken; and three observers if both altitudes are taken. As both the bodies are apparently moving, they must be observed at the same instant, and a theodolite is not suitable for the measurement of the distance. A sextant must be used.

The Greenwich mean time of the observation is found by interpolating between the registered lunar distances in the Nautical Almanac. Longitude in time is then deduced from the difference between the local mean time and the Greenwich mean time of the observation and then converted into degrees.

This observation is affected by all instrumental errors, and is not recommended to any one not practised in the use of the sextant.

When no altitude is taken, local mean time must be very accurately known, as 1 second time = ¼ minute arc.

Any error in the observed lunar distance results in about thirty times as much error in the calculated longitude. Thus an error of 20 seconds in the observation of distance gives about 600 seconds or 10 minutes error in longitude. A small error in observed altitude will not affect the calculation very much.

To get anything like a good result, several sets of lunar distances east and west of the moon are necessary, and a considerable time must be spent in taking the observations.

A complete pair of lunars consists of the following observations *in addition to those for latitude*:—

1. Read thermometer in air.
2. Adjust horizon glass of sextant.
3. Take two pairs of observations for index error of sextant.
4. Take three altitudes for time star east.
5. Take three altitudes for time star west.
6. Take three altitudes of moon.
7. Take five lunar distances star east of moon.

8. Take five lunar distances star west of moon.
9. Take three altitudes of moon.
10. Take three altitudes for time star west.
11. Take three altitudes for time star east.

Solution of Spherical Triangles.

Right-angled Spherical Triangles (Fig. 229).

(1.) Given one side a and one angle C—

$$\text{Cot } b = \frac{\cos \text{C}}{\tan a}$$

$$\text{Tan } c = \frac{\sin a}{\cot \text{C}}$$

$$\text{Cos A} = \sin \text{C} \times \cos a$$

(2.) Given the hypotenuse b and one angle C—

$$\text{Tan } a = \frac{\cos \text{C}}{\cot b}$$

$$\text{Sin } c = \sin b \times \sin \text{C}$$

$$\text{Cot A} = \frac{\cos b}{\cot \text{C}}$$

(3.) Given one side c and the opposite angle C—

$$\text{Sin } a = \cot \text{C} \times \tan c$$

$$\text{Sin } b = \frac{\sin c}{\sin \text{C}}$$

$$\text{Sin A} = \frac{\cos \text{C}}{\cos c}$$

(4.) Given the hypotenuse b and one side a—

$$\text{Cos } c = \frac{\cos b}{\cos a}$$

$$\text{Sin A} = \frac{\sin a}{\sin b}$$

$$\text{Cos C} = \cot b \times \tan a$$

(5.) Given two sides a and c—

$$\text{Cos } b = \cos c \times \cos a$$

$$\text{Cot A} = \frac{\sin c}{\tan a}$$

$$\text{Cot C} = \frac{\sin a}{\tan c}$$

(6.) Given the two angles A and C—

$$\text{Cos } a = \frac{\cos \text{A}}{\sin \text{C}}$$

$$\text{Cos } b = \cot \text{A} \times \cot \text{C}$$

$$\text{Cos } c = \frac{\cos \text{C}}{\sin \text{A}}$$

Oblique Spherical Triangles (Fig. 230).

Rule of Sines.—In every spherical triangle the sines of the angles are proportional to the sines of the opposite sides.

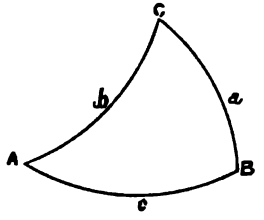

Fig. 229.
Right-angled Spherical Triangles.

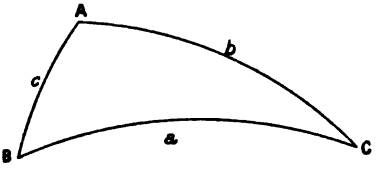

Fig. 230.
Oblique Spherical Triangles.

(1.) Given the three sides a, b, c—

Let $s = \frac{1}{2}(a+b+c)$

$$\text{Sin } \tfrac{1}{2}\text{A} = \sqrt{\frac{\sin(s-b) \times \sin(s-c)}{\sin b \times \sin c}}$$

$$\text{Cos } \tfrac{1}{2}\text{A} = \sqrt{\frac{\sin s \times \sin(s-a)}{\sin b \times \sin c}}$$

(2.) Given two sides a, b and the included angle C—

$$\text{Tan } \tfrac{1}{2}(\text{A}+\text{B}) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \tfrac{1}{2}\text{C}$$

$$\text{Tan } \tfrac{1}{2}(\text{A}-\text{B}) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \tfrac{1}{2}\text{C}$$

$\text{A} = \frac{1}{2}(\text{A}+\text{B}) + \frac{1}{2}(\text{A}-\text{B})$
$\text{B} = \frac{1}{2}(\text{A}+\text{B}) - \frac{1}{2}(\text{A}-\text{B})$

NOTE.—A and B must be located in the triangle by placing the greater angle opposite the greater side.

$$\text{Tan } \tfrac{1}{2}c = \frac{\sin \frac{1}{2}(\text{A}+\text{B}) \times \tan \frac{1}{2}(a-b)}{\sin \frac{1}{2}(\text{A}-\text{B})}$$

$$\text{Or } \sin c = \frac{\sin \text{C} \times \sin a}{\sin \text{A}}$$

(3.) Given two sides a, b and the angle A opposite one of them (ambiguous case)—

$$\text{Sin B} = \frac{\sin b \sin \text{A}}{\sin a}$$

$$\text{Tan } \tfrac{1}{2} c = \frac{\sin \tfrac{1}{2}(\text{A}+\text{B})}{\sin \tfrac{1}{2}(\text{A}-\text{B})} \tan \tfrac{1}{2}(a-b)$$

$$\text{Cot } \tfrac{1}{2} \text{C} = \frac{\sin \tfrac{1}{2}(a+b)}{\sin \tfrac{1}{2}(a-b)} \tan \tfrac{1}{2}(\text{A}-\text{B})$$

(4.) Given one side c and two adjacent angles A and B—

$$\text{Tan } \tfrac{1}{2}(a+b) = \frac{\cos \tfrac{1}{2}(\text{A}-\text{B})}{\cos \tfrac{1}{2}(\text{A}+\text{B})} \tan \tfrac{1}{2} c$$

$$\text{Tan } \tfrac{1}{2}(a-b) = \frac{\sin \tfrac{1}{2}(\text{A}-\text{B})}{\sin \tfrac{1}{2}(\text{A}+\text{B})} \tan \tfrac{1}{2} c$$

$$a = \tfrac{1}{2}(a+b) + \tfrac{1}{2}(a-b)$$
$$b = \tfrac{1}{2}(a+b) - \tfrac{1}{2}(a-b)$$

$$\text{Sin C} = \frac{\sin c}{\sin a} \sin \text{A}$$

(5.) Given one side a and two angles A and B, one adjacent to given side and one opposite—

$$\text{Sin } b = \frac{\sin \text{B}}{\sin \text{A}} \sin a$$

Having then given two sides a and b and the angles A and B, the third side c and the third angle C may be found from case (3), viz., given two sides and the angle opposite one of them.

Reduction of Sides of Spherical Triangles to Actual Linear Distances.—In the above formulæ the sides are given in terms of degrees, minutes, and seconds of arc subtended at the earth's centre or the centre of the sphere. To find the actual length of any side convert the angle subtended at the centre of the sphere into circular measure and multiply it by the radius of the sphere. When much accuracy is required, the earth's radius corresponding to the mean latitude of the side of the triangle whose length is required is to be used and will be taken from Geodetic Tables.

According to Clarke's spheroid, Report 1877, United States—

Earth's equatorial radius = 6378206.4 metres = 20926061.8 ft.
Earth's polar radius = 6356583.8 metres = 20855120.8 ft.

Clarke's spheroid is the standard of the United States Coast and Geodetic Survey, in which 1 minute of a great circle of a *true sphere* whose surface area = surface area of earth is taken as being equal to 1853.248 metres = 6080.27 ft. = 1.15157 statute miles.

Taking the earth's mean diameter as 41,781,000 ft. = 7,913 statute miles, we get the following :—

Mean length of 1 minute of arc = 6077 ft.

Angle in seconds subtended at the earth's centre by an arc on its surface $= \frac{\text{length of arc in feet}}{101.279}$ nearly

Length in feet of an arc on the earth's surface = angle of arc in seconds × 101.279 nearly.

As more accurate formulæ where n'' = length in feet of 1 minute of arc, d = difference between latitude of place and 45°, the plus or minus signs being used according as the latitude is greater or less than 45°, we have :—

When the arc is measured on a great circle making an angle α with the meridian—

$$n'' = n \cos^2 \alpha + n' \sin^2 \alpha$$

where n = length in feet of 1 minute of arc measured on a meridian, and n' = length in feet of 1 minute of arc measured on a great circle perpendicular to the meridian, and

$$n = 6076.76\left(1 \pm \frac{\sin 2d}{200}\right)$$

$$n' = 6076.76\left(1 + \frac{1}{300} \pm \frac{\sin 2d}{600}\right)$$

These formulæ are based on the supposition that the earth is a true spheroid of revolution with polar and equatorial diameters in the ratio of .996672.

When the actual length in linear measure of one of the sides is known, the best practical method of calculating the lengths of the sides of spherical triangles when they do not exceed about 100 miles is, however, by Legendre's method (see Chapter XII.).

APPENDIX.

EXPLANATION OF ASTRONOMICAL TERMS IN CHAPTER X.

Altitude.—The altitude of a celestial body is the vertical angle between the horizon and the body.

Apparent Time.—Apparent time or solar time is time according to the sun, and is that which would be indicated by a sun-dial. The difference between apparent time and mean time is called the equation of time, and varies at different seasons of the year from about 16 minutes to 0.

Astronomical Mean Time.—This is the same as civil time, with the difference that the civil day begins at midnight, while the astronomical day begins on the *following* noon, and the hours are reckoned up to 24. Thus Jan. 1, 9 A.M., 1899 civil time = Dec. 31, 21 hours, 1898 astronomical time; but Jan. 1, 3 P.M., 1899 civil time = Jan. 1, 3 hours, 1899 astronomical time.

Azimuth.—The spherical angle at the zenith between the meridian and the great circle through the observed object.

Circumpolar Stars.—Stars whose polar distance or codeclination is less than the latitude of observer. Circumpolar stars, therefore, culminate twice and do not set.

Civil Mean Time = local mean time (see Local Mean Time).

Coaltitude.—Coaltitude = 90° − altitude, therefore coaltitude = distance from zenith to observed body (see Altitude).

Codeclination. — Codeclination = 90° − declination, therefore codeclination = polar distance or distance from pole (see Declination).

Colatitude.—Colatitude = 90° − latitude, therefore colatitude = distance from zenith to pole (see Latitude).

Contraction.—In the angular measurement of the sun or moon's diameter or semidiameter the unequal refraction of the

upper and lower limb produces an error called *contraction* of semidiameter. The amount is very small, and is given in the Nautical Almanac.

Conjunction.—Two celestial bodies are said to be in conjunction when they are on the same meridian, and therefore culminate or elongate at the same time (see Culmination and Elongation).

Culmination.—Every celestial body appears to cross each meridian circle twice in twenty-four hours, once above the pole and once below it. These transits are called the upper and lower culminations.

Declination.—The distance of a celestial body north or south of equator measured on a meridian. Declination thus corresponds to terrestrial latitude.

Elongation.—When a celestial body in its apparent rotation round the pole is at its greatest distance east or west of the pole it is said to be at elongation.

Equation of Time.—The difference between apparent or solar time and local mean time or civil time. Therefore solar time $\pm$ equation of time = local mean time. The equation varies from 0 to about 16 minutes at different seasons of the year, and is given in the Nautical Almanac.

Equator.—A great circle midway between the poles and whose plane is perpendicular to the earth's axis. The celestial equator is the great circle formed by the intersection of a plane through the terrestrial equator with the celestial sphere.

Great Circle.—A great circle of a sphere is any circle described on it whose plane passes through the centre of the sphere and whose centre coincides with the centre of the sphere.

Horizon.—The horizon is a plane tangent to the earth's surface at the point of observation. The true horizon is a plane parallel to the horizon but passing through the centre of the earth

Hour Angle.—The spherical angle at the pole contained between the meridian of the observer and the meridian of the observed body. The hour angle converted into time is therefore the interval of time between the instant of observation of a celestial body and its culmination on the observer's meridian.

Latitude.—The distance of any point on the earth's surface north or south of the equator measured on a meridian. Celestial latitude = terrestrial latitude, and is the distance of the zenith of observer north or south of celestial equator.

Limb.—The edge of the circumference of a celestial body. Thus we have upper limb $\overline{\odot}$, lower limb $\underline{\odot}$, right limb $\odot|$, left limb $|\odot$.

Longitude.—The spherical angle at the pole between the meridian of Greenwich and the meridian of any place on the earth's surface is the longitude of that place. Longitude may be measured in degrees of arc on the equator or on any parallel of latitude.

Local Mean Time.—The time used in everyday life, and is the same as civil time. It is measured by the length of the mean solar day divided into twenty-four hours, and differs from apparent or solar time by an amount called the equation of time. Local mean time varies directly as the longitude (see Apparent Time, Astronomical Time).

Meridian.—The meridian of the observer at any point on the earth's surface is the great circle passing through the poles and the observer's position or the point on the earth's surface. The celestial meridian is the great circle passing through the celestial poles and the zenith of the observer or point on the earth's surface.

Occultation.—The disappearance of one celestial body by the intervention of another between it and the observer.

Parallax.—See page 358, Chapter X.

Polar Distance.—See Codeclination.

Poles.—The two points of intersection of earth's axis of rotation with its surface. The celestial poles are the two points of intersection of earth's axis of rotation with the surface of the celestial sphere.

Prime Vertical.—A great circle of the celestial sphere passing through the observer's zenith and east and west points of the horizon.

Refraction.—The correction applicable to the observed altitudes of celestial bodies owing to the rays of light proceeding from them being *refracted* or bent in their passage through the earth's atmosphere.

Right Ascension.—The right ascension of a celestial body corresponds to the longitude of a point on the earth. In the same way that the meridian of Greenwich is arbitrarily selected as the starting point from which to measure terrestrial longitude, so the meridian through the first point of Aries has been selected in the heavens as the celestial meridian from which to measure the right ascensions of celestial bodies. Right ascension is measured from west to east, and is expressed in hours, minutes, and seconds, 24 hours being = 360°. The co-ordinates of any celestial body are thus its right ascension and declination. When these are known the star's position in the heavens may be found at once, right ascension and declination in fact corresponding to terrestrial longitude and latitude.

Sidereal Time.—Every celestial body appears to perform a complete revolution round the poles once a day. The actual time in which the complete revolution is performed is 23h. 56m. 4.0906s. mean time. This space of time is called a sidereal day of 24 sidereal hours. Thus 24 hours sidereal time = 23h. 56m. 4.0906s. mean time, whence the rules for conversion of sidereal time to mean time and *vice versâ* given on pages 376, 377.

Twenty-four hours of sidereal time is in fact the time of a complete revolution of the earth upon its axis. It is a perfectly regular measure of time, but could not be used in place of local mean time or ordinary civil time, as it would be totally different from apparent or solar time, to which local mean time or civil time approximate as nearly as possible consistently with being at the same time a regular measure of time. The meridian of the first point of Aries is the starting point of the measurement of right ascension of celestial bodies (see Right Ascension). When this point is on the meridian of the observer it is 0 hours or 24 hours sidereal time or sidereal noon, just as when the sun is on the observer's meridian it is apparent noon. The mean time at which this takes place is called the "mean time of sidereal noon," and is given in the Nautical Almanac as the "mean time of

transit of first point of Aries." Conversely the "sidereal time at mean noon" is the sidereal time at ordinary noon

The sidereal interval between mean noon and a star's culmination is therefore equal to its right ascension less "sidereal time at mean noon." This is converted into mean time and a small correction for longitude applied, and we then have the mean time interval between mean noon and the star's culmination, or in other words, mean time of star's culmination at any place (see example, page 353).

Spherical Angle.—The angle formed on the surface of a sphere by the intersection of two great circles.

Zenith.—The point on the celestial sphere directly over the observer, *i.e.*, the intersection of a vertical through the observer's position with the surface of the celestial sphere.

Zenith Distance.—The zenith distance = coaltitude, and is the distance from zenith to observed body (see Altitude and Coaltitude).

CHAPTER XI.

*SURVEYS ABROAD IN JUNGLE, DENSE FOREST, AND UNMAPPED OPEN COUNTRY.**

THE surveys which the engineer is called upon to make abroad may be divided into two classes:—

I. *Surveys for a long line of communication, such as a road or railway.*

II. *Surveys of a limited area of country.*

(Surveys of greater extent necessitating triangulation from a measured base are dealt with in Chapter XII.)

I. Surveys for a Long Line of Communication, such as a Road or Railway, &c.—These again may be divided into two classes—(*a*) *In open country;* (*b*) *In jungle or dense forest.*

The operations of a survey of this kind are as follows:—

1. *Reconnaissance or exploration of various proposed routes.*
2. *Route survey or trial location on one or more routes.*
3. *Preliminary location survey on route selected.*
4. *Final location survey and setting out of line.*

1. Reconnaissance or Exploration of various Proposed Routes.—In order to decide upon the relative merits of various proposed routes it is necessary first to examine each either by riding or walking over it.

Reconnaissance in Open Country.—In open country it will be possible either to ride or walk along the route within a few chains of the probable final location of the centre line, and owing to the possibility of getting a clear view of the surrounding country a very fair idea of the best location along each route explored should be obtained. A rough survey of the ground actually ridden or walked over can be made with prismatic compass and aneroid without interrupting the march. The method of doing this is to take a bearing to the farthest visible point on the line of march ahead, and note the time on leaving

* For details of surveying and setting out not mentioned in this chapter, see Chapter V.

the commencement of each line and again on reaching its termination. By also noting the estimated rate of travelling between the two points, *i.e.*, 4 miles an hour, 3 miles an hour, &c., as the case may be, we get an approximate measurement of the distance. Pedometer, range finder, or perambulator may also be used to get the distances. Aneroid barometer readings at each end of the lines will enable us to plot an approximate longitudinal section of the route actually travelled, while the best approximate location of the line of the railway or road which is visible from the actual line of march may be roughly sketched in the survey book. Aneroid readings should be taken at all important river crossings, passes, &c., and every available piece of information as to towns, villages, sources of water supply, particulars with regard to large rivers, geological features of the country, nature of timber and cultivation if any, population and trade, industry, climate, rainfall, floods, &c., should be carefully recorded. The result of such a reconnaissance would then be a rough plan of the actual route traversed, with an approximate location of the line and possible deviations therefrom sketched roughly on it, and also a rough longitudinal section of the route traversed, which will give an approximate idea of the probable longitudinal section of the line, besides the other general information indicated above. See instruments for approximate measurement of distances, paced survey with prismatic and pocket compass, Chapter II.

Reconnaissance in Thick Jungle or Dense Forest.—A reconnaissance or exploration of a given proposed route in thick jungle or dense forest is to be conducted upon the same lines as already indicated for open country, and the same information should be obtained with the exception of the rough indication of the location of the line. This is of course out of the question in jungle work, as all information obtained will be strictly confined to the route actually travelled over, it being impossible to see anything farther than a yard or two from the actual line of march. With reference to the best route to explore in this description of country, there are two courses open—(1) To cut through the jungle or bush in a straight or approximately straight line for the objective point; (2) to confine the exploration or reconnaissance to existing roads, trade routes, or paths. With

reference to this point, Mr F. Shelford, in a paper on "Railway Surveying in Tropical Forests," * says, "After considerable experience in unmapped countries covered with dense forest, the author has no hesitation in stating that the existing trade route, road, or path is almost invariably the best base line from which to locate the railway." This amounts to saying that the native route between any two given points in country covered with dense forest is generally the best and most direct that the local features of the country will permit of. It being distinctly understood that these remarks apply only to the general route and not to the final location, it is believed that most of those who have had experience of surveying in tropical countries will concur in this view. The probability is that in most cases a direct line between two given points will run into difficult country, with the result that after spending a considerable time in running trial lines the final route will approximate more or less closely to the existing native trade route. With reference to the rough compass and aneroid survey of a route in jungle or dense forest, in a tortuous path the lines are excessively short, and in that case the best plan is to send a man ahead and judge of the direction from a shout or gun-shot. Distance by pedometer, perambulator, or time should be corrected by allowing for the twists in the path, which after a little practice can be fairly estimated. A range finder for distances is of course out of the question for jungle work.

The very roughest kind of exploration or route survey will consist of a series of detached observations of latitude and longitude taken with the sextant and artificial horizon on camping at night while exploring or travelling, together with barometer altitudes.

The longitudes may be taken by the method described on page 378, Chapter X., by finding the error of watch on local mean time at each place. The latitudes by any of the methods described in Chapter X. For measurement of heights by barometer see Chapter III. For the problem of "required distance and bearing of objective point whose latitude and longitude is known," and the converse problem, see page 409.

2. **Route Survey or Trial Location.**—When there is doubt as to the relative merits of one or more routes the only

* Minutes of Proceedings Institution of Civil Engineers, vol. cxxxiii.

way to decide the point is to make a route survey or trial location over each.

Route Survey or Trial Location in Open Country.—This consists in running a line and taking levels along it, the reconnaissance survey being taken as the basis. As a rule the theodolite will be used in open country, as long sights will be taken. The bearings, however, should be compass bearings referred to magnetic north, as such a survey will be more accurate than a traverse in which merely the angles between the lines are measured, even if a considerable amount of care is taken in measuring the angles. The reason of this is of course because the errors of the bearings in a compass survey are independent of each other, the total error being the algebraic sum of the separate errors, while in the other case the errors are cumulative and multiplied throughout the whole survey. The line will be chained either with a chain or measuring band. Levels with the ordinary spirit level will be taken along the line traversed. As a rule cross sections and contours will not be taken on a trial location of this kind, or at all events only a few in the most difficult parts of the line. There will be no attempt at pegging or marking the line, as a survey of this kind must be conducted with a certain amount of speed. The leveller should therefore follow up the theodolite as closely as possible. In open country the leveller will not have time to check his levels, and there should be a check leveller. The check leveller will do as much cross sectioning and surveying of rivers, streams, paths, &c., crossed, as his time will permit of, and the more of this work done, the more valuable will the survey be as a guide to the preliminary location and setting out *in the event of that route being selected.* When the survey is only one of several rival routes the fact must not be lost sight of that it will be a loss to spend too much time in cross sectioning and subsidiary surveying, as this will be of no use in the event of the survey being discarded on the general merits of the route itself. At the same time in difficult country it will be an absolute necessity to do a certain amount of cross-section work for contours and survey of details in order to properly compare different proposed routes. The party should consist of (1) theodolite or transit man, (2) leveller, (3) check leveller. Usually the chief will take the theodolite, otherwise the

party will consist of four. The instruments should be—Two 6 in. theodolites, three 14 in. levels, two clinometers, two prismatic compasses, two or three steel bands or measuring chains, three levelling staffs; ranging rods, axes, cutting knives, &c.

In the above list one spare theodolite and one spare level is provided, as also spare steel bands or chains and a spare levelling staff. This is important in foreign work, as the work may be stopped from a trivial accident if spare instruments are not carried.

For foreign work the diaphragms of the telescopes should be of glass, with the collimation line marked by lines etched on the glass and not cross hairs.

Messrs Cooke's instruments have usually glass diaphragms, and they save a great deal of trouble and time in replacing broken cross hairs.

For rapid survey of this kind in open country a tacheometer or stadia survey will be very suitable (see Tacheometry, Chapter VI.).

Route Survey or Trial Location in Jungle or Dense Forest.—A survey of this kind is to be conducted in the same manner as already described for open country. As, however, the sights will be as a rule much shorter, the best and most rapid instrument to use is the ordinary surveying compass or miner's dial. It is a very common practice to make these surveys with prismatic compass mounted on light legs (Fig. 65). The objections to a prismatic compass are as follows:—The needle either takes a long time to settle, or in hot damp countries becomes foul and slow. Moisture condenses inside the glass and the needle is apt to stick to it and give a false reading.

The mining dial has the advantage of a much larger compass and longer needle, and it is steadier and not so liable to get out of order. In point of lightness, speed, and portability it compares favourably with the prismatic compass, and it takes a good deal of rough usage to damage a mining dial. From the latter point of view it compares very favourably with the prismatic compass. Fig. 61 shows Whitelaw's theodolite or mining dial, invented by the author's father. Fig. 231 shows a plain form of ordinary mining dial. Whitelaw's theodolite or mining dial will be found a very good instrument for this class of survey, and is described by the inventor as follows:*—

* Paper read before Royal Scottish Society of Arts.

"The difference of the instrument, the subject of this paper, consists in fitting the miner's dial compass or the ordinary theodolite with two arcs, one of which is rigidly fixed to the compass box in the line of sight or direction of observation, and this arc has the usual graduations for taking angles of elevation or depression and reductions from the hypotenusal to horizontal distances; also in attaching a large level tube to said arc. The other of the two arcs is free to move in its bearings in a line at right angles to

Fig. 231.—Plain Surveying Compass or Mining Dial.

the line of observation. On the upper part of this movable arc are carried the frame and sights, and telescope, which may be used instead or conjointly. The vernier for vertical angles is formed in this movable arc.

"The instrument may also be used for levelling, in same manner as is sometimes done with an ordinary theodolite, by clamping it at zero of the rigid arc. This arc is also pierced by slits, whereby the other sights can be checked. The illustration represents a

5 in. theodolite for surface or underground surveys. With a vernier working inside the compass box—that of a mining theodolite—and without an external vernier and the clamp, it would represent an ordinary dial with the improvements applied thereto.

"The under part of frame has a groove with spring fixed to the under side of it, which makes it automatic in its action, for retaining it in whatever position it may be when an observation is being taken, and saves using the clamp.

"The spirit level L is attached by snugs for levelling the instrument and ascertaining its horizontal adjustment.

"Slits are pierced in the opposite sides of the rigid arc in the same vertical plane as the sights and telescope, and these serve for ranging and checking horizontal lines. Angles of elevation and depression up to 90° can be taken without difficulty, thereby making it very suitable for connecting surface and underground surveys, and particularly for steep workings.

"The arrangement of the frame with the sights admits of the compass being the same diameter as the under limb in theodolite, whereby rapidity, greater accuracy, and a more perfect check is obtained between the compass and verniers.

"The verniers of the under limb are placed near to the eye and right side of the observer, which enables him to read them without changing his position, as is required by him with an ordinary theodolite.

"The improved instrument may be used for taking horizontal angles, as with the mining compass or dial, by using the magnetic needle only, the lower limb or inside vernier being clamped. In this position it is also available for and especially adapted—in addition to its recognised merits for mine-surveying—*for rough and closely wooded countries where there is difficulty in obtaining long views*, as it is fitted with open sights fixed below the telescope so that one or other can be used.

"The improvements described are also applicable to ordinary mining or other compasses.

"The improvements may be noted as consisting of—(1.) In the general arrangement and construction of the parts constituting the instrument for angular measurement.

"(2.) Providing the compass box for angular measurement with a rigid graduated arc fixed to the sides and over the said box, and with a movable arc turning upon bearings on the sides of a com-

pass box, which carries at its top a frame provided with sights above the arc, whereby angles of elevation and depression up to 90° can be taken; and the placing of the verniers on the right of the line of sights instead of at the centre of the sides of the instrument, which obviates moving from the position for taking the bearings.

"(3.) The combination of a level with the rigid arc referred to in the preceding description of the drawings, &c.

"(4.) Forming slits in the rigid arc in the line of sights for the purpose of ranging and checking horizontal lines; also of connecting surface and underground surveys.

"I have now improved on the instrument, as shown in Fig. 61, in so far as *the folding sights are now made to fold close to bed-plate, and made so that when erect they can be used with the telescope on;* the holders of telescope being made of such height as to allow sights to be taken through below it—say a back sight.

"*The telescope holders are now also made to fold down close to plate, and then the telescope being taken off the instrument is used with the hair sights only.*"

The use of the mining dial for this class of survey is strongly advocated by Mr F. Shelford in the paper already alluded to (see page 395). In running a survey through thick jungle or dense forest every yard of the way has to be cut. A couple of ranging rods should always be lined in at the head of the line, and the natives who do the cutting keep themselves in line by means of these. The instrument should, however, be kept as close up to the cutters as possible, as they are liable to get off the line. As a rule the natives prefer to do the cutting with knives, called "machetes" in Mexico, Central and South America, and called "dhows," &c., in India. Small trees are cut down with axes, which as a rule must be supplied to the natives engaged for the cutting. They, however, often prefer to use their own knives, although an ample supply of these should also be provided. When very large trees are encountered, they must be triangulated round, although in this class of survey it is generally near enough simply to shift the instrument to the other side of the tree and set it up in the line again as nearly as can be judged. As trees will be encountered over 10 ft. in diameter, and having buttresses in addition extending up the tree for about 20 ft., somewhat after an Eiffel Tower shape, it is out of the question to cut these down

on a route or trial location survey. Such a tree would afford three or four days' work for two or three men to cut it down. Three to four miles a day, and sometimes upwards of six miles a day, can be surveyed, depending principally on the amount and rate of the cutting. As regards the accuracy of a compass survey of this kind, lines have been run 20 miles which came in to 1 chain, which is well within the limits of accuracy required.

Fifty or sixty natives may be easily employed in cutting, carrying, shifting camp, making huts, &c., and it is better to have even a greater force of natives if possible, as the whole of the work would be delayed for want of a sufficient number of men. Every yard of cross section will have to be cut, and the aneroid will be found a very suitable instrument for rough cross sections. It is simply necessary to walk down the cross section cut and note the readings, which may be checked on the way back. The clinometer and taped distances will of course be more accurate.

The dial should be set up at every intersection of two lines, and a back sight as well as a fore sight taken on each line. If the back and fore sights do not agree, the difference is due to an error in reading the bearing or else to local magnetic attraction. In the latter case the angle between the two lines must be measured on the vernier in Whitelaw's dial, or if there is no vernier the difference between the bearings will correctly represent the angle between the two lines in spite of any local magnetic attraction. When the back and fore sights again correspond, the magnetic bearings may be again adopted.

A steel band will be found most suitable for jungle work, as it does not catch in twigs and branches so much as the links of a chain. A steel band must be well oiled every night in damp tropical countries, or it will not last long.

The amount of cutting done is only just sufficient to run the line through. The line as cut partakes more of the nature of a tunnel than open cutting, consequently when any depressions or elevations occur the view is interrupted, which is one reason why the sights are as a rule short. Large trees in the way is another cause of the sights being short. A comparatively small anthill (*i.e.*, about 6 ft. high, which is small for some countries) will also block the line of sight.

It will easily be understood that although the inclined line of sight for the compass has been cleared, the leveller's line of sight,

which is necessarily horizontal, may be anything but clear. The leveller therefore requires some men to cut down intervening twigs and branches.

Cross sections should be cut by a special gang at points which may be indicated by a cleft stick stuck in the centre line, with a paper inserted with directions to cut a cross section, and in what direction and about what length. In India a native surveyor may be put in charge of cutting cross sections. A number of cross sections may then be taken in a short time by the check leveller when he gets his levels checked up on the main line. Otherwise the native surveyor or assistant who cuts the cross sections may survey them with the clinometer or aneroid at the time they are cut.

When the line runs into a hill, the best plan is to run one or more cross sections right over the hill. This will indicate which side of the hill is the best for the line, and may therefore save time and trouble in cutting cross sections on the wrong side of the hill. The best form of protractor for plotting the traverse is a large circular cardboard protractor, about 12 or 18 in. diameter having the circle inside the graduations cut out (see page 123).

It is perhaps scarcely necessary to state here that the object of cross sections for location purposes is to enable the contours to be plotted on the plan and the line thereby located. This is fully described in Chapter V.

The party should consist of (1) engineer running compass lines, (2) leveller, (3) check leveller.

The instruments will consist of two mining dials, three 14 in. levels, two clinometers, two aneroids, two prismatic compasses, two or three steel bands, three levelling staffs; ranging rods, axes, cutting knives, &c.

3. **Preliminary Location Survey on Route Selected.**—A survey of this description will be conducted on practically the same lines whether it be in open country or in jungle, due consideration being given to the remarks already made on work in jungle or dense forest. This class of survey is attended with a considerable amount of cross sectioning and surveying of details, such as roads, rivers, paths, &c., and the plan produced should show full contours of the ground and a well-located line marked on it. This being so, it is consequently of much importance that the exact line which has thus been carefully located at con-

siderable expense and trouble should be capable of being retraced with accuracy after the lapse of a considerable interval of time. The only satisfactory method of doing this is to survey the lines carefully with a 6 in. theodolite and refer their bearings to astronomical or true north. Any one who has had experience of trying to find marks left on trees in jungle or dense forest after a considerable interval has elapsed since the marks were made will appreciate the force of this. In open country it may be possible to refer the line to marks which may be easily found, but a survey whose bearings are correctly referred to true north is always more satisfactory. This class of survey should therefore invariably be executed with the theodolite, whether in open country or in jungle. For open country tacheometer or stadia work is the best method, see Chapter VI.

The first thing to do is to take an observation for true north at the starting point and determine the true bearing of the first line (see Chapter X.). Next set the instrument so that the needle of the compass points to zero, the vernier being also at zero. The needle of the compass should be set to zero by means of a small pocket microscope. Now sight on the first line and read *on the vernier* its magnetic bearing. This may be checked by reading the needle, which should indicate the same bearing as nearly as it can be read. The magnetic variation is then the difference between the true and magnetic bearings. The instrument should be kept clamped at the *magnetic* bearing, and kept at this bearing when shifted to the commencement of the second line. Being set up there, sight back on the first line, turn over the telescope, unclamp, and sight on to the second line. The bearing of the second line may now be read by the vernier, and this bearing should correspond with the bearing indicated by the needle. This virtually amounts to measuring the angle between the lines, only the magnetic bearings are recorded, so that the needle is a check on the vernier reading. This is of much importance, as the needle may often save a mistake of a degree or so in reading the vernier. The magnetic bearings are thus booked, but the true bearings are of course known as the true bearing of the first line has been determined at the starting point, the object of booking and working to magnetic bearings being simply to get the check afforded by the needle of the compass.

As the observation for true north at the starting point will be

as a rule correct to about 1 minute with a 6 in. theodolite, the probable angular deviation of the recorded bearings from the actual bearings of the line traversed will be 1 minute. This represents a divergence of about 18 in. in 1 mile. Another observation for true north should therefore be taken at about every mile. A second observation having been taken, the error in the bearing at that point is ascertained and distributed over the lines run between the two observations for true north.

When the second observation for true north is taken, if there is any great discrepancy between the magnetic bearing as indicated by the needle and that indicated on the vernier, a new magnetic bearing should be adopted. This ensures correspondence between needle and vernier again, at least for some distance, and gives us the desired check on the vernier readings.

The reader must not confuse the above with a survey referred to magnetic bearings, as it is as above described a survey referred to true north, the use of the magnetic bearings being purely conventional. Neither is it to be confused with a survey in which the magnetic variation of the compass on the theodolite is determined say at camp, and the true bearings of the survey lines simply deduced from their magnetic bearings $\pm$ the variation of the compass. The essential feature of the method described is that the true bearings of the actual survey lines are found from astronomical observations taken on the lines themselves at intervals of about a mile.

In distributing the errors over the lines intervening between the points at which observations for true north are taken, the total error in bearing is divided by the number of lines, and the corrections made accordingly. This assumes the angular error to be the same for a short line as for a long line. Strictly speaking this may not be the case, but the above is usually near enough for all practical purposes.

As regards the other details of a survey of this class, it may be said that generally speaking the survey is conducted on the same lines as already described for 2, "Route Survey or Trial Location over one or more Routes," and the reader is referred to the remarks already made in that connection. The essential differences are:—

(1.) The bearings are more carefully taken and are referred to astronomical or true north about every mile.

(2.) Temporary pegs are driven in with point marks or tacks in them as change points on which to set up the theodolite.

(3.) Chaining or stadia work is more carefully done.

(4.) Levelling is more carefully done, and the stipulated difference between leveller and check leveller's levels per mile is less.

(5.) More cross sectioning is done.

(6.) More detail is surveyed, such as roads, rivers, paths, villages, &c.

(7.) The engineer in charge of the location will go back upon the survey lines in the event of the line running into difficult country where the contours and levels show that the line would be very expensive, and he will try a new line with levels and cross sections until he has explored the country to his satisfaction and decided on the best line.

Generally speaking we may say that the whole operation is a repetition of the second class of survey, viz., "Route Survey or Trial Location over one or more Routes," conducted with greater care and more accuracy and detail.

The instruments should consist of two 6 in. theodolites, three 14 in. levels, three or four clinometers, three or four prismatic compasses, two or three plane tables and accessories (for native assistant surveyors in British India), two or three steel bands, three levelling staffs; ranging rods, axes, cutting knives, &c.

As before, spare instruments have been provided in the above list in case of accident. The party should consist of one theodolite or transit man, one leveller, one check leveller, two or three assistants for cross sectioning and surveying detail. The number of the latter will depend on the nature of the ground. Usually the chief will run the theodolite, otherwise the party will consist of four engineers besides assistants.

As regards taking astronomical observations, the extra meridian observation of sun for true north (see Chapter X.) may be taken either in the forenoon or in the afternoon about three or four hours before or after mid-day, and need not occupy longer than about five minutes. The calculation takes some little time, but that may be done in camp without interrupting the field work. When the party is camped on the survey lines, the method by observation of a circumpolar star at elongation is quick and easily computed (method (5), Chapter X.), or the method by two stars at elongation (method (4), Chapter X.). Star observations are,

however, inconvenient when the camp is not on the survey lines, but at some little distance, as is often the case.

It has been above recommended that observations for meridian should be taken about every mile. When the lines are very short, and the bearings consequently numerous, it may, however, be necessary to take observations for meridian more frequently. The frequency of observations for meridian will be determined by these and local considerations, instrumental error, &c. See also in running long lines, convergence of meridians, page 366.

In describing on page 403 the operation of taking the bearings of the lines, it is stated "sight back on the first line, turn over the telescope, unclamp, and sight on to the second line." This will introduce an error if the instrument is not in adjustment for collimation. To obviate the necessity for turning over the telescope, proceed as follows:—Set the vernier to the bearing of the back line + 180°, which will be the bearing of that line as looked at from its termination to its commencement. Now in place of turning over the telescope vertically, simply unclamp and rotate the upper part of the instrument horizontally and sight on to the forward line. The bearing of the forward line may then be read off the vernier without any error due to collimation line being out of adjustment. The same result will be attained if both faces of the instrument are used, and the mean taken. In running long lines, if the adjustment of the instrument is bad, use both faces, and take the mean of the two points thus given as the correct position of the line (see page 83).

4. **Final Location Survey and Setting out of Line.**—This will differ but little from 3, "Preliminary Location Survey on Route Selected." Its essential feature is that pegs are driven in at every chain, see Chapter V., the curves are set out, and bench marks are carefully established about ½ mile apart. As in tropical countries pegs quickly rot and are eaten up by ants, they should be driven in flush with the ground, and every tenth peg should be larger than the others, and be protected by a piece of tin nailed on the top and stamped with the number of the station. Kerosene tins may be cut up and utilised for this purpose.

The only satisfactory way to mark a line so that it may be readily found after the lapse of some considerable time is to "nick out" or "dogbell" the centre line throughout. This

survey is executed on the basis of the line located by the "Preliminary Location Survey on Route Selected," and the bearings are to be carefully referred to true north, so that the line may be retraced with accuracy in the event of anything happening to delay the construction for some time. Astronomical observations are of course necessary in any case to trace out accurately the line located by the "Preliminary Location Survey on Route Selected."

Bench marks are cut on trees, and should be clear of the line. In jungle or dense forest the line is usually cleared of all trees and undergrowth for a distance of 100 ft. on each side of the centre line prior to construction. The bench marks should therefore be clear of this, *i.e.*, more than 100 ft. from the centre line. This entails about 2 chains of cutting for each bench mark, which somewhat impedes the leveller's progress. With reference to bench marks on trees, there appeared some time ago in a letter to the editor of *The Engineer* a statement to the effect that in levelling for an important railway bridge in India it was discovered some time after the works for the bridge had been commenced that the tree on which the bench mark was cut had grown since the date of the survey, and the value of the bench mark had thereby been altered about 2 ft. This statement was of course absurd, and was promptly corrected by another letter in the next issue of *The Engineer*, which stated that exogenous trees increase in height by successive additions to the top. It is of course to be understood that in executing the "Final Location Survey and Setting out" no pains are to be spared in making any improvements to the location of the line which may present themselves. These alterations are to be conducted precisely on the principles already explained. In British India the native surveyors who will act as assistants to the engineers are usually trained to survey with the plane table. It is advisable, therefore, to include some plane tables and accessories in the list of instruments, as these surveyors are usually at a loss if asked to do much surveying by any other method than the plane table. In damp tropical countries the paper becomes spoiled by damp, ants, flies, perspiration dropping on it, &c., and the plane table is on the whole not a very good instrument for jungle work.

The number of engineers in the party will be the same as for (3), "Preliminary Location Survey on Route Selected," and the instru-

ments the same. As the line is to be pegged out it may save time if there are two theodolite men, one to run ahead simply cutting the line, and the other coming after chaining and putting in pegs. In this case there will be another engineer and three theodolites.

Scales.—The usual scales for plotting the work are—For preliminary location surveys about 400 ft. to 1 inch or about 6 chains to 1 inch or about $\frac{1}{8000}$; for working surveys about 200 ft. to 1 inch or about 3 chains to 1 inch or about $\frac{1}{2500}$.

It may be stated that the rapidity with which undergrowth

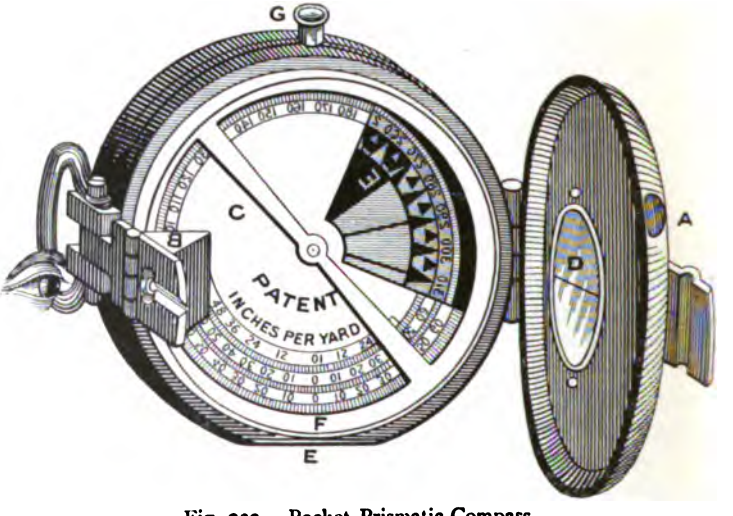

Fig. 232.—Pocket Prismatic Compass.

springs up in wet tropical countries is almost incredible. Suppose the line to have been cleared of all timber and undergrowth for a width of 200 ft. If the stumps of the trees have been left in, during the interval between the end of one dry season and the commencement of the next, in a very hot damp climate the undergrowth will spring up to a great height, thick and impenetrable without cutting every step, the only indication that the line had been cleared being the gap in the *large* trees, which is only visible if the observer is raised above the undergrowth.

No engineer should go abroad to take up survey work in unmapped country, jungle or dense forest, without having a pocket

compass in case of accident. There is a pocket form of prismatic compass (Fig. 232), not much larger or bulkier than a watch, which is very good. The prism folds down where the handle of a watch usually is, and the fore sight is formed by a circular piece of glass in the cover, having a vertical and horizontal scratch on it. The prismatic shown in Fig. 232 may also be used as a clinometer. In any case the compass should be protected in a case similar to the hunting case of a watch.

In the absence of a compass a north and south line may be approximately found by a watch as follows:—Point the hour hand towards the sun, the meridian will then be half way between the sun and the XII on the dial.

Distance and Bearing of Objective Point of a Long Traverse or Straight Line.

Given the Latitude and Longitude of A, and the Latitude and Longitude of B, to find the Bearing of B from A, Distance AB, and Bearing of A from B.—In running a long line of traverse between two points we have sometimes given the latitude and longitude of the starting and objective points as data from which to work. These may be either from observation or taken from a map. In any case the problem then is, "Given the latitude and longitude of A, and the latitude and longitude of B, to calculate the bearing of B from A, the distance AB, and the bearing of A from B." The latter being required when one survey party works from A towards B, and the other from B towards A.

The formulæ are as follows:—

$$\text{Tan } \tfrac{1}{2}(\text{B}+\text{A}) = \cot \tfrac{1}{2}\text{P} \frac{\cos \tfrac{1}{2}(\text{colat of A} - \text{colat of B})}{\cos \tfrac{1}{2}(\text{colat of A} + \text{colat of B})}$$

$$\text{Tan } \tfrac{1}{2}(\text{B}-\text{A}) = \cot \tfrac{1}{2}\text{P} \frac{\sin \tfrac{1}{2}(\text{colat of A} - \text{colat of B})}{\sin \tfrac{1}{2}(\text{colat of A} + \text{colat of B})}$$

$$\text{A} = \tfrac{1}{2}(\text{B}+\text{A}) - \tfrac{1}{2}(\text{B}-\text{A})$$

$$\text{B} = \tfrac{1}{2}(\text{B}+\text{A}) + \tfrac{1}{2}(\text{B}-\text{A})$$

where A = bearing of line AB

B = 360° − bearing of line BA

P = difference of longitude of A and B

For the distance AB we have—

$$\text{Tan } \tfrac{1}{2} \text{ arc AB} = \frac{\tan \tfrac{1}{2} (\text{colat of A} - \text{colat of B}) \sin \tfrac{1}{2} (\text{B} + \text{A})}{\sin \tfrac{1}{2} (\text{B} - \text{A})}$$

Another formula for distance AB is—

$$\text{Cos arc AB} = \frac{\cos \text{colat of A} \times \cos (\text{colat of B} - \phi)}{\cos \phi}$$

where $\tan \phi = \tan$ colat of A $\times \cos$ difference of longitude of A and B.

The problem is simply a solution of the spherical triangle PAB, Fig. 233, in which PA = colat of A, PB = colat of B, AB = required distance, and the angles A and B are the angles which the line AB makes with the meridians at A and B, while P represents the pole and the angle at P = difference of longitude of A and B.

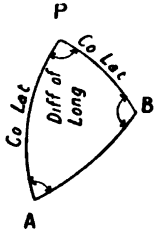

Fig. 233.
Distance and Bearing of Objective Point.

Given the Latitude and Longitude of A, and the Bearing and Distance AB, to Calculate the Latitude and Longitude of B and the Bearing of A from B.— Similarly if any other data of the spherical triangle PAB are given, as for instance, "Given the latitude and longitude of A, and the bearing and distance AB, required the latitude and longitude of B and the bearing BA," the problem may be solved by applying the formulæ for solution of spherical triangles given on page 385, this being a case where we have given the two sides and included angle at the point A.

The solution is as follows :—

$$\text{Tan } \tfrac{1}{2} (\text{B} + \text{P}) = \cot \tfrac{1}{2} \text{A} \, \frac{\cos \tfrac{1}{2} (\text{colat of A} - \text{AB})}{\cos \tfrac{1}{2} (\text{colat of A} + \text{AB})}$$

$$\text{Tan } \tfrac{1}{2} (\text{B} - \text{P}) = \cot \tfrac{1}{2} \text{A} \, \frac{\sin \tfrac{1}{2} (\text{colat of A} - \text{AB})}{\sin \tfrac{1}{2} (\text{colat of A} + \text{AB})}$$

* The angle at $\text{B} = \tfrac{1}{2} (\text{B} + \text{P}) + \tfrac{1}{2} (\text{B} - \text{P})$.

Difference of longitude $\text{P} = \tfrac{1}{2} (\text{B} + \text{P}) - \tfrac{1}{2} (\text{B} - \text{P})$.

To find the colatitude of B = PB we have—

$$\text{Tan } \tfrac{1}{2} \text{ PB} = \frac{\tan \tfrac{1}{2} (\text{colat of A} - \text{AB}) \sin \tfrac{1}{2} (\text{B} + \text{P})}{\sin \tfrac{1}{2} (\text{B} - \text{P})}$$

* This is 360° – bearing BA.

where AP = colatitude of A.

AB = given distance (reduced to degrees, minutes, seconds).

A = given bearing of line AB.

The distances are, however, expressed in degrees, minutes, and seconds of arc, and have to be reduced to actual distance in feet, yards, chains, miles, or metres, as the case may be, or *vice versâ* (see page 386, Chapter X.).

Distance and Bearing of Objective Point from Intermediate Point of Traverse.—In running a traverse between any two points A and B, Fig. 234, whose latitude and longitude are known, it may become necessary at any time to calculate the bearing and distance of the objective point B from the point which the traverse has reached. Suppose the traverse to start at A, and that the objective point is B, and that we have reached a point of the traverse C. Knowing the latitude and longitude of A, we may deduce the latitude and longitude of C by account (see pages 347 to 350), and then the distance and bearing CB as above described. Otherwise we may find the latitude and longitude of C from independent astronomical observations, and from this deduce the distance and bearing CB as above described.

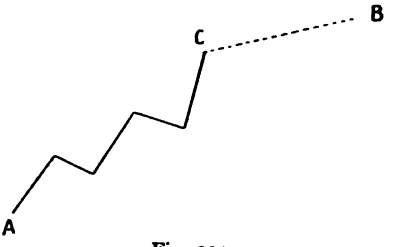

Fig. 234.
Distance and Bearing of Objective Point from Intermediate Point of Traverse.

In the first case we rely upon the accuracy of the traverse from A to C, and in the second case upon the accuracy of the astronomical observations taken at C. If the latitudes and longitudes of the starting and objective points A and B are accurately known from geodetic measurements, and if the traverse from A to C has been carefully executed, it will as a rule be better to adopt the latitude and longitude of C from account, as the observations which can be taken at C will generally with the instruments at the surveyor's disposal be approximate only, at least for longitude. In any case the latitude and longitude of the point C can be worked out from account and checked by observation.

Running a Long Straight Line.—Similarly in running a long *straight* line between two points A and B, Fig. 235, whose latitudes and longitudes are known, the accuracy of the alignment may be checked from time to time by calculation of the true bearing at any point in the line AB, as for instance at C, Fig. 235. The bearing of the straight line AB changes continually owing to the convergence of the meridians to the pole (see page 366). To calculate the bearing of B from any point C there are two methods open to us. First, we may work out the latitude and longitude of C by account, and either calculate the amount of the convergence between A and C (see page 366) or calculate the bearing at C from the respective latitudes and longitudes of C and A. Whichever way we adopt the resulting bearing at C should be the same. Secondly, we may take independent observations for latitude and longitude at C, and from these and the latitude and longitude of B work out the distance and bearing CB. In the first case we rely upon the accuracy of the line run from A to C, and the result of the calculation will only be a guarantee that we are now running with the calculated bearing in a direction *parallel* to CB. If the line has been correctly aligned between A and C the actual bearing of the straight line run should of course agree with the calculated bearing at C, but the mere fact of this being the case is no guarantee that we are exactly on the straight line AB; we may have deviated either to the right or left of C and still happen to be on the correct bearing, as shown by the curved line AC'B', Fig. 235. As already stated, if we now run on the calculated bearing at C, we run on the dotted line C'B' *parallel* to the true line CB. On the other hand, the result of independent observations for latitude and longitude at C' would be that we should run the line C'B straight for B. The circumstances of the case must decide whether to adopt latitude and longitude by account or from observation.

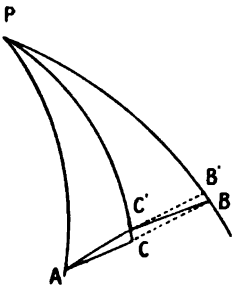

Fig. 235.
Running a Long Straight Line.

In commencing a long traverse say for a railway between two distant points, the first part, until we reach the outskirts of the cultivated and populous district in which the line may be supposed

to commence, will be run with the view of suiting it to the features of the country. It is when we get into the jungle, dense forest, bush, or bare open country, where it is flat, and no features in the ground determine the location of the line, that it becomes necessary to determine the distance and bearing of the objective point and run the line straight for it. These problems are also to be worked out when two parties run to meet each other, and in that case it is necessary to send messengers round by the nearest available route to communicate the respective positions of the points reached on each traverse from time to time.

A rough shot at the required distance and bearing may of

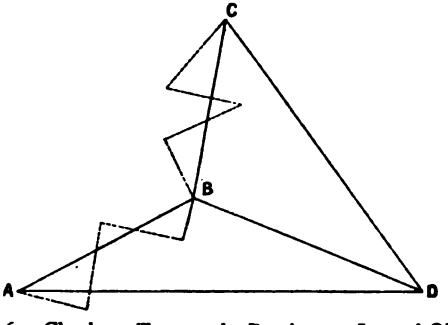

Fig. 236.—Check on Traverse by Bearings to Lateral Objects.

course be made by plotting the starting and objective points on a sheet of paper to a small scale, and keeping the traverse plotted up on it from day to day.

Checking Traverse by Bearings to Lateral Objects.—In making a traverse of any considerable extent, if any conspicuous point, such as a well-defined peak, becomes visible, the opportunity of checking the measurement of the lines of the traverse should not be lost. Thus the point D in Fig. 236 being visible from A and also from B, the bearings AD and BD were taken. From the length AB and the angles DAB and ABD as deduced from the bearings the length BD was calculated. On arriving at C, D was again visible, and the bearing CD taken. From the bearings BD and CD the angles DBC and DCB were deduced, and from these and the calculated length DB the distance BC was calculated. The

actual measurement of BC should then agree with the calculated length.

When the lines of the traverse are broken between A and B and between B and C, as shown by the dotted lines in the figure, the distances and bearings AB and BC may be calculated by reckoning up the latitudes and departures of the survey lines, and the desired check on the measurements obtained.

Classification of Curves by "Degree."—In British India and the United States and South America curves are designated by their "degree." The "degree of a curve" is the angle subtended at the centre of the curve by a chord of 100 ft. (In British India and the United States the 100 ft. chain is used.) The deflection angle for 1 chain under the "degree" system is therefore half the "degree" of the curve. This simplifies matters considerably. In countries where the metric system is in use, the 20 metre chain is usually used, and the "degree of curve" is the angle subtended at the centre of the curve by a 20 metre chord. In this connection the following *memoria technica* are useful:—

.10 metre or 10 centimetres = 4 in. nearly.
.15 metre or 15 centimetres = 6 in. nearly.

American Method of Cross Sectioning.—For those to be engaged in the survey or construction of roads or railways on the American system in Mexico, Central or South America, the following American method of cross sectioning for final earthwork quantities will be useful.

Let Fig. 237 represent the cross section of an embankment. Fig. 238 represents the field cross-section book, in which there is a central column. In this central column are first entered all the heights of bank or depth of cutting on the centre line at every chain. Thus in Fig. 238 there is shown in the central column $\frac{-1.00}{0}$. This indicates that at that point there is a 1.00 metre bank or 1 metre "fill" as it is usually called. The numerators of the decimal fractions represent heights of bank or depths of cutting as the case may be, while the denominators represent the distances out from the centre line at which these depths occur. The depths of cut or heights of bank are taken on the ground with a "hand level" (see page 148), and the method of procedure

is as follows:—Stand at *a* at the centre peg, and hold up the hand level at a convenient height above the ground, say 1.50 metre, as indicated by a nick cut in a ranging rod in which the hand level may be conveniently inserted. If now the staff-holder holds the staff at *b* and the reading is 2.00 metre, this indicates a fall of .50 metre. The bank on the centre line at *a* being 1.00 metre, the bank at *b* will therefore be 1.50 metre. The distance out as measured either with the tape or the staff being 1.20 metre, there is booked in the cross-section book, Fig. 238, $\frac{-1.50}{1.20}$ indicating that there is 1.50 metres of bank at a distance out of 1.20

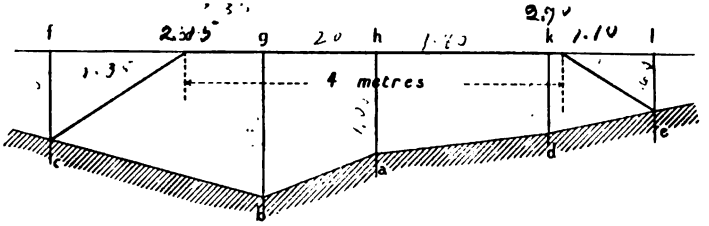

Fig. 237.—American Method of Cross Sectioning.

$\frac{-0.90}{3.35}$	$\frac{-1.50}{1.20}$	$\frac{-1.00}{0}$	$\frac{-0.85}{1.80}$	$\frac{-0.60}{2.90}$

Fig. 238.—Cross-Section Book.

metres to the *left*, the entry being to the *left* hand of the central column. Minus is used to indicate bank or "fill," and plus to indicate cutting. The points *c* and *e*, where the bottom or top of slope comes, are located (to locate top or bottoms of slope see Chapter V.), and the distances out and height of bank or depth of cut entered in the cross-section book. In the case of Fig. 237, these are $\frac{-0.90}{3.35}$ and $\frac{-0.60}{2.90}$, the slopes being $1\frac{1}{2}$ to 1, and the formation width or "width of road bed" as it is called being 4 metres. The other intermediate readings, where the slope of the ground changes as at *d*, Fig. 237, are booked as shown in Fig. 238. This method of cross sectioning is very rapid after a

little practice, and one great advantage of it is that the area of the cross sections and cubic contents of earthwork may be calculated from the figures in Fig. 238 *without plotting the cross sections.* The areas and cubic contents of earthwork are usually entered in special columns on the opposite page of the cross-section book, which page is reserved for that purpose. To calculate the area of the cross section shown in Fig. 237 and booked in Fig. 238, proceed as follows:—The area of the trapezium *abgh* is the mean of the depths *ah* and *bg* multiplied by *gh*, *i.e.*, $\frac{1}{2}(1.00 + 1.50) \times 1.20 = 1.50$. Similarly the area of *bcfg* is the mean of the depths *bg* and *cf* or $\frac{1}{2}$ $(1.50 + 0.90)$ multiplied by *fg*, *i.e.*, $\frac{1}{2}(1.50 + 0.90) \times 2.15$. The distance *gh* is 1.20 as indicated by the denominator of the first fraction to the left, Fig. 238, and the distance $fg = hf - hg = 3.35 - 1.20 = 2.15$, or the difference between the denominators of the two fractions to the left of the central column, Fig. 238. It now remains to deduct the triangle *fxc*, of which the area is $\frac{1}{2}(1.35 \times 0.90)$.

The area of the left hand part of the cross section is therefore $\frac{1}{2}(1.00 + 1.50) \times 1.20 + \frac{1}{2}(1.50 + 0.90) \times 2.15 - \frac{1}{2}(1.35 \times 0.90) =$ 3.47 superficial metres.

The other half of the cross section is treated similarly, and the total result is the area of the whole cross section.

Cubic contents are then got by adding up the superficial areas of the cross sections and multiplying by the common distance apart of the cross sections in the usual manner. Thus there is no necessity for any plotting, and as the quantities are calculated from the actual levels and measurements taken in the field, errors of plotting and scaling for the areas are obviated. With a little practice both the field work of cross sectioning and the calculation of the areas is very rapid.

II. **Surveys of a Limited Area of Country.**—As a rule all survey work of very large areas is conducted by Government, the primary triangulation of the country being taken as the basis of the work. The surveyor may, however, in his private capacity be called upon to make surveys of considerable area abroad for private individuals, syndicates, or companies. In this chapter we shall confine ourselves to surveys of moderate extent, reserving for Chapter XII. larger surveys necessitating triangulation from an accurately measured base.

Methods to be Adopted.—After making a careful examina-

tion of the ground, in most cases the surveyor will decide to run a traverse round the boundaries of the property. At convenient points on this traverse he will leave marks from which to run lines throughout the interior, which he will divide up into well-conditioned triangles, the most convenient for picking up all interior detail. From these interior lines and subsidiary chain lines he will survey all the details of the property. The traverse round the boundary will form the basis of the work, and all other lines will "check in" upon it. Whether or not a complete traverse is first made round the whole property, or whether it is divided up by smaller traverses partly along the boundaries and partly through the interior, will be decided by the circumstances and local conditions of each individual case. In other cases it may be found better to project lines right through the property as bases from which to survey the ground. Tacheometry or stadia work is much used for the details or topography of surveys abroad (see Chapter VI.). In any case the main principles to be observed are precisely those set forth in Chapters I., II., III. The essential difference is that the bearings must be referred by astronomical observations to true north, and thus checked and corrected about every mile, as already explained for a long line of communication, while the chaining must be executed with the greatest care and every allowance made.

Steel Tape Measurements.*—As regards chaining, the steel band will be found the best to use, and it may be from 1 chain to 10 chains in length, according to the character of the country. On sloping ground more accuracy is attained by measuring on the slope and reducing to horizontal distance by calculation. When the range of temperature is considerable, allowance must be made. In the colony of Victoria, Australia, for instance, the measurement of a line 1 mile long measured with a steel band 66 ft. long in winter may vary nearly $5\frac{1}{2}$ links from the measurement of the same line with the same band in summer, from the effect of change of temperature only. The increase of length of a steel band per degree Fahr. is 0.000006886 ft. per foot of steel band. Thus for a 66 ft. steel band the increase of length is 0.000006886×66 ft. per degree Fahr. Error due to unequal tensile strain on the steel band should be provided against.

* See also page 443.

The extension in 1 mile with a 66 ft. steel band having an effective sectional area $\frac{1}{2} \times \frac{1}{40}$ in. is 0.667 link when a stress of 30 lbs. is used, and the extension will vary with the stress put upon the steel band, which should therefore be uniform. This is effected by means of a spring attachment to indicate the pull on the chain. When measurements are made on inclined ground by "stepping" and holding up the chain, an error of considerable amount is introduced. "A steel tape weighing 6 lbs., used with a tensile

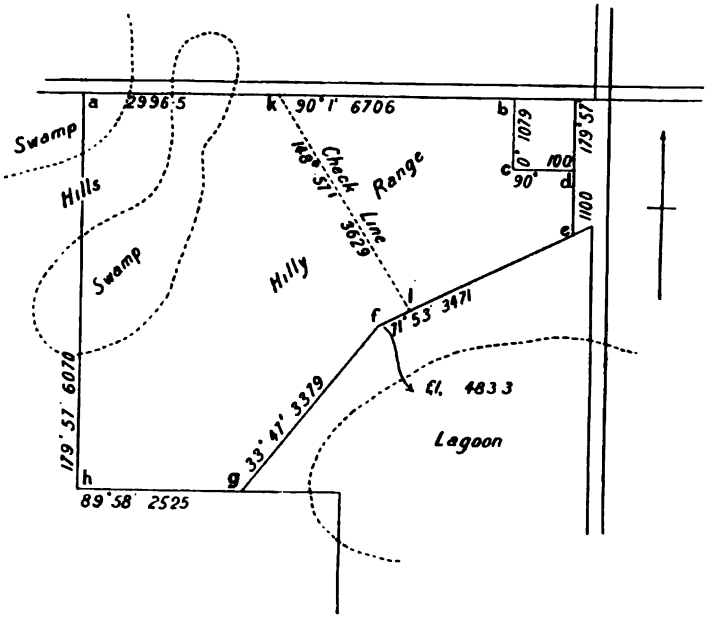

Fig. 239.—Example of Survey.

stress of 30 lbs., will give an error of 13.44 links per mile due to 'sagging' or curvature of the tape."

In practice, of course, some of the errors in chaining counterbalance and neutralise each other to a greater or less extent.

As a survey of this kind must close in on its starting point, great care must be taken with the chaining and all sources of error as above guarded against. In a long line of communication such as a railway, there is no necessity to close in to a nicety on any given point, neither does the line return to its starting point as

in a survey of an area of country whose exact superficial area is required. The chaining, therefore, on a railway survey need not be conducted with excessive care. See Chapter XII. for accurate steel tape measurements.

Example of Survey.—Fig. 239* is an example of a survey of 320 acres in the heart of steep, scrubby, and heavily timbered ranges in Australia. The survey was made with a Cooke's 5 in. theodolite and a 66 ft. steel band. The check line *kl* which was projected through it showed the "closing error" to have been 30″ in angle and 0.1 link in linear measurement. This great accuracy was no doubt due to errors of chaining and angular measurement counterbalancing each other. The following is the calculation of the latitudes and departures or northings, southings, eastings, and westings of the survey, and also of the check.

TRAVERSE (Fig. 239).

Bearings.	Distance.	North.	South.	East.	West.	W.L.	Areas.	
							North.	South.
	Links.	Links.	Links.	Links.	Links.	Links.		
S. 89° 59′ E.	6706	...	2.0	6706.0	...	6706.0	...	0.13412
South - -	1079	...	1079.0	...	...	13412.0	...	144.71548
East - -	1001	...	...	1001.0	...	14413.0	...	...
S. 0° 3′ E.	1100	...	1100.0	0.9	...	15414.9	...	169.56390
S. 71° 53′ W.	3471	...	1079.0	...	3298.0	12117.8	...	130.75106
S. 33° 47′ W.	3379	...	2808.5	...	1878.9	6910.9	...	194.93518
S. 89° 58′ W.	2525	...	1.5	..	2525.0	2537.0	...	0.03805
N. 0° 3′ W.	6070	6070	...	...	5.2	6.8	0.41276	...
	...	6070	6070.0	7707.9	7707.1	...	0.41276	640.13779
								0.41276
							2	639.72503

The area is calculated from the latitudes and departures as described in Chapter II.

Area, 319 acres 3 roods 18 poles.

Note.—The bearings marked on Fig. 239 are measured from 0° round to 180°. The bearings recorded above are measured from N. to 90° E. and W., and from S. to 90° E. and W., this being more convenient for calculation as it is the sines and cosines of these bearings which are used to calculate the latitudes and departures.

A	319.86251
R	3.45004
P	18.00160

* Minutes of Proceedings Inst. C.E., vol. xcv., "The Practice of Surveying in the Australasian Colonies." By S. K. Vickery, Assoc. M. Inst. C.E.

CHECK TRAVERSE (Fig. 239).

Bearings.	Distance.	North.	South.	East.	West.
	Links.	Links.	Links.	Links.	Links.
S. 89° 59′ E. - -	2996.5	...	0.9	2996.5	...
S. 31° 3′ E. - -	3629.0	...	3109.0	1871.8	...
S. 71° 53′ W. -	483.0	...	150.2	...	459.1
S. 33° 47′ W. -	3379.0	...	2808.5	...	1878.9
S. 89° 58′ W. -	2525.0	...	1.5	...	2525.0
N. 0° 3′ W. -	6070.0	6070.0	...	...	5.2
	...	6070.0	6070.1	4868.3	4868.2

The original traverse consisted of the distances and bearings *abcdefgha*. The check was made by measuring the distance *ak*, distance and bearing *kl*, and distance *lf*, the remaining distances and bearings *fg*, *gh*, and *ha* being taken from the record of the original traverse. The angle at *a* was measured and compared with the difference between the original bearings of *ha* and *ab*, which gave the "closing error" in angle.

Regulations of the Victoria Survey Department, Australia.—The regulations of the Victoria Survey Department are that the dimensions furnished on the plan must close geometrically, *i.e.*, they must have been adjusted to do so as described in Chapter II., and the maximum difference between the plan and the actual field measurements is not to exceed 2′ in arc or the tangential equivalent on the length of any line. Chaining by stepping is allowed, but when the slope exceeds 5° it is recommended that the angle of slope should be measured on the vertical arc of the theodolite and the distance measured on the slope and reduced to its horizontal projection.

With a 5 in. or 6 in. theodolite and a steel band the above limits of error may be easily complied with.

Surveying Regulations in the Australian Colonies.—In the Australian Colonies almost every branch of the civil engineer's profession is practised directly under the control of the State, and more especially those branches in which surveying plays an important part.

Surveys under the control of the Departments of Lands and

Survey are performed by salaried officials or by those who are "authorised" on either Crown lands or sold lands which will be brought under the operation of the Transfer of Lands Statute. Plans of surveys made by "unauthorised" persons are not officially recognised. All "authorised" surveyors must comply with the regulations issued as departmental instructions or bearing the force of an "Order in Council." Particulars of the examinations for selecting "authorised" or certificated surveyors are given at the end of this chapter.

Each colony is divided into survey districts, each of which is under the control of a salaried inspecting officer. These again are subdivided into divisions, and each is assigned to an "authorised" surveyor, to whom orders for departmental work are sent. The "authorised" surveyors are not salaried, but are remunerated by fees fixed by regulation, and they are removable at the pleasure of the Surveyor-General. When a vacancy occurs selection is made from those holding a certificate of competency obtained by examination. The inspecting officer of each district makes periodical inspection of "authorised" surveyors' work in the field, their instruments, field books, &c., and certifies to all accounts. Inspecting surveyors are under immediate control of the Surveyor-General.

The higher classes of surveying, including minor triangulation, are performed exclusively by staff officers; the ordinary sectional or block surveying and laying out of towns and roads is performed by "authorised" surveyors. The system of examination and supervision is so complete that a very high standard of efficiency has now been attained.

SUBJECTS OF EXAMINATION OF "AUTHORISED" SURVEYORS TO LANDS DEPARTMENTS, COLONY OF VICTORIA, AUSTRALIA.

a. Construction, adjustment, and use of theodolite, level, and other modern instruments.

b. Principles and practice of subdivisional, topographical, and road surveying.

c. Practical trigonometry.

d. Computation.

e. Plotting by co-ordinates and otherwise.

f. Drawing.

Additional special subjects are:—

a. Practical geodesy, including determination of latitude, longitude, and meridian, and computation of relative latitude and longitude from triangulation.

b. Spherical trigonometry.

c. Setting out curves and computations in connection therewith

d. Levelling and mensuration of earthwork.

These examinations generally extend over five to seven days of eight hours each. About 30 per cent. of the candidates pass. The examinations in the other colonies are practically the same. Certificates are granted in some instances without examination, the fact of their being so obtained being stated on them. In the colony of Victoria certificates are granted without examination only under the following conditions:—

1. Having passed some examination equivalent in the opinion of the Board to that prescribed in Victoria, in Great Britain, United States of America, India, or some British colony, and having been in practice for not less than one year.

2. Having been engaged in the colony under some authorised surveyor for not less than six months, and having a favourable report on his qualifications from the Surveyor-General or the inspecting surveyor in charge of the district in which the applicant has been employed.

CHAPTER XII.

TRIGONOMETRICAL OR GEODETIC SURVEYS.*

Ordinary Small and Large Surveys.—A small survey may be defined as one in which the length of the main chain lines does not as a rule much exceed a mile. An ordinary large survey may be defined as one of greater extent or area, but in which the length of the main lines is still limited to about a mile. In these cases the lengths of the lines are actually measured and the stations are easily visible from each other, so that no special signals or signalling apparatus are required. This class of survey is dealt with in Chapters I. and II. *et sequitur.*

Trigonometrical Survey.—A trigonometrical survey is one undertaken for the mapping of a large extent of country, and in which the length of the sides of the main triangles is great, the maximum being about 100 miles, while the average will be about 40 to 60 miles.

Base Line and Primary Triangles.—The whole of a trigonometrical survey is based upon the accurate measurement of a "base line."

The base line is laid out in the most favourable situation, accurately measured, and from its extremities angles are observed to surrounding stations, which stations form with the base line triangles. These triangles are gradually and systematically extended in size as hereafter described (page 456), until they reach approximately the size of the largest triangles to be measured in the survey. The sides of these triangles are computed from the actually measured length of the base and the observed angles, and they then serve as bases upon which to form other triangles of like extent whose angles only are measured, the sides being com-

* Strictly speaking the term geodetic applies only to measurements undertaken to determine the figure of the earth. It is, however, commonly applied to trigonometrical surveys.

puted from the angles and the previously calculated sides. In this manner the whole country to be surveyed is covered with a net work of "primary" triangles averaging 40 to 60 miles in the side, the whole being computed from the one accurately measured base and the observed angles.

All the angles of each triangle are measured so that the errors of measurement of the angles may be ascertained and corrected.

In laying out and deciding upon the stations of these "primary triangles" any available maps or sketches should be utilised, and these stations should be determined on by a careful reconnaissance of the country and examination of each proposed station.

Base of Verification.—During the progress of the "primary triangulation" a "base of verification" should be measured from time to time. Its length as calculated from the original base and the angles of the intervening primary triangles is compared with its length by actual measurement, and this gives a check upon the measurement of the angles of the primary triangulation. A similar "base of verification" is also to be measured at the conclusion of the primary triangulation.

Secondary Triangles.—Within the "primary triangles" are formed "secondary triangles." As many of these as possible should be fixed upon and their angles observed with the large theodolite during the progress of the primary triangulation. The remainder may afterwards be observed with a smaller instrument. The sides of these "secondary triangles" will average 10 to 12 miles in length, and are computed from the observed angles.

Tertiary Triangles.—Within these secondary triangles "tertiary triangles" are formed, averaging in open country 3 to 4 miles in the side, and in enclosed country perhaps 1 or 2 miles in the side. The sides of these triangles also are computed from the observed angles.

Interior Filling-in.—When the above skeleton triangulation of the country has been completed, the interior filling-in, survey of roads, streams, villages, towns, &c. &c., is performed by traversing with the theodolite and chain, compass, plane table, and ordinary chain surveying, as described in Chapters I. and II. *et sequitur*, this work being made to "check in" between the stations of the "tertiary triangles" within certain stipulated limits of accuracy.

Triangulation Stations.—All prominent points, such as churches, mills, &c. &c., may of course be taken advantage of as stations for the tertiary triangulation, and also for the primary and secondary triangulation when suitable. In the absence of such existing marks the stations must be fixed by suitable marks as hereafter described. In the case of churches, &c., being selected as stations the instrument cannot as a rule be set up over the centre of the station, in which case it is set up as near it as possible, and the observed angles taken with the instrument at that station are "reduced to the centre of the station" as hereafter described.

Whole Survey Reduced to Sea Level.—The measurement of the base is reduced to mean sea level or other fixed datum and to a standard temperature. All the computed lengths of the sides of the triangles are therefore also as reduced to mean sea level and to the standard temperature. The consequence is that the result of the survey is a representation of the country as projected on to the surface of a sphere whose radius is that of the earth at mean sea level, the whole being as at the standard temperature adopted. Apart from considerations of temperature, a line 4 miles long, at a height of 1 mile above mean sea level, actually measured on the ground, would thus differ from its length as represented on the map by about 6 ft. On a scale of $\frac{1}{2500}$ or 25 in. to the mile it would be scarcely possible to scale a distance of 4 miles to 6 ft. Upon reducing the measured length to mean sea level it should of course agree exactly with the length as represented on the map. The above is merely cited to show that for all practical purposes actual measured distances will agree with their scaled lengths on the map.

Base Lines of the Ordnance Survey of Great Britain and Ireland.

Hounslow Heath Base.—The first base measured was the Hounslow Heath Base, about 5.20 miles long. The length of this base as measured with glass rods was determined in 1784 to be 27404.0137 ft. reduced to mean sea level and to a temperature of 62° F., the standard temperature of British imperial linear measures. The height of the base above mean sea level was taken as 54 ft. This base was remeasured in 1791 with a steel

chain, when the length was made to be 27404.32 ft., a difference of only 0.31 ft. or less than 4 in.

Romney Marsh Base of Verification.—The next base measured was Romney Marsh Base of Verification, about 5.40 miles long. The apparent length as measured by the chain was 285.36736 chains = 28,536 ft. 8.835 in. The reduction for temperature was 12.8 in., reduction of hypotenusal to horizontal distance 3.023 in., for height above mean sea level taken as 15½ ft. 0.166 in., increase for half ascertained wear of chain 3.282 in. The ultimate length of the base at 62° F. and mean sea level was 28,535 ft. 8.128 in. or 28535.66 ft. By calculation from the sides and angles of the primary triangles of the survey the length was made to be 28533.3 ft. or a difference of 2 ft. 4 in. The mean of these was used in the subsequent computations.

Salisbury Plain Base of Verification.—This was measured with the chain in 1794, and was nearly 7 miles long. The measured length of the base reduced to 62° F. and mean sea level was 36574.4 ft. The length of this base was also computed from the angles and sides of seventeen triangles intervening between it and the Hounslow Heath Base. Five different results were obtained, but the most unexceptionable was considered a computation of 36574.7 ft. The difference between the calculated and measured length of this base of verification was thus little over 3½ in.

King's Sedgemoor Base.—Unsuccessful attempts were made to find a suitable place for the measurement of another base of verification to the west in Cornwall and also in Devonshire. King's Sedgemoor in Somersetshire was therefore ultimately selected for a new base. The length of this base as measured with the chain and reduced to 62° F. and mean sea level was made to be 27680.14 ft. or nearly 5¼ miles. The greatest probable error was calculated to be not less than 6 in. and not more than 9 in.

Misterton Carr Base.—This base was measured in 1801 with the chain, and was nearly 5 miles long. It was measured horizontally, and was made to be 26342.712 ft. at 62° F. and mean sea level. The greatest probable error was calculated to be 2 in.

Rhuddlan Marsh Base.—This base was measured in Rhuddlan Marsh, North Wales, in 1806, and was the last base measured in Great Britain. Similarly to the others it was measured with the steel chain, and its length was made to be 24514.26 ft. or about 4½ miles. The calculated length of this base as computed from the Misterton Carr Base and the intervening triangles differed from the above actual measurement about 9½ in.

Method adopted for Measuring Base Lines. — In measuring all the preceding bases with the chain a drawing-post and a weight-post were employed, one end of the chain being fixed to the drawing-post while a 56 lb. weight was hung to the other end of the chain at the weight-post. The chain was made to lie in deal coffers carried by trestles, so as to be perfectly straight and without "sag." Thermometers were placed at different points along each chain measured so as to reduce for temperature. The chain used was 100 ft. long, with 40 links, ½ in. square.

Base on Shores of Lough Foyle, Ireland: Colonel Colby's Base-measuring Apparatus.—This was perhaps the most accurately measured base of the Ordnance Survey. The preceding bases were all ultimately measured with the steel chain, although glass rods and deal rods were also used. This, however, was measured in 1826 with a special compensating apparatus devised by Colonel Colby which counteracted the effects of expansion and contraction. It is shown in Figs. 240, 241, 242, Plate XIV., and is described by Bourns as follows:—"Two bars, one of iron and the other of brass, 10 ft. long, placed parallel to each other, were riveted together at their centres, it having been previously ascertained by numerous experiments that those metals expand and contract in their transitions from cold to heat and the reverse in the proportion of 3 to 5. The brass bar was coated with a non-conducting substance to equalise the susceptibility of the two metals to change of temperature, and across each extremity of these combined bars was fixed a tongue of metal with a minute dot of platinum almost invisible to the naked eye, and so situated on these tongues that under every degree of expansion or contraction of the rods the dots at each end always remained at the constant distance of 10 ft. apart. This will be better understood by reference to Fig. 240. B is the iron bar

(about $\frac{5}{8}$ in. wide and $1\frac{1}{2}$ in. deep) the expansion of which is represented by 3; C the brass bar, of the same size, the expansion of which = 5; the two being riveted together at the centre *d*. EF and *eg* are the tongues, pinned on so as to allow the bars to expand. The platina dots are near the extremities E and *e*. The tongues are by construction made perpendicular to the rods, with the dots 10 ft. apart, at a mean temperature of 60° F.; and the expansion or contraction taking place from the common centre *d*, when B expands any quantity which may be expressed by 3, C expands, at the same time, a quantity = 5, and the position of the tongues is changed to EG and *ef*, the dots E and *e* remaining unalterably fixed at their original distance apart.

"(In most methods of measuring a base line the temperature requires to be noted as every chain or rod is laid. The mean of all the readings of the thermometers is ultimately taken, the temperature in which the chain or rod was originally proved being known, the difference between it and this mean is readily found; and the expansion or contraction for 1° of heat being also known, that quantity is multiplied by the number of degrees constituting the difference, and the resulting quantity is added or deducted as the case may be. For example, in the measurement of one base on the English survey, the sum of all the degrees shown by the thermometers placed by the side of the 100 ft. chain was 98511, wherefore—

$$\frac{98511^{\circ}}{5} - 54^{\circ} \times 272.8 \times \frac{0.0075}{12} = 3.1069 \text{ ft.}$$

There having been five thermometers for each chain's length, the chain having been originally measured in 54° of temperature, the length of the base being 272.8 chains, and 0.0075 in. being the expansion of 100 ft. of blistered steel to 1° F.)

"It is evident, from the construction, that these dots could not, if desired, be brought into either contact or coincidence; but a more correct plan was adopted, which consisted of laying each bar so that the dot at its extremity should always come within a fixed distance of the end of the next bar. This was effected by means of a small apparatus consisting of two microscopes, shown at *b*, *b*, Fig. 242, each furnished with cross hairs, attached to a similar compound bar *c*, 6 in. long, mounted on a stand by means of which they could be laid perfectly horizontal by a spirit

level *d*, the cross hairs of the microscopes attached to these bars occupying the position of the dots on the longer ones.

"The starting point of the base was a stone pillar with a platinum dot let into its centre. A transit instrument was set up over this, and the coffers carefully adjusted to the line by means of sights at their extremities. When the first coffer was levelled by the levels shown at K, K, Figs. 241, 242, and the microscope stand adjusted so that the cross hairs of one microscope were over the starting point, the adjacent point of the first bar was brought under the cross hairs of the other microscope by means of three slow motion screws attached to the coffer in which each bar was fixed, so that the coffer could be moved on one side, backwards or forwards, as required.

"The microscopes were usually 6 in. apart, but when the uneven surface of the ground made it difficult to bring the bars to the same level at this distance, the interval was altered. Microscopes of different lengths were also used when the ground made it necessary to lay the coffers at different levels, so that the platina dots might be brought into the focus of each microscope.

"The coffers were laid on trestles S, S, Fig. 242, equidistant from their centres, so that they might always have the same bearing. A stand with screws was similarly adjusted at the other end of the first bar, the second coffer laid and levelled, and its adjacent dot brought under its microscope; the third, fourth, and fifth bars were similarly adjusted, and thus 52 ft. were measured. The first coffer was then carried forward and similarly placed in front of the fifth bar, and the operation repeated, and so on.

"An average distance of about 250 ft. was measured per day; five coffers or a length of 52 ft. being levelled and laid together. About 400 ft. of the base was across the river Roe, in which piles 5 ft. 3 in. centre to centre were driven to support the coffers. At the end of each day's work a triangular stone was sunk at the end of the last bar laid, with a cast-iron block fitting into it, having a brass plate, adjustable by screws, with a dot let into it. This dot was brought exactly under the cross hairs of the extreme microscope and served as a starting point on the next day. A sentinel was always left in charge of the stone, which was secured by a wooden cover screwed over it.

"The total length of base thus measured was about 8 miles. Two miles were afterwards added by the method of prolonging a

base described on page 455, making the total length of base rather more than 10 miles."

In 1849 the base on Salisbury Plain was again measured with Colonel Colby's apparatus. The length of the Lough Foyle Base was then calculated from this measurement and the intervening triangles. The difference between the computed and measured lengths of the Lough Foyle Base was found to be only 5 in., thus affording a final proof of the accuracy of the English triangulation.

The base lines of the Trigonometrical Survey of India were measured with Colonel Colby's base-measuring apparatus. To test the apparatus, the last base at Cape Comorin, 8,912 ft. long, was measured four times with a view to ascertaining the actual error in measurement. The difference between the several measurements and the mean of the four, +.0017, −.0049, −.0015, +.0045 ft. According to Colonel Walker, Surveyor-General of India, the average probable error of a single measurement of a base with Colonel Colby's apparatus is ± 1.5 millionths. The average rate of measurement with Colby's apparatus in India was about 1 mile in five days; the length of bars used in this case was 10 ft.

United States Coast and Geodetic Survey Base-measuring Apparatus.—A similar base-measuring apparatus is used in the United States Coast and Geodetic Survey. It also consists of a bar of brass and a bar of iron. They are, however, connected at one end, so that the entire contraction or expansion takes place at the other end. The whole apparatus is covered by a double tin tubular case, only the ends of the sliding rods being open. The observations are made through glasses in the sides. The length of the bars is 6 metres, and two tubes are employed in measuring a base, each tube being supported by two trestles. On one base 7 miles long, the greatest possible error was calculated to be less than $\frac{6}{10}$ in. On another base $6\frac{3}{4}$ miles long, the probable error was under $\frac{1}{10}$ in., and the greatest possible error less than $\frac{3}{10}$ in.

With the above apparatus 1.06 mile was measured in $8\frac{1}{2}$ hours, and under favourable circumstances 1 mile a day can be measured. To test the apparatus to the utmost the base at Atlanta, Georgia, was measured twice in winter and once in summer, 1872-73, at

temperatures 51°, 45°, 90° F.; the difference of first and second measurements was +.30 in., and of second and third measurements +.34 in., the actual length and computed probable error in metres being 9338.4763 ± 0.0166.

The Great Theodolite of the United States Coast and Geodetic Survey.—The great theodolite of the United States Coast and Geodetic Survey was 2 ft. 6 in. in diameter, divided to 5 minutes, and read to single seconds by three micrometer microscopes, placed 120° apart. The focal length of the telescope was 4 ft.

The Great Theodolite of the Ordnance Survey.—The great theodolite was used for measuring the angles of the primary triangles. It was made by Ramsden, and the circle or divided limb was of brass, 3 ft. in diameter. It had only two verniers. The focal length of telescope was 36 in., aperture 2½ in., magnifying power 54. Its weight was about 200 lbs. or nearly 2 cwt. The stand, steps, stools, tent, &c., weighed about another 200 lbs. It was found that the effect of heat and cold on the limb would produce considerable errors if the instrument was subjected to currents of air. This was avoided by keeping up the walls of the tent in windy weather, and leaving only an opening sufficient to take the observation. In calm weather the tent was put down.

There were altogether four large instruments used on the Ordnance Survey of the United Kingdom, two 3 ft. diameter, one 2 ft. diameter, and one 1 ft. 6 in. These were all made by Ramsden in 1798, except the 2 ft. diameter, which was made by Troughton & Simms. They are still to be seen in good condition in South Kensington Museum.

Portable Scaffold.—A portable scaffold was made for the purpose of raising the instrument above the ground. It consisted of an inner scaffold for supporting the instrument and an outward one for the observers, not touching or in any way connected with each other. These scaffolds were in two parts, altogether 30 ft. high; so that one half could be used when it was only necessary to raise the instrument 15 ft. A wooden three-sided vertical shaft which turned on an axis protected the silk thread which suspended the plumb bob from wind.

Portable Signal.—A portable signal was made consisting of a tripod ladder about 35 ft. high, which carried at its top either a globe lamp, a white light, or a flagstaff as necessary. A heavy plumb bob was used to centre this over the station to be observed.

Portable Crane.—A portable crane was used for raising the great theodolite to the top of churches, towers, or other buildings, or to the top of the scaffolding.

Permanent Station Marks.—The main stations were marked by permanent marks so that they might be found and the observations repeated at any time, or new work connected with the original primary triangulation.

12 in. Theodolite of the Ordnance Survey.—The angles of the secondary triangles were observed with a 12 in. theodolite.

7 in. and 5 in. Theodolites of the Ordnance Survey.—The angles of the tertiary triangles were measured with 7 in. and 5 in. theodolites, principally with a 7 in. instrument.

Triangles of the Ordnance Survey of Great Britain and Ireland.

Primary Triangles.—The length of the sides of the primary triangles averaged from 40 to 60 miles, and the total number of stations was about 250. The longest side of one of the largest triangles and connecting the West Coast of Scotland with Ireland was about 95.20 miles. Some of the triangles in Ireland exceeded 100 miles in the side.*

Secondary Triangles.—These averaged 10 or 12 miles a side.

Tertiary Triangles.—These averaged from 1 to 3 miles a side.

Having given the chief principles of a trigonometrical survey and a brief outline of the Ordnance Survey of the United Kingdom, which is one of the best examples of a trigonometrical

* The largest triangle has one angle at Snowdon in Wales, another on Slieve Donard in Ireland, and a third at Scaw Fell in Cumberland. Each side is over 100 miles, and the spherical excess is 64″.

survey, these will serve as a guide to the main features. We now proceed to describe the various operations in detail.

General Form of Triangulation.—The surface of Great Britain is uniformly covered with triangulation. The most usual method, however, is that of chains of triangles in the direction of the meridian, and perpendicular to it. The principal triangulations of France, Spain, Austria, and India are so arranged. Oblique chains of triangles are formed in Italy, Sweden, and Norway, also in Germany and Russia and in the United States. These chains are composed merely of consecutive triangles, sometimes and more frequently in India, of combinations of triangles forming consecutive polygonal figures. In this method of triangulation the sides of the triangles are generally from 20 to 30 miles in length, seldom exceeding 40 miles.

Reconnaissance and Selection of Stations.—One of the first points to be noticed in the selection of stations is that no triangle should have an angle less than 30° or more than 120°. A small angle may be measured as accurately as a large one, but a given error, say 1 second, in a small angle, gives a much greater error in the resulting calculated distance than an error of 1 second in a large angle. The distances are proportional to the sines of the angles, and the differences of the sines of small angles near 0° are very much greater than the differences of the sines of large angles near 90°, which are nearly zero; the error in distance due to a given error in a small angle is therefore much greater than the error in distance due to the same error in a large angle. The best-conditioned triangle is the equilateral triangle, and the best-conditioned quadrilateral is the square.

As the accuracy, time of execution, and cost of the survey depend in the first instance on a judicious selection of the stations, this is one of the most important duties of the chief engineer of the survey, and should be personally attended to.

The points to be particularly noted are:—That triangles are well conditioned; the amount of cutting and clearing necessary to see between stations selected; probable amount of damage; height, expense, and convenience of access of stations; liability to movement of stations during progress of survey; preservation of stations after survey is completed.

Lines should be avoided in which factory chimneys or other similar objects intervene, which may interfere by smoke or give rise to excessive refraction by heat. Stations selected may be marked by flags, and it will be found a good mode of procedure, when any maps of the ground to be surveyed are available, to first fix the stations on the map and then examine the vicinity of each point for a suitable position for the station. When a station is selected the highest trees are climbed, and by the aid of field glasses it is ascertained if the other stations are visible. When there are no trees or buildings, ladders spliced together and elevated by means of ropes may be used. When forest, jungle, or undergrowth obstructs the line between stations, the stations are usually raised by erecting a wooden scaffold so as to clear the greater part of the obstruction, and the remainder is cut and cleared out.

Determining necessary Heights of Stations.—The reconnaissance party should determine the heights of the scaffold-

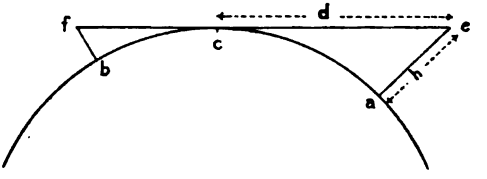

Fig. 243.—Heights of Stations.

ing required at each station. The stations must be high enough to overcome the earth's curvature as well as to clear intervening obstacles. Scaffoldings are of course not required when the ground at the station is of sufficient elevation.

In Fig. 243 let a and b be two points where stations are selected, and let it be proposed to erect at a a scaffolding ae whose height is to be h. From e draw a line ec tangent to the earth's surface at c, then for distance d we have—

$$d = \frac{\sqrt{h}}{\cdot 7575}; \text{ also } h = \frac{d^2}{1.7426}$$

where h is in feet and d in statute miles. In these formulæ refraction is allowed for. Knowing the total distance between

the stations *a* and *b*, the height *bf* of the signal at *b* necessary to overcome earth's curvature for the remaining distance may be calculated from the second of the above formulæ. The actual height of the signals or instrument scaffolds at *a* and *b* will of course be made a few feet more than as found above; any intervening obstacle between *a* and *b* or rise in the ground will of course also necessitate a corresponding increase in heights of signals or instrument scaffolds.

Clearing out Line between two Stations not visible from each other.—When two stations A and B, Fig. 244, are not visible from each other on account of forest, jungle, or undergrowth between A and B, the direction of the line AB may be determined as follows:—Select two points C and D visible from each other, and from each of which both A and B are visible. Measure the two angles at each point C and D, and represent the distance CD by unity. Solve the triangle ACD for AC and BDC for BC. Then in the triangle ABC we have two sides AC and BC and included angle ACB to find the other angles. These being found, the line AB may be cut from either A or B, its direction being laid off by means of the angles DAB and CBA. The following instruments are required by the reconnaissance party:—Pocket sextant, prismatic compass, aneroid barometer, field glasses. Climbing irons for climbing trees are also required. The elevations of the proposed stations are determined by aneroid barometer (see Chapter III.), and the necessary heights of signals and scaffolds deduced as above explained.

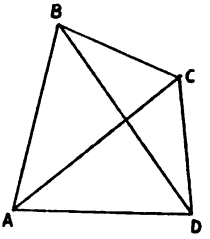

Fig. 244.
Clearing Line between Stations.

Selection of Base Lines.—The ground for measuring a base line should be as nearly level as possible, and free from ruggedness, sudden inequalities, and variations in slope. It should be favourably situated with reference to the best available stations for triangulation points in the neighbourhood. The base line and the adjacent triangulation stations are to be so situated with reference to each other that favourable conditions are secured for the "extension of the triangulation from the measured base";

and secondly, so that these triangulation stations adjacent to the base are favourably situated for carrying on the triangulation in the directions required for the survey, due attention being paid to having the triangles well conditioned.

The question has been raised whether the advantage of a *long* base line is great enough to warrant the expenditure of the time it requires to measure it, or whether as much precision is not obtainable in the end by careful triangulation from a short base. No general rule, however, applies; it must depend on the circumstances of each case.

The total number of base lines measured in Europe up to 1879 was about eighty, fifteen of which do not exceed 2,500 metres in length, or about $1\frac{1}{2}$ miles. Two, one in France and one in Bavaria, exceed 19,000 metres. The base lines in Spain, measured by Porro's method, were as follows:—Central base of Madridejos, length 14664.500 metres, probable error ± 0.17 millionth. This was the longest base. There were seven others, six of which are under 2,500 metres long. The last base, measured in Barcelona, is 2483.5381 metres according to the first measurement, and 2483.5383 metres according to the second measurement.

The following are base lines, with the length of each, measured in the primary triangulation of 500,000 sq. miles of country, executed by the United States Geological Survey. By reference to a map the location and distribution of the base lines may be examined. Wingate, New Mexico, 4.20 miles; Bozeman, Montana, 4.56 miles; Austin, Texas, 6.40 miles; Fort Smith, Arkansas, 2.84 miles; Little Rock, Arkansas, 3.72 miles; Spearville, Kansas, 7.10 miles; Albany, Texas, 9.00 miles; Sierra Blanca, Texas, 4.60 miles; Boise, Idaho, 4.75 miles; Aspen, Colorado, 1.00 mile; Laramie, Wyoming, 2.50 miles; Rapid, South Dakota, 5.00 miles. These base lines were measured with 300 ft. steel tapes, as described in this chapter.

Recent improvements in methods of measuring bases with steel tape will probably lead to more and longer bases being measured, and shorter intervals between them, than has hitherto been the rule, as may be noted in the case of the work of the United States Geological Survey given above.

Permanent Marking of Stations and Base Lines.—

One of the best ways of marking the station points permanently is to set stones about 2 ft. × 6 in. × 6 in. from 2 to 3 ft. below the surface of the ground. The point may be marked by a small hole drilled in the stone, or by a mark on a piece of copper let into the stone. Another stone similar to the first, projecting about 6 in. above the surface, should also be set, the lowermost stone being used only when the upper one is lost or disturbed. When the station is on rock the point may be marked on a piece of copper let into the rock. Three reference posts projecting about a foot above the surface should be put in within 200 or 300 ft. of the station. The distances and bearings of these reference points should be taken and recorded along with a sketch of any features of the immediate vicinity.

Instrument Stations and Scaffolds.—When the elevation of the ground at a station is sufficient to enable the other stations to be seen without raising the instrument on a scaffold, such a station is called a "ground station."

The common instrument legs will, however, seldom be found satisfactory to use for trigonometrical surveys. Extra heavy and stiff legs of the ordinary pattern have given good results. A table formed of three or four short posts framed and braced together, firmly planted over the station and having a hole in its centre for the plumb line, will answer well. On this table the instrument is set up on a brass three-armed support having a steel point under each arm, which points are pressed into the table. This support is also used in place of tripod legs when the instrument is raised on a scaffolding and in other situations. A similar support is commonly used in ordinary survey work for setting up the instrument on a wall, &c.

When it is necessary to elevate the instrument at a station a triangular framed scaffold is made to support the instrument. Round this another scaffold usually of square or rectangular shape is erected for the observer. These must be *entirely independent* so as not to disturb the instrument. A wooden tube runs from top to bottom of the instrument scaffold inside which the plumb line hangs, and is thus protected from wind. This may be made with one side open and to revolve upon a vertical axis, so that it may be turned to bring the open side on the lee side.

In the primary triangulation of the United States Survey of the Great Lakes the total number of stations at which scaffoldings were raised was 243. The average height of scaffolding was 58 ft. Of these 22 were under 10 ft. high, 68 from 10 to 50 ft. high, 118 from 50 to 100 ft. high, 35 from 100 to 124 ft. high. When used to sight on, the signals on these scaffolds were raised from 5 to 30 ft. more than the above heights.*

Many cases of very high scaffolding occurred on the Ordnance Survey of the United Kingdom. At Thaxted Church, where the tower is 80 ft. high, surmounted by a spire 90 ft. high, the scaffold for the observatory was carried up from the base to the top of the spire, the scaffold for the instrument was raised from timbers passing through the spire at 140 ft. above the ground, both scaffoldings being entirely independent.

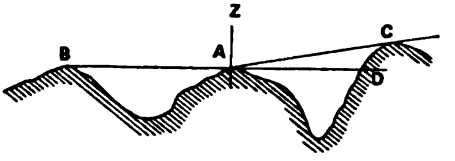

Fig. 245.—Signals.

Signals.—Experience shows that a signal to be easily seen must subtend an angle of at least half a minute. The height of a signal must therefore be at least $\frac{1}{7000}$ of the distance. The diameter at base should be about $\frac{1}{3}$ the height. If at any station A the sum of the vertical angles BAZ, CAZ is equal to or greater than 180°, then the signal at A will be seen from B projected against the sky; if the sum of these vertical angles is less, the signal must be elevated until the sum equals 180° (see Fig. 245).

Signals must be high, easily seen, and so constructed that the instrument can be placed exactly under them. Three or four timbers forming a pyramid, well framed together as shown in Fig. 246, will answer. A mast is placed in the centre and kept vertical by means of stay ropes.

Figs. 247, 248 show another good form of signal, consisting

* Johnson, Theory and Practice of Surveying.

simply of three strong timbers, forming a tripod, framed with the flagstaff by a bolt which passes through their ends and through the flagstaff. Fig. 247 shows the signal put up, the bottom being fixed by a pile of stones. Fig. 248 shows the flagstaff

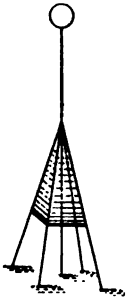

Fig. 246.

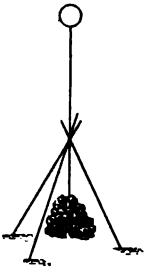

Fig. 247.

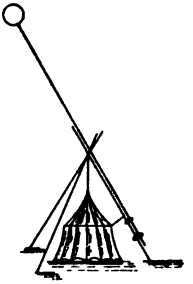

Fig. 248.

Signals.

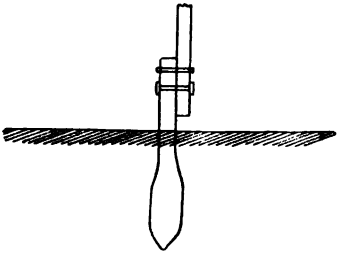

Fig. 249.

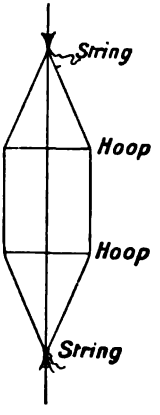

Fig. 250.

Signals.

turned to one side and the theodolite tent underneath. Signals of this description have been used of heights from 15 to 80 ft.

A very good simple form of signal is shown in Fig. 249. It consists simply of a strong post sunk into the ground. The flagstaff is attached by means of the collar and bolt shown. When the

collar is opened the flagstaff may be put down and the theodolite may then be placed exactly over the previous position of the flagstaff.

White and red flags are best seen against a ground background and red and green against the sky. Flags are little use in calm weather. A circle of sheet iron with a hole in it, which may be revolved so as to face each station in succession, is very easily seen.

Fig. 250 shows an inexpensive and good signal. It simply consists of two hoops over which is stretched a piece of muslin. The loose ends are tied to the flagstaff and the whole then forms a barrel or double cone. Such a signal would be phaseless (see page 443) and gives very good results.

Sun Signals or Heliographs and Heliostats.—The simplest form of sun signal is merely a few pieces of burnished tin fixed to the staff at various angles.

A cone of burnished tin will reflect the sun's rays in almost any direction. On the Ordnance Survey the station on Hanger Hill Tower could not be seen from Shooter's Hill, although the distance was only 10 miles, owing to the dense London smoke. It was, however, rendered clearly visible by means of tin plates attached to the signal post. By adopting the same plan the station on Leith Hill near Dorking was rendered distinctly visible from a distance of 45 miles although the hill itself was never once seen.

Heliostats and Heliographs.—For long distances special instruments are used to flash the sun's rays. A heliostat reflects a continuous beam of light, while a heliograph has a spring to the mirror by means of which it is made to flash long or short flashes like the dots and dashes of the Morse Code. A heliostat may be also used as a heliograph by covering and uncovering the mirror.

The simplest form of heliostat is a frame of coloured glass, made to rotate upon a horizontal and also on a vertical axis, so that any desired inclination may be given to it. The rays may be directed by observing the station of the observer through a hole in a circular disc of polished block tin or brass, at the same time causing the disc to be illuminated by the rays reflected from the glass.

Gauss' Heliotrope.—The most perfect heliostat is Gauss'.

It is formed by a mirror a few inches square mounted on a telescope near the eyepiece. This mirror is turned so as to reflect the sun's rays through two perforated discs, one near the mirror, the other near the object end of the telescope. The line joining the centres of the openings in these discs is parallel to the axis of the telescope, and the light is consequently reflected in the required direction if the telescope has been previously directed to the station of the observer. A "heliotroper" remains at the instrument, and he keeps the mirror turned so as to reflect the light through the two discs. Gauss' heliotropes have been seen from a distance of 80 or 90 miles when the outlines of the places on which they were situated could not be seen.

Good heliographs are Galton's and Mance's. To the observer the heliostat presents the appearance of a star of the first or second magnitude, and is generally a pleasant object for observing.

Wharton's Improvised Heliostat.—Heliographs and heliostats are somewhat expensive instruments. The following description of a heliostat made by an ordinary ship's blacksmith is given in Wharton's "Hydrography." It is shown in Fig. 251.

"The arm *n*, of light iron, is carried separately, and slips over the shaft of the standard, clamping when required with a screw *g*. Into a circular socket in head of standard shaft the leg of the frame holding the mirror is slipped; this is also to be tightened by a retaining screw *f*. The mirror *e*, which can be of any size from 2 to 6 in. or more in diameter, revolves on its retaining screws as an ordinary toilet table glass, and can be held in any position by the screws. The ring *c*, of flat wood, is made as light as possible, so as to exert less strain in wind. Across it are nailed crossed strips of copper with a white cardboard disc, about an inch in diameter, fastened to their centre. The rod that carries this ring slips up and down in a hole at the end of the arm, and is clamped by a retaining screw. In the centre of the back of the mirror a hole of about $\frac{3}{4}$ in. diameter is scraped in the tinfoil, being careful to leave a sharp edge. A similar hole is cut out of the wooden back of the glass frame. This we shall call the blind spot. To direct the flash to an object, bring the mirror vertical, and looking through the hole in the centre, revolve the arm until in the direction of the

object nearly, clamp it, and adjust the disc rod as nearly as may be for elevation or depression. Then slightly loosening the screw, clamping the arm, finally adjust the latter so that the object, as regarded through the hole in the mirror, is obscured by the white cardboard disc in the centre of the ring. By turning the mirror so that the dark shade caused by the blind spot is thrown on to the disc, the flash will be truly directed, and must be kept so by slight alterations of the position of the mirror, which should therefore be clamped only sufficiently to hold it steady and yet admit of gentle movement. The shadow of the blind spot should be slightly smaller than the disc, so as to ensure having it truly in the centre of the latter.

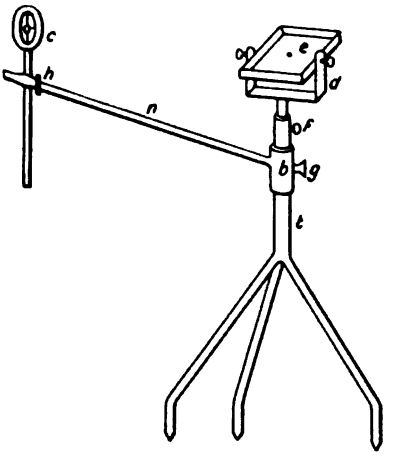

Fig. 251.
Wharton's Improvised Heliostat.

"The mirror must be of the best glass, with its faces parallel, or the shadow of the blind spot will be very indistinct when the mirror is at a large angle, and also the beam of light will be dispersed before it has traversed many miles. It is well to have the mirror a fair size, say 6 in. square, as in practice it will be found generally necessary, in order to save time, after once adjusting the flash, to leave a man to keep it on while the surveyor is taking his angles; and although a man will soon pick up the knack, a larger mirror will allow for eccentricities on his part, and also, on a dull day, a faint flash will be detected from a large mirror where a small one would not carry any distance. On a bright day a flash from a 3 by 2 in. mirror has been seen 55 miles and more. In hazy weather angles have been got when the place from which the flash was sent was entirely invisible; and thus whole days have been saved by this simple contrivance. Only those who have spent hours, or even days, in straining their eyes to see a distant mark can appreciate the value of a heliostat."

Night Signals.—Bengal lights were first used by General Roy on the Ordnance Survey. These were succeeded by Argand lamps and parabolic reflectors. These again were succeeded by a large plano-convex lens, prepared by Messrs Fresnel & Arago, and used by them in co-operation with Colonel Colby and Captain Kater. By means of this a station 48 miles distant was observed. The most powerful night signal is, however, Drummond's light. This was invented by Lieutenant Drummond, and consists of a ball of lime about $\frac{1}{4}$ in. diameter placed in the focus of a parabolic reflector and raised to an intense heat by a stream of oxygen gas directed through a flame of alcohol. This produces a light eighty times the intensity of an Argand burner. In boisterous and hazy weather this light was brilliantly visible at 67 miles distance, and would evidently have been so at a much greater distance.

Phase of a Signal.—The "phase" of a signal is the effect of the sun shining on one side of the signal so that the telescope is directed to one side of it instead of to its centre. This source of error must be avoided. The error may be calculated as follows:—

If the signal is a tin cone for example, let R = its radius, θ = angle at station of observation between the sun and the signal, d = distance of signal from station.

$$\text{Correction to observed angle} = \pm \frac{R \cos^2 \frac{1}{2} \theta}{d \sin 1''}$$

Measurement of Base: Measurement with Steel Tape.—Any error in measurement of the base is *multiplied* throughout the whole of the work. Extreme care must therefore be taken in measuring the base. For most practical purposes in trigonometrical surveys of moderate extent a careful measurement with a steel band will suffice, expensive and complicated base-measuring apparatus not being as a rule at the surveyor's disposal. Spring attachments should be provided to the steel band so as to regulate the tension on it, and the length of the band for a given tension should be ascertained. Observations for temperature should be taken as each length is measured out. On the Ordnance Survey, in some cases, five thermometers were placed alongside each chain, and the mean of the readings taken.

A 66 ft. steel band having an effective section of $\frac{1}{2} \times \frac{1}{10}$ in. under a stress of 30 lbs. will stretch 0.667 link per mile.

Each chain should be measured on the actual slope of the ground, and the total distance reduced to its horizontal equivalent by levelling a careful section of the whole base.

The ground selected for the base should be as smooth and nearly level as possible, and the extremities of the base must be visible from the principal stations in the neighbourhood which are to be used as stations of the primary triangulation. Each extremity of the base should be marked by a stone sunk into the ground with an iron plate let into the stone on which a fine point mark may be made. When possible two or three measurements of the base should be made and the mean value adopted.

In the measurement of a base an elaborate apparatus such as the United States Coast and Geodetic primary base-measuring apparatus, described on page 430, or Colby's compensating bars, described on page 427, is used only when very great accuracy, as 1 in 1,000,000, is desired, and when the base is for a very large primary triangulation.

For the measurement of bases for trigonometrical surveys of moderate extent *the steel tape* is much the most convenient, quickest, and cheapest means to adopt. By it an accuracy of 1 in 300,000 may be attained, and under the most favourable circumstances even 1 in 1,000,000. In the future, as improvements in methods of measuring with the steel tape are introduced, it will probably be adopted even for the measurement of bases of large primary triangulations.

The tape used may be from 300 to 500 ft. long, a usual length is 300 ft., and the area of cross section of the tape from $\frac{2}{1000}$ to $\frac{3}{1000}$ sq. in.; a tape $\frac{1}{10}$ in. wide and $\frac{2}{100}$ in. thick is often used. An ordinary measurement with a steel tape, in all weathers, simply held and stretched by hand in as nearly a horizontal position as possible by estimation, and with the pull on the tape by the chainmen kept as nearly constant as can be estimated, the end of the tape being marked by a plumb bob on uneven or sloping ground, and an occasional reading of the thermometer, will give an accuracy of 1 in 5,000 if due care be taken.

If an accuracy of 1 in 50,000 is desired the slope of each tape length must be determined by levelling, the tape being stretched over pegs or other points accurately aligned. The pull on the tape should be regulated with spring balances, and the temperature of

each tape length determined to the nearest degree Fahrenheit, the work being done in cloudy weather.

To attain an accuracy of 1 in 500,000, great precautions must be taken. The slope of the tape must be found by stretching it over points whose levels are accurately determined, and if supported at intermediate points the levels of these must also be known, the pull on the tape must be accurately known, and all friction must be avoided. The mean temperature must be accurately known. The work must be done in *cloudy days* only. It is impossible to find by mercurial thermometer the mean temperature of a tape exposed to the sun. In windy weather the work must be stopped owing to the variable tension on the tape and the uncertain variations of temperature due to wind.

The following method is that used by Mr O. B. Wheeler, U.S. Assistant Engineer, on the Missouri River Survey:*—

Pegs are driven with their sides on the line, from 20 to 100 ft. apart. From nails in the sides of these pegs are suspended hooks in which the tape rests, the object of this being to eliminate friction. These nails may be set on a uniform gradient, but if they are not, the level of each point of support must be determined. There should not be a low point of support between two higher ones, otherwise the pull on the tape may raise it from the low support. "Marking pegs" are driven on the line with their tops about 2 ft. above the surface of the ground, at distances apart equal to the length of the tape. On the tops of these pegs are nailed zinc strips $1\frac{1}{2}$ in. wide on which the extremities of the tape are marked with a fine point. In front of the forward "marking peg" three "table pegs" are set to support the straining apparatus. These are 2 or 3 ft. from the "marking peg," and just low enough to let the tape rest on the zinc strip on the top of the marking peg when it is stretched. Behind rear marking peg is set a "straining peg" to which the rear end of the tape is fixed.

Straining Apparatus.—The straining apparatus is shown in Fig. 252. A chain is fastened to the end of the tape, and this is hooked over the point C of the block ABC. There is a knife

* Report of the Missouri River Commission.

edge at B round which the block rotates, and the weight W is applied at A. By means of the screw E working in the nut D the bearing at B is adjusted so that the bubble of the level L is at the centre of its run. The arms AB and BC of the block are so proportioned that when the surface of the block is horizontal as indicated by the level L, then the pull on the tape is equal to the weight W. In order that this may be so we have $BC = AB + \frac{w}{W}d$; where w = weight of block ABC, W = weight applied at A, d = horizontal distance from the knife edge at B to a vertical through the

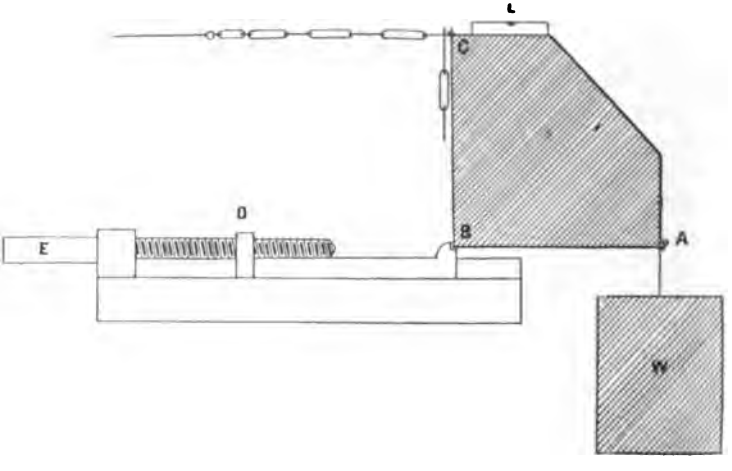

Fig. 252.—Straining Apparatus for Measurement of Base Lines.

centre of gravity of the block ABC. The centre of gravity of the block is at the intersection of two verticals through two different points of suspension, and may readily be found by suspending the block from two different points of its cross section ABC.

The rear end of the tape is hooked on to a slide worked by an adjusting screw similar to that in Fig. 252. This slide is fixed on the "straining peg," and the rear graduation of the tape is made to coincide with the mark on the zinc strip on the rear "marking peg." When this is effected and the block ABC, Fig. 252, adjusted, the forward end graduation of the tape is marked on the forward

"marking peg." The thermometers are then observed and the tape is taken forward and the operation repeated, and so on. A check measurement is made by going over the line again in the *same* direction, the marking pegs and zinc strips being left for this purpose.

Three thermometers or more should be used with a 300 ft. tape, and they should be equally distributed along the tape, *i.e.*, if three thermometers are used they should be situated at 50 ft., 150 ft., and 250 ft. on the tape. Each thermometer reading is to be corrected, the correction for each thermometer being found by comparing it with a standardised thermometer.

Absolute Length of Tape.—The absolute length of the tape or the temperature at which the length of the tape is correct must be determined. It may be found by comparison with another tape whose length is known, or by comparison with a correct standard of length if one is available. It may also be found by measuring a base line whose length is accurately known, and from the measurement computing the temperature at which the length of the tape is correct. The absolute length of the tape is usually to be had from the Government Survey Department on payment of a small fee.

The Coefficient of Expansion for steel tapes varies from .0000055 to .0000070 per degree F. If the coefficient of expansion cannot be determined, .0000065 may be used as an average value. When the tape is used at nearly its correct temperature an exact value of the coefficient of expansion is not required. For the most accurate work, however, the coefficient of expansion must be carefully found. It may be found by stretching the tape between two well-fixed points, such as stones let into the ground with a fine mark on each, and noting the difference in measurement at different temperatures. Care must be taken that the pull is the same for each measurement, and that all the other conditions, such as points of support of the tape, &c., are the same for each measurement.

The Modulus of Elasticity is to be found by putting different weights or pulls on the tape and noting the expansion.

Let F_1 = maximum load in pounds.
F_o = minimum load in pounds.
b = observed increased length of tape when pull is increased from F_o to F_1.
L = length of tape in inches, *i.e.*, 12 × 300 ft. or 500 ft. as case may be.
A = area of cross section of tape in square inches.*
E = modulus of elasticity in pounds.
y = distance apart of supports in inches.
w = weight of tape in pounds per inch of its length.

The effect of sag in the distance between supports, *i.e.*, the difference between the length of the curve assumed by the tape and the linear distance between supports, is $\frac{y}{24}\left(\frac{wy}{F}\right)^2$; if there are n intervals between supports there are n sags, then $ny = L$, and the effect of sag for the whole tape length is $\frac{L}{24}\left(\frac{wy}{F}\right)^2$, for any given pull F.

In these formulæ the curve assumed by the tape is considered to be a parabola, which it may be without appreciable error. The curve assumed by the tape is a catenary.

If f_1 and f_o are the effects of sag for the pulls F_1 and F_o, then the total stretch of the tape $b = c + (f_o - f_1)$ where c = increased length of tape due to pull only, whence $c = b - f_o + f_1$, and we get—

$$E = \frac{(F_1 - F_o)\,L}{A\,(b - f_o + f_1)} = \frac{F_1 - F_o}{A\left\{\frac{b}{L} - \frac{(wy)^2}{24}\left(\frac{F_1^2 - F_o^2}{F_1^2\,F_o^2}\right)\right\}}$$

If the same weight and conditions as to supports are used as were used in finding the absolute length of the tape, the stretch of the tape due to pull need not be computed, and the modulus of elasticity E is not required. E may also be dispensed with if the elongation per pound of pull is noted by stretching the tape with different weights and noting the variations in length, due allowance being made for different sags under different pulls,

* The cross section A of the tape may be found by weighing the tape and calculating its volume, taking 1 lb. = 3.6 cub. in.; the cross section of the tape is then the volume divided by the length of the tape.

as above. If E is not found by experiment, it may be taken as 28,000,000 lbs.

Correction for Sag.—The correction for sag is—

$$\text{Sag} = -\frac{L}{24}\left(\frac{wy}{F}\right)^2$$

where F is the pull used.

If the absolute length of the tape is given for a pull F_o and distance apart of supports y_o, and in taking the measurements the pull F and distance apart of supports y is employed, then the correction for sag is—

$$\text{Sag} = \frac{Lw^2}{24}\left(\frac{y_o^2}{F_o^2} - \frac{y^2}{F^2}\right) = \frac{L}{24}\left(\frac{w}{F_o F}\right)^2 (y_o^2 F^2 - y^2 F_o^2)$$

Correction for Pull.—The correction for pull is—

$$\text{Pull} = +\frac{(F - F_o)\,L}{AE}$$

When the absolute length of the tape is given for no pull, *i.e.*, corrected for pull, we have—

$$\text{Pull} = +\frac{FL}{AE}$$

where F is pull used in field, F_o pull under which absolute length of tape is determined, in pounds.

To Eliminate Corrections for Sag and Pull.—The correction for sag being negative and the correction for pull positive, these corrections may be eliminated by making the pull such that the extension from pull is equal to the shortening from sag. The pull to effect this is given by—

$$\text{Pull in pounds} = \sqrt[3]{\frac{W^2 L}{24 s m^2}}$$

where W = weight of whole tape in pounds.
L = total length of tape in inches.
s = stretch of tape for a pull of 1 lb.
m = number of intervals between supports.

If the tape is not supported at intermediate points, $m = 1$, and

$$\text{Pull in pounds} = \sqrt[3]{\frac{W^2 L}{24 s}}$$

If the length of tape is correct at a temperature T and under a pull F_o, distance between supports y_o (*i.e.*, if the absolute length of the tape has been determined under a pull F_o and distance apart of supports y_o, and the tape found to be of correct length at a temperature T), then if the pull F be used, the tape will be of correct length at a temperature $T + c$ under this pull, where—

$$c = \frac{1}{\beta}\left\{\frac{F_o}{AE} - \frac{1}{24}\left(\frac{wy_o}{F_o}\right)^2\right\}$$

where β = coefficient of expansion for 1° F.

When, therefore, the effects of sag and pull are eliminated by using a pull F, the measurements must be reduced to the temperature $T + c$, that being the temperature at which the tape is of correct length when the pull is such that the effects of pull and sag are eliminated.

When a steel tape is sent to a Government Survey Department to have its absolute length determined, the information usually supplied is the temperature at which the length of the tape is correct, there being *no pull* and *no sag*. In this case, therefore, what is necessary is simply to correct for sag and pull, or use the pull which eliminates these corrections, and then reduce the measurements for temperature.

Correction for Temperature.—When mercurial thermometers are used each reading must have its correction applied, these corrections being found by comparison of the thermometers used with a standard thermometer. The average of the corrected readings may be taken for all the entire tape lengths measured and the correction for that part of the line applied at once. The correction for any fractional part of a tape length is to be calculated separately.

Let D = length of part of line composed of entire tape lengths.
T = temperature at which length of tape is correct.
θ = mean corrected temperature of all entire tape lengths.
β = coefficient of expansion for 1° F.

Then correction for temperature = $+\beta(\theta - T)$ D.

The correction for the residual part of the line is—

$$+\beta(t_m - T)l$$

where β = coefficient of expansion for 1° F., t_m = mean corrected

temperature of residual part of line, T = temperature at which length of tape is correct, l = length of residual part of line.

Corrections for Fractional Part of Tape Length.—To correct any fractional part of a tape length for sag and pull, the following formulæ may be used. For sag—

$$\frac{y}{24}\left(\frac{wy}{F}\right)^2$$

where y = distance between supports in inches, w = weight of 1 in. in length of tape in pounds, F = pull used in pounds. The above formula applies only to any part of a tape between two supports, distant y apart. If there are m intervals between supports, all distant y apart, the total correction is m times above. For a different value of y the correction is to be computed separately, using proper value of y. For pull—

$$+\frac{(F - F_o)l}{AE}$$

where F = pull used in field in pounds, F_o = pull under which absolute length of tape is known in pounds, l = length of fractional part of tape in inches, A = area of cross section of tape in square inches, E = modulus of elasticity in pounds.

If the absolute length of the tape is given for no pull, *i.e.*, corrected for pull, we have for pull—

$$+\frac{Fl}{AE}$$

The formulæ already given for pull necessary to eliminate corrections for sag and pull apply to any fractional part of a tape length, L being taken equal to the *fractional part* of tape length, W = weight of *fractional part* of tape length, s = elongation of *fractional part* of tape for a pull of 1 lb.

Measurement of Bases with Steel and Brass Wires.—Professor E. Jaderin, of Stockholm, has used steel and brass wires with much success. The wires are 25 metres in length, and are stretched over tripods set in line, a very strong spring balance being used at one end and a very accurate one at the other end. The tension is measured on the forward spring balance

only. On the top of each tripod there is a fixed mark, and at the rear end of the wire a single mark. At the forward end there is fixed to the wire a scale about 10 centimetres long, divided to millimetres. When only one steel wire or steel tape was employed it was found that the work had to be carried out in calm and cloudy weather or at night, to get the best results; if two wires were employed, one of steel and the other of brass, work could be done during the whole day, even in sun and wind, and an accuracy of 1 in 1,000,000 obtained.

When the coefficients of expansion and lengths at a certain temperature of the two wires have been determined, the temperatures of the wires themselves may be deduced from the difference of the measurement of the same distance by the two wires. Mr Jaderin obtained a probable error in a single measurement of 1 in 600,000, the probable error of the mean being 1 in 1,000,000 as compared with the length of the line as measured with the regular primary base-measuring apparatus, in three measurements of a line 2 kilometres long, using steel and brass wires.

Accuracy of Steel Tape Base Measurements.—Mr O. B. Wheeler, U.S. Assistant Engineer, using the method described on page 445, measured the following lines on the Trigonometrical Survey of the Missouri River.* One steel tape 300 ft. long was employed. Base I.—First measurement, 7923.237 ft.; second measurement, 7923 403 ft. Probable error of a single measurement, 1 in 100,000; probable error of mean, 1 in 140,000. Base II.—First measurement, 9870.443 ft.; second measurement, 9870.388 ft. Probable error of a single measurement, 1 in 380,000; probable error of mean, 1 in 533,000. Base III.—First measurement, 9711.915 ft.; second measurement, 9711.892 ft. Probable error of a single measurement, 1 in 900,000; probable error of mean, 1 in 1,250,000. Base IV.—First measurement, 10821.9658 ft.; second measurement, 10821.9065; mean = 10821.9662 ± .0002 ft.

Professor J. B. Johnson has measured with a steel tape a number of bases about ½ mile long, and in each case the pro-

* Report of the Missouri River Commission.

bable error of the mean of three or four measurements was less than 1 in 1,000,000. The work was confined to densely cloudy days.*

Professor R. S. Woodward measured a base line on the United States Coast and Geodetic Survey, 3,807 metres long, in four parts with two steel tapes, two measurements with each at night and one measurement in sunshine in the day being made. The probable error of the result was 1 in 1,280,000.†

Measurement with Rods.—Rods of iron, brass, and other metals may be used for measuring a base, but are greatly affected by changes of temperature. Wood, on the other hand, is affected by damp. Glass rods or tubes are less affected by either moisture or changes of temperature. When rods are used wood is, however, the most easily procured. They should be of straight-grained wood, well seasoned, dried, saturated in boiling oil, painted and varnished. They may be trussed or framed to prevent deflection; 10 or 15 ft. is a suitable length. Three rods are required, which may be painted different colours to obviate mistakes in reading. The length of the rods is to be carefully adjusted with a standard measure of length. Supports consisting of posts driven in on the line at a distance apart equal to the length of the rods must be used. They may be driven or sawn off to the correct level.

Portable tripods or trestles with adjusting screws for raising or lowering may also be used. Legs attached to the rods and capable of sliding up or down upon themselves may be adopted. These legs are then simply driven in on the line. The rods may be levelled with an ordinary mason's spirit level. The rods are brought into contact by bringing them end to end, and the third rod should be placed before the first is removed. The ends of the rods must be spherical and of a radius equal to the length of the rods and be covered by metal caps. Round-headed nails even may be used to make contact. When one rod is higher than another a plumb line must be used to make contact, and its thickness allowed for. To prevent disturbance from contact a graduated wedge may be used. This is inserted between the rods, and the reading of the graduations will give the thickness to be

* Johnson, Theory and Practice of Surveying.

† Transactions American Society of Civil Engineers, vol. xxx.

added for the space between the rods. The rods may be laid alongside each other, and lines drawn across each brought into coincidence. Double microscopes, whose cross hairs bisect marks at the end of each rod, similar to those described in Colonel Colby's base-measuring apparatus, may also be used.

Expansion per degree F.—Brass bar, 0.00001050903; iron bar, 0.00000696353 5; platinum, 0.0000051344; glass, 0.0000043119; white pine, 0.0000022685.

Broken Base.—In Fig. 253 let the lines measured be AC and BC owing to some obstacle opposite C. Let the angle BCD $= \beta$ expressed in minutes. Let BC $= a$, AC $= b$, and AB $= c$.

Then—

$$AB = a + b - 0.000000042308 \times \frac{ab\beta^2}{a+b}$$

$$\text{Log } 0.000000042308 = 2.6264222 - 10$$

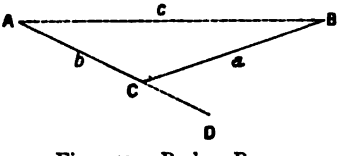

Fig. 253.—Broken Base.

If β is greater than from 3° to 5° AB must be calculated by ordinary sine formula.

Reduction of Base to Horizontal.—When the base is measured on the slope or on different slopes the actual measured length is to be reduced to its horizontal projection. The horizontal projection is equal to the square root of the difference of the squares of the measured length and the difference of level, or to the measured length × cosine of the angle which it makes with the horizontal. As this angle is usually small, when it is not more than 2° or 3° the difference between the measured length and its horizontal projection is equal to the measured length × the square of the angle expressed in minutes × 0.000000042308.

Reduction of Base to Mean Sea Level.—In Fig. 254 let AB $= d$ be the measured base.

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$A'B' = d'$ the base reduced to mean sea level.
h = height of measured base above mean sea level.
r = radius of earth at mean sea level.

If r is taken as 20,890,592 ft., then log r is 7.3199507
Then—

$$\frac{d'}{d} = \frac{r}{r+h}$$

$$\text{and } d' = d\frac{r}{r+h}$$

$$d - d' = d\frac{h}{r} - d\frac{h^2}{r^2} + d\frac{h^3}{r^3} - \&c.$$

As h is very small compared with r it is usually near enough to use the first term only, then $d - d' = d\frac{h}{r}$

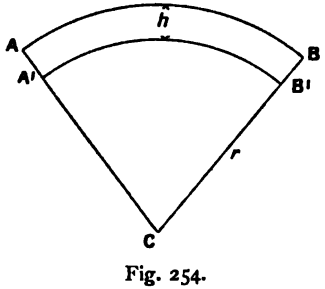

Fig. 254.
Reduction of Base to Mean Sea Level.

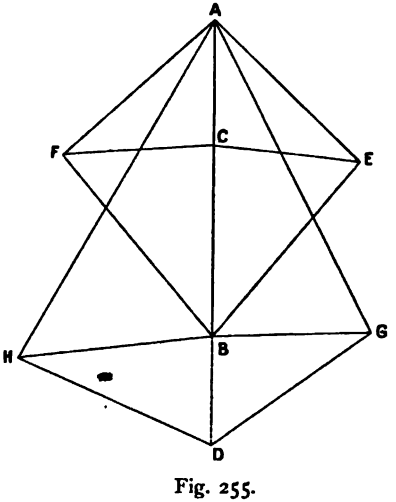

Fig. 255.
Extending a Base.

In reducing the base to mean sea level, if the surface slopes uniformly, h may be taken as the height of the lower extremity of the base. If the surface is broken and irregular the base may be divided into several parts and the reduction to mean sea level made for each separately, different values of h being used; otherwise the mean height of the base above sea level may be used.

Extending a Base.—When one extremity of a base, which is otherwise favourably situated, is in an inconvenient situation

for observing the proposed neighbouring main stations, it may be extended as shown in Fig. 255.

Let AB be the measured base, and let it be desired to extend it to the point D.

Let C be a point marked during the measurement of AB, and let EF be prominent stations visible from C selected so that the angles at C are nearly right angles and the triangles ACE, ACF as well conditioned as possible. G and H are stations selected similarly with reference to B. At A the angles to E, F, G, and H are observed, and the angles at C and B to those stations, as well as the angles at E, F, G, and H. From AC and the observed angles CE and CF are calculated. From each of these sides CE and CF and the angles the distance CB may be calculated and the mean of the results accepted. This will check the agreement of the measurements of the portions AC and CB of the measured base AB. From AB and the observed angles, BG and BH may now be calculated, and from each of these again BD may be calculated and the mean of the results adopted.

The first portion of the above may of course be omitted and the distance BD calculated from AB and the observed angles at A, B, G, H, and D.

Extension of the Triangulation from the Measured Base.—The *fewer* the number of primary triangles the better. As the measured base will as a rule be only about 5 miles long, while the sides of the primary triangles will be from 20 to 30 miles in length, the triangulation must be extended from the measured base in a systematic manner, until the sides of the triangles reach about the estimated length of the sides of the largest triangles in the survey.

Thus in Fig. 256 let AB be the measured base, C and D the nearest stations selected so that ABC and ABD are nearly equilateral. All the angles of the triangles ABC, ABD are observed, and the sides AC, BC, AD, BD calculated from these and the measured base AB. Then in the triangle ADC we have the two sides AD, AC, and the included angle from which to calculate CD. Similarly CD may be calculated from BC, BD, and the included angle, and the mean of the results may be taken as the length of CD.

The angles of the triangles CDE and CDF are then observed and EF calculated in exactly the same manner. Then the process

is repeated, and GH found. If GH is of the required length, the primary triangulation may now be proceeded with, and the main triangles laid out from GH as GKH, &c. If not, the process is continued until the required length is attained.

Instruments.—As already stated, on the Ordnance Survey of the United Kingdom a 3 ft. theodolite was used to measure the angles of the primary triangles, a 12 in. theodolite being used for the secondary triangles, and 7 in. and 5 in. instruments for the tertiary triangles; a 2 ft. 6 in. theodolite reading to single seconds

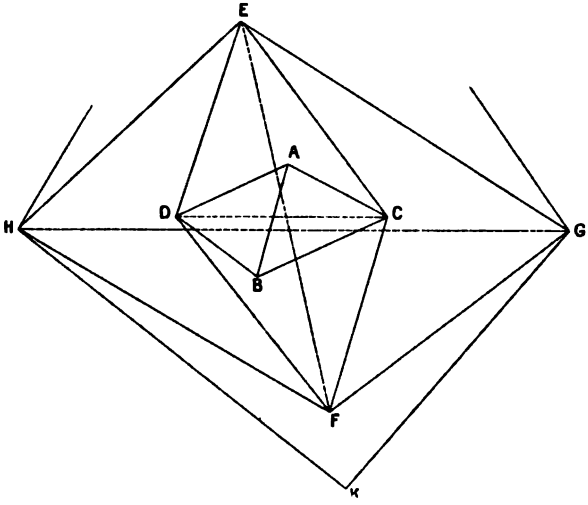

Fig. 256.—Extension of Triangulation from Measured Base.

was used on the United States Coast and Geodetic Survey. It was previously supposed that the larger the diameter the greater the accuracy which could be attained, but it is now found that there is no gain in having the diameter more than 12 in.

Fig. 257 shows Messrs Troughton & Simms' 12 in. theodolite. Each circle is read by two micrometer microscopes to 1 second of arc. Pointer microscopes are furnished for reading the figures. The focal length of the telescope is about 19 in., aperture of object glass $2\frac{1}{10}$ in. These instruments are furnished with three astronomical eyepieces, one diagonal eyepiece, and an eyepiece

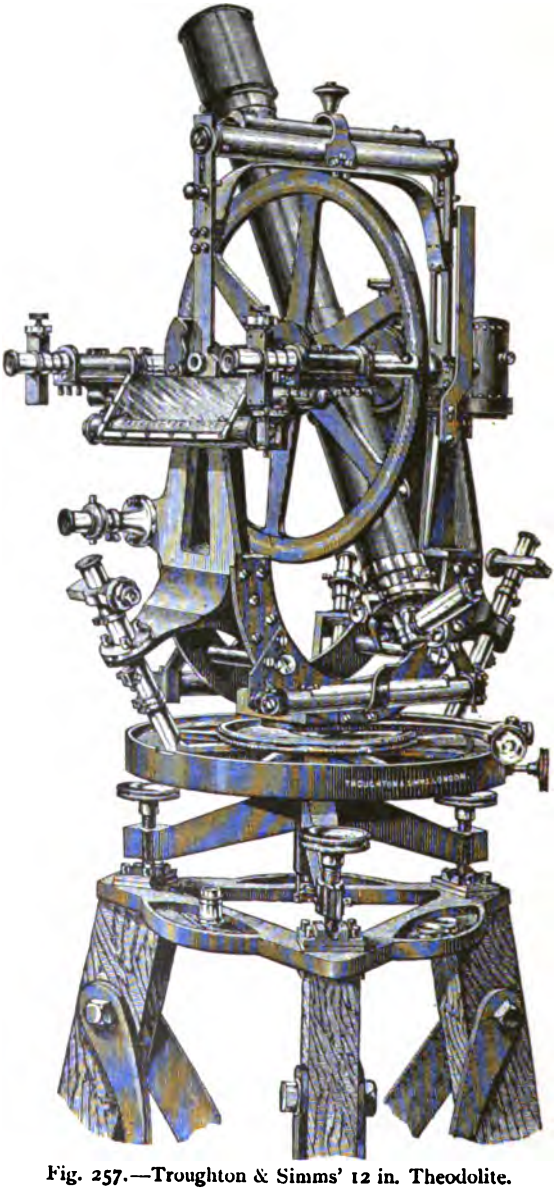

Fig. 257.—Troughton & Simms' 12 in. Theodolite.

micrometer. In the figure there is also shown a striding level for levelling the horizontal axis of the telescope, and on the right hand of the axis a lamp for illuminating the cross hairs.

With a 12 in. theodolite, one or two 7 in. instruments, and several 5 in., a fairly accurate and extensive trigonometrical survey can be made.

Observation of the Angles: Method of Repetition.—On the Ordnance Survey the angles were observed a great many times, and the mean of the observations taken. This is the best practical method. There is another method which consists in "repeating" the observation say 10 or 20 times, the total as read by the vernier being divided by 10 or 20 as the case may be. In this method the angle is only read once on the verniers at the end of the last observation, and it is supposed to eliminate all errors of centring and dividing. It is much used by French engineers and on the Continent Borda's repeating circle was used for the primary and secondary triangles. In theory this method is apparently perfect, but from defects of materials and workmanship, and the insufficiency of the tangent and clamping screws to produce absolute stability, it is found in practice to *repeat the error* along with the angle measured, so that a small repeating theodolite is not equal to a larger and heavier ordinary theodolite.

Method of observing Horizontal Angles adopted on Ordnance Survey of United Kingdom.—Let A, B, C be stations to be observed in order of azimuth. The telescope is first directed to A, and the cross hairs are made to bisect the object presented by A, then the microscopes or verniers of the horizontal circle are read and recorded, as are also those of the vertical circle if necessary. The telescope is then turned to B, which is observed in the same manner; then C and the other stations if any. Coming round by continuous motion to A, it is again observed, and the agreement of the second reading on A with the first is some test of the stability of the instrument.* In taking this round of angles, or "arc" as it is called, the interval of time between the first and second observations of A should be

* This is called "closing the horizon."

as small as possible consistent with due care in taking observations. Before taking the next "arc" the horizontal circle is moved through 20° or 30°. Thus a different set of divisions of the horizontal circle is used for each "arc," which tends to eliminate the errors of graduation of the horizontal circle. Each "arc" at a station should have a "referring point" to which all the angular measurements are referred; the observations of each "arc" commencing and finishing with this point. Usually from among the points which have to be observed that which affords the best object for precise observation is selected as the "referring point." For mountain tops a "referring object" is constructed of two rectangular plates of metal in the same vertical plane, their edges parallel and placed at such a distance apart that the light of the sky seen through appears as a vertical line about 10 seconds in width. The best distance for this object is from 1 to 2 miles.

General Method of observing Angles.—When the observer has "closed the horizon," if the first and last readings of the horizontal circle do not agree, the error is divided among the angles in proportion to their number, irrespective of the size of the angles. It is very questionable whether this correction adds to the accuracy of the result, and therefore it is a common practice to read the angles without closing the horizon.

When the station is high there is usually a twisting of its upper part, in clear weather, in the direction of the sun's motion. This has been observed to amount to 1 second in a minute of time on a station 75 ft. high. To eliminate the error due to this the angles are read both to left and to right—that is, the telescope is first directed to the extreme left hand object, and the angles are measured round from left to right. Then the telescope is directed to the extreme right hand object, and the angles are read round from right to left.

To eliminate errors of adjustment of collimation line and levelling of horizontal axis both "faces" of the instrument are used (see page 83, Chapter II.).

To eliminate errors due to eccentricity opposite verniers or micrometers are read.

To eliminate errors of graduation of horizontal circle each angle is read on different parts of the horizontal circle. To

effect this the limb is moved after each set of readings an angle equal to $\frac{180^\circ}{n}$, where n is the number of sets of observations to be taken.*

For fully detailed instructions for observing angles in accordance with the best modern practice see Instructions for Secondary Triangulation, page 471.

If circumstances should arise in which it is necessary to do triangulation work with an ordinary theodolite reading to 30 seconds only, the method of repetition may do well *if there is no movement due to the clamping.* The repetitions should be first to the right and then to the left.

All the angles of every triangle are to be observed, but in any case where this is not possible the unobserved angle should be as nearly 90° as possible.

Time for observing Angles.—In clear weather it is not possible to get even fair results during the greater portion of the day. Up till about 4 P.M. in summer the air is so tremulous from heat that the accurate direction of the cross hairs on a distant object is very difficult. From about 4 P.M. till dusk, in clear weather in summer, observations may be made. In cloudy weather, with clear air, good observations can be made all day. Work with heliostats can be done in clear weather only. A line of sight high above the ground is not so much affected by atmospheric conditions as a low line.†

Night observations are slightly more accurate than day observations, while the cost of night signals is less than that of good heliostats. Night signals can usually be attended to by the

* When the instrument used is not a transit and the telescope cannot be rotated on its horizontal axis through 180° nor lifted out of its bearings and turned end for end, then *both faces* cannot be used. In this case the limb should be moved by $\frac{360^\circ}{n}$ after each set of readings; this will eliminate *most* of the error due to collimation line being out of adjustment and also of horizontal axis.

† If the country in which the observations are taken is flat, owing to the stratum of air near the surface being disturbed by exhalations and inequalities of temperature and density, accurate observations are not possible unless the instrument is raised above the surface of the ground. See portable scaffold of Ordnance Survey, page 431.

same men as attend the heliostats, while the available time of observing, in clear weather, is more than doubled by working at night.

Accuracy of Measurements of Angles.—The probable error of a single measurement with Ramsden's 3 ft. theodolite used on the Ordnance Survey of the United Kingdom was 0.20 second. On the great Trigonometrical Survey of India theodolites 3 ft. and 2 ft. diameter, read by five equidistant microscopes, were used. Every angle was measured twice in each position of the zero of the horizontal circle, of which positions there were generally ten; the total number of measurements of an angle was never less than twenty. An examination of 1,407 angles showed that the probable error of an observed angle was on the average ± 0.28 second. In the observations of six triangles by Professor Bache on the United States Coast and Geodetic Survey the maximum error was 0.60 second.

Reduction of Angles to the Centre of Station.—In many cases it will be impossible to set the instrument precisely over the stations when church spires, &c., are selected as stations. When this is so, the angle is to be reduced to the centre of the station as follows:—

In Fig. 258 let C be the centre of the station, B the right hand object, A the left hand object, and BCA the required angle. Set up the instrument at D as near to C as possible, and measure the angle BDA. The angle BDA will be less than, equal to, or greater than the angle BCA according as D lies without the circle passing through B, C, A; upon its circumference; or within it.

Observe the angle ADC called the "angle of direction," between the left hand object and the centre of the station; also measure DC.

Calculate the distances CB and CA with the angle BDA; as BDA will not differ much from BCA, this will give these distances near enough to reduce the angle to the centre of the station.

Then—

$$\text{BCA} = \text{BDA} + \frac{\text{CD} \sin (\text{BDA} + \text{ADC})}{\text{CB} \sin 1''} - \frac{\text{CD} \sin \text{ADC}}{\text{CA} \sin 1''}$$

In other words, the last two terms of the above will be the correc-

tion *in seconds* to be added or subtracted. The algebraic signs of the sines must be attended to. Log sin 1″ = 4.6855749.

The point D should be selected so that all the signals such as B and A can be seen from it. It will then only be necessary to measure the one angle of direction ADC and the one distance DC. The angle between A, C, and each of the other signals is then calculated as above, and the angles between the several stations obtained by subtraction. Thus in Fig. 258 let E be another signal, the angles ACE and ACB are calculated, then the angle ECB = ACB − ACE.

If the centre of the tower C cannot be seen or measured to as in Fig. 259, set the theodolite at B and turn the telescope so that its line of sight is tangent to the tower at K and L. Measure on

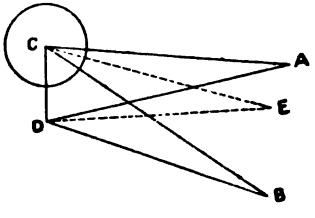

Fig. 258.
Reduction of Angles to Centre of Station.

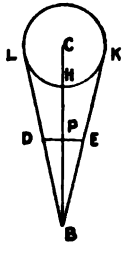

Fig. 259.

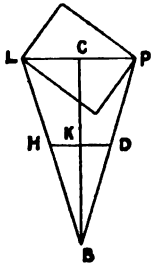

Fig. 260.
Sighting to Centre of Tower.

these tangents equal distances BE, BD, and direct the telescope to P, the centre of the line DE. The telescope will then be directed to the centre of the tower.

The distance BC = BH + radius of tower. The radius of tower may be deduced by measuring its circumference.

If the tower or building is rectangular, as in Fig. 260, measure BL, BP. Take any point H on BL, and make $BD = BH \frac{BP}{BL}$. Then DH will be parallel to the diagonal of the tower PL and a line joining B and K, the centre of DH, will bisect the diagonal PL at the centre of the tower C.

The distance $BC = BK \frac{BL}{BH}$

Correction of the Errors of the Angles.—The most accurate adjustment of the angles of a system of triangulation is that by the "method of least squares." The most rigid adjustment by this method is effected when the whole triangulation is adjusted in one mass, all the angles being simultaneously considered. The angles should be adjusted so that not only the sum of the angles in any triangle is equal to 180° plus the spherical excess, but also so that the calculated length of any side found through independent sets of triangles shall be the same in all cases. The method of least squares is most laborious even for the simplest cases. The total number of equations of condition in the case of the triangulation of the Ordnance Survey of the United Kingdom is 920, and if the whole had been reduced in one mass, by the method of least squares, as it should have been, the solution of an equation of 920 unknown quantities would have occurred as a *part* of the work. To avoid this the triangulation was divided into 21 parts or figures. Four of these, not adjacent, were first adjusted by another method, and the corrections determined in these figures were carried into the equations of condition of the adjacent figures. The average number of equations in a figure was 44; the largest equation was one of 77 unknown quantities.

The triangulation of the great Trigonometrical Survey of India is worked out by the method of least squares. Although this entails an enormous amount of work, it is thoroughly carried out.

By considering the quadrilateral figures formed by pairs of adjacent triangles, and adjusting the angles of each quadrilateral, an adjustment of considerable accuracy may be obtained without using the method of least squares, but even this is too laborious for ordinary practice.

As a general rule the surveyor will have to be satisfied with a simple adjustment of the angles of each triangle as follows:—

Calculation of Spherical Excess.—The first operation is to calculate the "spherical excess." The sum of the three angles of a spherical triangle exceeds 180° by an amount which depends on the area of the triangle, and is called the "spherical excess." For a triangle about 76 sq. miles in area, of equilateral shape

and having sides 13 miles long, the spherical excess is 1 second. For a triangle with sides 102 miles long the spherical excess is 1 minute. Approximately the spherical excess in seconds is—

$$\frac{\text{Area in square miles}}{75{\cdot}5}$$

The formula for the spherical excess is—

$$\text{Spherical excess } e \text{ in seconds} = \text{S} \times \frac{648000}{\pi r^2}$$

where S = area of triangle, r = mean radius of earth; S and r must be in the same measure, *i.e.*, square feet and lineal feet, square metres and lineal metres, &c. As S is small compared with r^2, the area of the triangle may be obtained with sufficient accuracy for computing the spherical excess by treating the triangle as if it were plane—that is, the area may be calculated simply from the observed angles and the known side by plane trigonometry. If the earth's mean radius is taken as 20,888,761 ft., then—

$$\text{Log } \frac{\pi r^2}{648000} = 9.32540, \text{ whence } \log e = \log \text{S} - 9.32540.$$

Taking the mean radius of the earth as 20,888,629 ft., $\log \frac{648000}{\pi r^2} = \overline{10}.6746069$, whence $\log e = \log \text{S} + \overline{10}.6746069$.

When two sides and the included angle are known—

$$\text{S} = \tfrac{1}{2}\, ab \sin \text{C}.$$

When two angles and the adjacent side are known—

$$\text{S} = \tfrac{1}{2}\, a^2 \times \frac{\sin \text{B} \sin \text{C}}{\sin (\text{B} + \text{C})}$$

Adjustment of the Angles.—Having calculated the spherical excess, the sum of the three observed angles of any triangle should be equal to 180° + the spherical excess. If it is not, the difference must be distributed among the angles.

If all the observations are considered to be equally accurate, $\frac{1}{3}$ of the total error is to be added or subtracted from each of the angles as the case may be. If the angles are the averages of unequal numbers of observations the errors may be distributed among the angles inversely as the number of observations or inversely as the square of the number of observations, *i.e.*, proportionately to the reciprocals or to the reciprocals of the squares of the numbers of observations.

Example of Correction and Adjustment of Angles of a Triangle.—As an example,* let A be East Lomond in Fifeshire, B Bencleuch in the Ochils, and C the Calton Hill at Edinburgh.

The observed angles were—

A	83° 55′ 46.06″	by 7	observations
B	45° 38′ 55.19″	6	,,
C	50° 25′ 15.83″	20	,,
	179° 59′ 57.08″		
$180° + e =$	180° 0′ 2.79″		
	Error 5.71″		

1. *Correction by reciprocals of number of observations—*

$\frac{1}{7} = 0.143$	Correction of A $= \frac{0.143}{0.360} \times 5.71'' = 2.27''$	
$\frac{1}{6} = 0.167$	Similarly correction of B	$= 2.65''$
$\frac{1}{20} = 0.050$	and correction of C	$= 0.79''$
0.360	Total correction	$= 5.71''$

2. *Correction by reciprocals of squares of number of observations—*

$\frac{1}{49} = 0.02245$	Correction of A $= \frac{0.02245}{0.05273} \times 5.71'' = 2.43''$	
$\frac{1}{36} = 0.02778$	Similarly correction of B	$= 3.01''$
$\frac{1}{400} = 0.00250$	Correction of C similarly	$= 0.27''$
0.05273	Total correction	$= 5.71''$

The length of the given side *a* being 146,335 ft., by the first method of correcting the angles for the other sides *b* and *c* we get 105230.2 ft. and 113423.3 ft. By the second method of correcting the angles the sides *b* and *c* would be 105230.29 ft. and 113423.0 ft.

As a rule it will be sufficient to distribute the error proportionally to the reciprocals of the number of observations.

Adjustment of Angles inversely as "Weights" of Observations.—A more accurate but more laborious method is to distribute the error inversely as the "weights" of the observations, for which Gauss' rule is generally used. Gauss' rule is as follows:—

* Ainslie's Land Surveying, by Galbraith.

Let l, l_1, l_2 = the observations of any angle A in seconds.
n = number of observations.
m = average of observations.

Then $m - l$, $m - l_1$, $m - l_2$ = errors of individual observations, and the weight of the observation of the angle A is—

$$\frac{\frac{1}{2}n^2}{(m-l^2)+(m-l_1^2)+(m-l_2^2)+\&c.}$$

Similarly the weights of the observations of the angles B and C are found. Putting x, y, and z for the weights of the observations of the angles A, B, and C respectively, the error in the sum of the three angles A, B, C is to be divided into three parts proportional to $\frac{1}{x}$, $\frac{1}{y}$, and $\frac{1}{z}$, which are to be algebraically added to A, B, and C.

Calculation of the Sides of the Triangles.—Three methods of calculating the sides of the triangles may be employed, viz. :—

1. Spherical trigonometry.
2. Delambre's method.
3. Legendre's method.

Calculation by Spherical Trigonometry.—One side of a triangle and all the angles being known, the other sides may be calculated by the ordinary rules of spherical trigonometry. This, however, is very troublesome, and the other methods give equally correct results, with less labour.

Calculation by Delambre's Method.—If the three angular points of a spherical triangle be joined by straight lines, these will be chords of the arcs, and will form a plane triangle. Reduce the given spherical side to its chord and the spherical angles to the plane angles between the chords. Calculate the other sides or chords by plane trigonometry, and then calculate their corresponding arcs.

Calculation by Legendre's Method.—Legendre's theorem is—"In any spherical triangle, the sides of which are very small compared with the radius of the sphere, if each of the angles be diminished by $\frac{1}{3}$ of the true spherical excess, the sines of these angles will be proportional to the lengths of the opposite sides, and the triangle may therefore be calculated as if it were plane."

The reduction of the spherical angles to the plane angles included between the chords in Delambre's method is laborious. Legendre's method is simple and is most commonly used. The difference between the results of computation by the three methods is only a small fraction in 100 miles. In the Ordnance Survey of the United Kingdom most of the triangles were computed by Delambre's method and checked by Legendre's method. In the French *Base du Système Métrique* all three methods were employed. Having corrected the angles, $\frac{1}{3}$ of the spherical excess is then deducted from each of the corrected angles, and with the known side and these last angles the triangle is calculated by plane trigonometry.

Summary of Operations in Computation of Triangles by Legendre's Method.—1. Calculate the area of the triangle from the observed angles and the given side as if it were plane.

2. From this area calculate the spherical excess.

3. Find the total error of measurement of the angles, which is the difference between the sum of the observed angles and 180° + spherical excess.

4. Correct the angles by distributing the error among them as already described.

5. From each corrected angle deduct $\frac{1}{3}$ of the spherical excess.

6. With these last angles and the given side calculate the other sides by plane trigonometry.

Calculation of the Astronomical Co-ordinates of the Stations.—These are the latitudes and longitudes. The latitude and longitude and direction of the meridian must be very accurately determined at one station by a number of astronomical observations, for which see Chapter X. In order to find the latitudes and longitudes and azimuths of the other stations we have then only to solve the problem, "Given the latitude and longitude of A, and the distance and bearing AB, to find the latitude and longitude of B," for each station in succession. This problem, as well as the converse, "Given the latitude and longitude of A and B, to find the distance AB and the azimuth from each to the other," has already been given on page 409, Chapter XI. These calculated latitudes, longitudes, and azimuths are called "geodetic" to distinguish them from the observed astronomical latitudes, longitudes, and azimuths. The computations are checked

and adjusted by taking astronomical observations for meridian, latitude, and longitude at other stations throughout the survey; the mean of these astronomical observations thus determines the astronomical co-ordinates of the survey considered as a whole.

When an astronomical observatory is included in the survey, advantage may be taken of it to get an accurate determination of meridian, latitude, and longitude. The observed and calculated latitudes and longitudes of a place will not always agree even if the earth is considered to be an ellipsoid of revolution. This is owing to the irregular form and density of the earth, resulting in local attraction of the plumb line. The difference between the "geodetic" and astronomical latitudes and longitudes is termed the "station error." At 32 stations of the primary triangulation of the United Kingdom the latitudes were determined with Ramsden's & Airy's zenith sectors. The zenith telescope was also used for determination of latitude. In a single night with this instrument a very accurate result with a probable error of about 0.3″ or 0.4″ can be obtained from say 20 pairs of stars; usually for the highest possible precision four or five nights are necessary. Observations for meridian were made with the theodolite at a large number of stations, the stars α, δ, and λ, Ursæ Minoris, and 51 Cephei being those observed and always at their greatest azimuths. At 6 stations the probable error of the result was under 0.4″, at 12 stations under 0.5″, and at 34 stations under 0.7″, so that the absolute azimuth of the whole network of triangles was determined with extreme accuracy.

Polar Spherical Co-ordinates.—The polar spherical co-ordinates of one point with reference to another point are the arc of a great circle passing through them, *i.e.*, the distance between the points, and the azimuth or true bearing of the arc.

Rectangular Spherical Co-ordinates. — The axes of rectangular spherical co-ordinates are a meridian and a perpendicular to it. For short distances these may be taken as being in one plane. For greater distances than 50 miles new meridians must be used.

Surveying of Interior Detail.—After the execution of the triangulation the details are surveyed by the ordinary methods with chain, compass and chain, theodolite, plane table, &c. &c.

Tacheometry or stadia work is very useful for this part of the survey. All surveys of detail, as for instance traverses, are made to check in on the stations of the triangulation. The various methods which will be adopted for the survey of the interior detail are fully described in Chapters I., II., and VI.

Levelling.—The heights of the very high principal stations and other very high points may be determined by trigonometrical levelling, for which see Chapter III. The principal levelling of the survey will, however, consist of lines of "precise levels" run across the area surveyed. For precise levelling see Chapter III. On these lines bench marks are left about 4 or 5 miles apart. From these bench marks other bench marks are established throughout the survey from ½ mile to 1 mile apart, by ordinary spirit levelling. From the bench marks all the levelling of the survey for contours, &c &c., is executed in the ordinary manner as described in Chapter III. The 6 in. Ordnance map of the United Kingdom gives contours at 25 ft., 50 ft., and 100 ft. Generally the contours should be at much less intervals (see Chapter III.). When it is not possible to level from mean sea level or assumed mean sea level at the coast, the datum of the levels of the survey must be fixed by assuming an elevation above mean sea level for one of the principal points of the survey. This may be determined from barometric observations, for which see Chapter III.

Cost of Trigonometrical Surveys.—In the case of the primary triangulation of 500,000 square miles of country executed by the United States Geological Survey, in which 1,295 primary stations were fixed, the cost of the triangulation was about £60 per station, including base measurements. The total cost of this primary triangulation was £80,000, and the average cost about 3s. 4d. per square mile; 8 in. theodolites reading by microscope to 2 seconds were used.

Below are given the Mississippi River Commission Instructions for secondary and tertiary triangulation.* These are the result of many years' experience, and show the best American practice.

* Johnson, Theory and Practice of Surveying.

Instructions for Secondary Triangulation.

Locating Stations.—In locating stations it is desirable to fix them at such points as give good conditioned triangles. The smallest angle in any triangle should never be less than 30°, and but few of these should be permitted to enter into the system. The triangles should lie in such a way that pointings can be made from any station to the stations immediately above and below on the same side of the river—that is, blind lines should always be avoided. Other things being equal, stations should be set where they can be readily found and where they will not be disturbed.

Reading Angles.—The angles will be read with T. & S. theodolites Nos. 1 and 2. The instruments will be mounted firmly and protected from sun and wind when in use. The value of the angle will be determined by eight combined results read as follows (see Fig. 261):—

The instrument being at A, carefully levelled and in good adjustment, with the vertical circle to the right or telescope direct and lower motion fixed, point successively to Δ 1, 2, 3, 4, recording the reading of both micrometers for each pointing. This gives a positive result for each angle. Then point to Δ 4, 3, 2, 1, and record readings as before. This gives a negative result for each angle. A mean of the two gives one combined result. The readings in a positive and negative direction will eliminate twist of station or instrument, provided that the readings occupy but a short period of time, during which the twist, if any, is uniform.

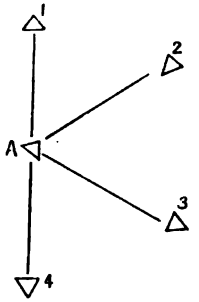

Fig. 261.
Observing Angles.

For the next combined result. The telescope will now be reversed, that is revolved through, leaving the pivots in the same wyes, and the whole will be revolved 180° in azimuth. The vertical circle will then be on the left; the limb will be shifted 22½°, and the stations will be read forward and back as before. The notes for this series will be headed circle left. Reversing the telescope will eliminate errors of collimation, small level errors, and inequality of pivots. Shifting the limb so as to read the angles at equal intervals around the circle will eliminate periodic errors and errors of graduation.

The same programme is followed until all the results are obtained, the limb being shifted and the telescope reversed after each combined result.

The micrometers should be adjusted so the run will be nearly zero. This should, however, be tested at the beginning of each day's work, and entered in the notebook.

Closing Triangles.—The error in closing a triangle should rarely reach and never exceed 6 seconds, and the average closure should be much below this. This will require great care in the centring of instruments and targets. A discrepancy of ¼ in. will give an error of a second in a distance of 1 mile. A transparent cloth phaseless target will be used, the size varying with the length of triangle sides.

Base Lines.—Base lines will be measured at intervals of about 75 miles. This will be done with the 300 ft. steel tape. The line should be carefully staked out and its grade determined instrumentally. Supporting stakes will be driven at intervals of 30 ft. The stakes marking the extremities of each tape will be firmly set and free from any disturbing influence due to tension of tape or otherwise. On these stakes strips of zinc will be fastened and remain until the whole measurement is completed. The temperature of the tape will be determined by three thermometers placed near the ends and in the middle of the tape. They will be attached to suitable supports and placed with their bulbs near the tape when measurements are being taken. Observers must be careful to keep sufficiently far away so as not to affect the thermometers.

The tape will be suspended in hooks at intervals of 30 ft., and attached in such a way that it may swing freely and eliminate friction as far as practicable. The tension of the tape will be kept uniform while measuring by attaching a weight of 16 lbs. The extremity of each tape length will be marked on the zinc strips with a fine line and suitably numbered. The preservation of these strips furnishes a ready means of comparison of each tape length at any future time.

The line should be measured two or more times, with a discrepancy when reduced of not more than 1 in 250,000. This can readily be done if measurements are made on cloudy days or at night.

Observations for Azimuth.—The azimuth of each base line will be determined by observing, with a triangulation instrument, two closely circumpolar stars at elongation on two different nights. The instrument and light should preferably be at the extremities of the base or a triangle side. The following order of observing will be used :—

First.	Second.	Third.	Fourth.
Circle right.	Shift limb 45°.	Shift limb 45°.	Shift limb 45°.
Point to light.	Circle left.	Circle left.	Circle right.
Point to star and note time.	Point to light.	Point to light.	Point to light.
Read level direct and reverse.	Point to star and note time.	Point to star and note time.	Point to star and note time.
Point to star and note time.	Read level direct and reverse.	Read level direct and reverse.	Read level direct and reverse.
Point to light.	Point to star and note time.	Point to star and note time.	Point to star and note time.
	Point to light.	Point to light.	Point to light.

On the second night repeat this programme, starting with a reading of limb 45° greater than the last reading of the previous evening.

It will probably be found most convenient in these observations to use Polaris, δ Ursæ Minoris, λ Ursæ Minoris, and 51 Cephei.

The time will be determined by observing the meridian passage of high and low stars.

Stone Line Bench Marks.—At intervals of about 3 miles along the river, lines of pipe and tile marks will be set for future surveys.

These lines will be numbered and located about as shown on maps on file in this office.

The marks nearest the river will be far enough back to be safe from erosion for many years; the others will be ½ mile farther back.

In cases where the bluffs are near the river the rear marks may be omitted. The marks will preferably be placed at property corners, along public roads, or on property lines, in places where they can be readily found and where they will not be liable to disturbance.

It is desirable to determine the azimuth and distance between the successive marks on the same line when practicable. The marks should also be as nearly in a line as the conditions of location above named will admit.

The marks will be connected directly with the secondary triangulation, where practicable, by three pointings from two or more secondary stations, and an equal number from the point to be located to two stations that will give a fairly good triangle.

Where the points cannot be located directly from the secondary work, a tertiary system may be used, starting and closing on a secondary line. In this work the angles may be read with a good 10 second theodolite, and the triangles should close within 15 seconds. A steel tape or chain may also be used, where desirable, in locating the point which is farthest from the river.

Cutting Timber.—Cutting timber to clear the lines of sight or for material with which to build stations should be avoided as far as practicable. Where cutting is necessary, a strict account must be kept of the number of trees cut, their size and kind of timber.

Descriptions of Stations.—A minute description of each station will be made and entered in notebook kept for that purpose. This description will be complete for each station, and will show what the geodetic point is and how marked. Its location with reference to surrounding objects will be shown by an accurate sketch giving azimuth and distance to bearing trees, houses, or other prominent objects.

A similar record will also be kept of the stone line marks.

TERTIARY TRIANGULATION.

Triangles.—Where the secondary stations are more than 3 miles apart a tertiary system will be carried giving points on either bank at intervals of a mile or less. This system will begin on a triangle side of the secondary system, or a carefully measured base, and all of the available secondary stations will be used in the tertiary chain. The tertiary work will also close on a line of known length as a check on its accuracy. The discrepancy should not exceed 1 in 3,000. The system should be laid out and the angles read in advance of the topographers,

so that the azimuths and lengths of sides can be used in checking stadia work.

Stations.—The station point may be marked by a pole 2 in. in diameter stuck into the ground, and bearing a red and white flag to distinguish it from the ordinary sounding flags. A strip of white cloth wrapped near the bottom of the pole will admit of the pointings being made so low down that errors arising from disturbance of the pole by the wind will be inappreciable.

Instrument Stations.—For observing, the instrument may be placed on an ordinary tripod centred over the hole after the pole has been removed.

Observing Angles.—The angles should be read with a 10 second instrument in good adjustment, and should be repeated at least three times on different parts of the circle to check errors of reading.

It is desirable to have the first series read on azimuth. Having pointed to the first station, read to all of the others in succession. Pointings should also be made to all of the sounding flags in the vicinity, as well as prominent objects on land, such as chimneys, houses, &c., the location of which will serve to check the topographical work.

For the second series slip the lower limb 60° and read to the stations in the opposite order from the first series. Slip the limb the same amount again and read the third series.

Marking Stations.—The river ends of the stone lines will be made points in the tertiary system, and whenever practicable the stones should also be located trigonometrically.

Tertiary points which are likely to remain undisturbed for some time should be plainly marked with a strong stake 2 ft. high, the number of the point, the initials of the observer and date being marked on it with red chalk.

Map Projections.

Projection to be Adopted.—The particular kind of projection to be used in showing parts of the earth's surface on a map depends on the *area* of the survey, the *purpose* for which the map is required, and on the *degree of accuracy* necessary.

Thus a method of projection which is accurate enough for a small area may be unsuitable for a large area; also as it is impossible to develop a sphere exactly, in every projection the relative lengths, bearings, areas, or latitudes and longitudes, must be to a certain extent inaccurate, and the purpose of the map must decide which of these is to be made correct, and the others sacrificed.

Rectangular Projection.—In this projection the meridians are all straight parallel lines, and the parallels of latitude are also

straight parallel lines perpendicular to the meridians. To construct a map on this projection draw a central meridian and divide it into degrees of latitude according to their value at the various latitudes as given in Geodetic Tables. Through these points of division describe the parallels of latitude as straight lines at right angles to the central meridian. On the middle parallel of latitude lay off the degrees of longitude according to their value for that latitude as given in Geodetic Tables, and through the points found draw the other meridians parallel to the central meridian.

This projection is to be used for surveys which are worked to a single meridian, and should not be used for sheets containing a larger area than 100 sq. miles. The plotting may be done either by polar or rectangular co ordinates, *i.e.*, distance and bearing or latitudes and departures. All straight lines, whether fixed by triangulation or ranged out by theodolite on the ground, will be straight on the projection, and all such straight lines will have a constant bearing on the map, the meridians being parallel. The greatest error in this projection is in taking the meridians to be parallel. In latitude 40° in a map of 10 miles square the error at the corners in longitude is about 20 ft. At the equator there is no distortion.

Such a map, if plotted from distances and bearings or latitudes and departures, and referred to a single meridian, gives correct distances, bearings (with reference to the meridian adopted), and areas. The latitudes and longitudes indicated on the map thus plotted will, however, be approximate, the error depending upon the distance from the equator.

A survey executed on a system of triangulation may be plotted on a rectangular projection, each sheet being referred to a meridian taken through one of the triangulation stations on it.

Trapezoidal Projection.—When the survey is still referred to a single meridian, but the sheets of the map contain a greater area than 100 sq. miles, the trapezoidal projection is to be used, in which the meridians and parallels of latitude are still straight lines, but the meridians are drawn to converge. To construct a map on this projection draw a central meridian and divide it into degrees and minutes of latitude by means of values of latitude

given in Geodetic Tables. Through these points draw parallels of latitude perpendicular to the central meridian. Take two parallels of latitude, about quarter the height of the sheet from the bottom and the top. These two parallels of latitude are then divided into degrees and minutes of longitude according to the values at their respective latitudes as given in Geodetic Tables. The meridians are then drawn as straight converging lines through the points of division on these two parallels of latitude. The map is thus ruled into trapezoids. The parallels of latitude are perpendicular to the central meridian only. The principal error in this projection is due to the parallels of latitude being drawn as straight lines. The error is about 30 ft. in 10 miles in latitude 40°.

The plotting may be done as before, either by means of polar or rectangular co-ordinates.

On a map plotted on a trapezoidal projection, the correct distances, bearings (referred to the central or adopted meridian), and areas are given. The longitudes indicated on the map thus plotted are also correct, and the latitudes nearly so.

Trapezoidal projection may be used for sheets containing an area of not more than 25 miles square. If the map is to be used for the purpose of giving the relative position of points in terms of their latitude and longitude then the computed latitudes and longitudes are to be used in plotting. This, however, will seldom be the case.

Simple Conic Projection.—In a simple conic projection positions on the spherical surface are first projected on to the surface of a cone tangent to the sphere. The map is then the development of the conical surface. The apex of the cone is taken as a point in the earth's axis produced, and is such that the surface of the cone is tangent to the earth's surface at the middle parallel of latitude of the map. The conical surface being developed, the meridians are straight and converge to the apex of the cone, and the parallels of latitude are arcs of concentric circles with the apex as their common centre. To construct a map on simple conic projection rule a central meridian and divide it into degrees and minutes by means of the values of the latitude given in Geodetic Tables. Through these points describe parallel circular arcs, taking the apex of the cone as a centre. The length of the

side of the tangent cone, which is the radius of the middle parallel of latitude, is given in Geodetic Tables.

On the middle parallel of latitude of the map, degrees and minutes of longitude are marked from values in Geodetic Tables. Through these the meridians are ruled as straight lines converging to the apex of the tangent cone. The meridians and parallels of latitude are perpendicular. In this projection the latitudes are practically correct and the longitudes are sufficiently correct for a map containing an area of several hundred miles square. The degrees of longitude are exaggerated on all the parallels of latitude except the centre one; the area shown on the map between the extreme parallels of latitude and the extreme meridians is greater than the true area on the sphere.

De L'Isle's Conic Projection.—This is the same as the simple conic projection; only two parallels of latitude, one at one-fourth and one at three-fourths the height of the sheet, are properly divided. The parallels are drawn as concentric arcs of circles as before. This projection is more correct than the simple conic projection, and the cone is not tangent to the sphere, but cuts it in the two divided parallels of latitude. The area between the two graduated parallels of latitude is too small, and the rest is too large. The area of the whole map is therefore nearly correct.

Bonne's Projection.—In this all the parallels of latitude are properly divided, and the meridians are drawn through the corresponding points of division. The parallels of latitude are concentric circles as before. The meridians are perpendicular to the parallels of latitude only in the centre part of the map. There is a slight error, due to the parallels of latitude being concentric circles, in the corners of the map. Bonne's projection is used for the 1 in. to the mile Ordnance Survey Map of Scotland.

Polyconic Projection.—In the last three projections only one tangent or intersecting cone is used, and for very large areas

it is better to make each parallel of latitude the development of the base of a cone tangent to the sphere at that latitude. The radii of the parallels of latitude therefore diminish towards the pole so that the parallels of latitude separate from each other towards the sides of the map. A degree of latitude at the edges, therefore, does not correspond with a degree of latitude on the middle meridian. The difference is, however, perceptible only on a map of a very large area.

The polyconic projection is employed for the maps of the United States Coast and Geodetic Survey.

Fig. 262 will show the use of Geodetic Tables in laying down the meridians and parallels of latitude for a polyconic projection. Draw the straight line c_4c_7 through the middle of the map and divide it into degrees of latitude by means of the values given in Geodetic Tables. Through these points draw the dotted lines $a_3c_4a^1_3$, &c., perpendicular to c_4c_7. The points a_1, a_2, a_3, a^1_1, a^1_2, a^1_3, &c., are then plotted by means of the meridian distances given in Geodetic Tables.

From the points a_1, a_2, a_3, &c., the divergences of the parallels of latitude, a_1p_1, a_2p_2, a_3p_3, &c., are plotted perpendicular to the line $a_3c_4a^1_3$. The parallels of latitude and the meridians are then drawn through all the points p_1, p_2, p_3, &c. They may be plotted as straight lines through these points if p_1, p_2, p_3, &c., are taken sufficiently near each other.

The position of the meridians and parallels of latitude on this projection being practically exact, the survey may be plotted by means of the calculated latitudes and longitudes if convenient or necessary. The ordinary methods of plotting by polar or rectangular co-ordinates are of course also applicable, and will generally be used for the plotting.

The circular arcs representing the parallels of latitude may be drawn by ordinates computed from their radii, which radii are the various lengths of side of tangent cones given in Geodetic Tables, the longitudes being then laid down on these arcs by means of the meridian distances in tables. These meridian distances must of course be laid down along lines perpendicular to the central meridian, as a_3, c_4, a^1_3, &c., Fig. 262. The points where perpendiculars from these points of division intersect the circular arcs representing the parallels of latitude are the required points of division of the arcs into longitude.

When a number of sheets of a survey are to be plotted in a single map drawn to a reduced scale, if the original large scale sheets are plotted on rectangular or trapezoidal projection, the small scale map should be drawn on a polyconic projection, if the total area of the survey is of any considerable extent.

Projection of Maps of the Survey of India.—The following method of projection is used for the maps of the Indian Survey. Suppose the intersection of two meridians by two parallels to form a small spherical quadrilateral, with sides of aliquot parts of a

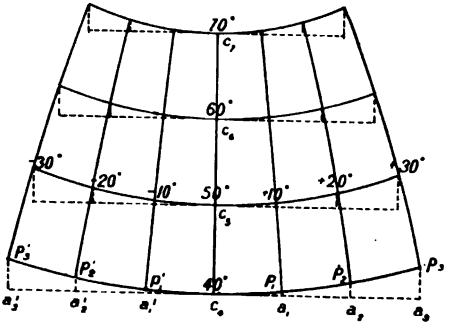

Fig. 262.—Polyconic Projection.

degree in latitude and in longitude; let m be the length of each of the meridianal arcs, p, p' the lengths of the arcs on the upper and lower parallels, and let q be a diameter, then $q = \sqrt{m^2 + pp'}$; thus m, p, and p' being given, q is calculated. With these data, which are tabulated for different arcs and scales, the corner points are laid off for a number of quadrilaterals in succession on either side of an adopted meridian, and lines are drawn through the points to indicate the collateral meridians and the parallels of latitude. The latter are always curved, the former are also curved but in a much less degree, being concave to their initial meridian and more so the further they are from it. When the area is small and the scale large, the meridians are practically straight lines, and the several sheets of a map each projected on its own meridian

will fit together closely when carried on in any direction. But when the area is large, exceeding 8 or 10 square degrees, and the scale small, the sheets will not fit together continuously unless they are projected with reference to a single meridian for the whole map, to which the meridians on each side will be increasingly concave, or unless all the meridians are made straight lines, by slightly contracting each of the intermediate arcs of parallel to a length which is exactly proportional to the lengths and relative distances of the upper and lower parallels of the map from it. There must be some distortion in either case. In the first, meridians which are actually straight lines are represented as being curved; in the second, straight meridians are obtained, but the distances between them are exact only on the upper and lower parallels, and are too small elsewhere, especially on the middle parallel, the length of which necessarily exceeds the mean length of the upper and lower parallels. Distortion is, however, inevitable in any projection of a spherical surface on to a plane.

When a map is constructed in rectangular sheets, some station is adopted as the origin and its meridian as the principal axis, to which the corner points of the sheets are to be referred: the co-ordinates of these points are given such dimensions as are most suitable for the size and scale of the map, and are equivalent to the rectangular spherical co-ordinates of imaginary points on the curved surface of the earth, at corresponding distances from the origin and its meridian. These being given, the distances of the points from the origin in latitude and longitude may be computed, and thus data become available for projecting the meridians and parallels within the rectangular marginal lines of each sheet, or for introducing the divisions of latitude and longitude on the marginal lines if preferred. Conversely when the latitudes and longitudes are given, the rectangular spherical co-ordinates are computed and the marginal lines projected around the meridians and parallels. Filling in is then commenced; the principal stations are laid down by their co-ordinates, and the topographical details pencilled around them by copying or tracing the field sheets of the survey; the names and the outlines are then inked in; the shading for delineating the features and general configuration of the ground is usually done last of all.

Projection of Maps of the Ordnance Survey of the United Kingdom.—Fully detailed instructions and tables for the projection of maps on the Rectangular Tangential Projection are given in a pamphlet published by the Ordnance Survey Department, "On the Construction and Use of Marginal Sheet Lines for the Uniform Projection of Maps in any part of the World."

TABLE I.

PART I.—HEIGHTS BY MERCURIAL BAROMETER.

Inches.	Feet.	Diff.	Inches.	Feet.	Diff.	Inches.	Feet.	Diff.
11.0	1396.9	236.4	16.0	11186.3	162.8	21.0	18291.0	124.1
11.1	1633.3	234.3	16.1	11349.1	161.8	21.1	18415.1	123.6
11.2	1867.6	232.3	16.2	11510.9	160.8	21.2	18538.7	122.9
11.3	2099.9	230.2	16.3	11671.7	159.8	21.3	18661.6	122.4
11.4	2330.1	228.2	16.4	11831.5	158.8	21.4	18784.0	121.8
11.5	2558.3	226.2	16.5	11990.3	157.9	21.5	18905.8	121.2
11.6	2784.5	224.2	16.6	12148.2	156.9	21.6	19027.0	120.7
11.7	3008.7	222.4	16.7	12305.1	155.9	21.7	19147.7	120.1
11.8	3231.1	220.5	16.8	12461.0	155.1	21.8	19267.8	119.6
11.9	3451.6	218.6	16.9	12616.1	154.1	21.9	19387.4	119.0
12.0	3670.2	216.8	17.0	12770.2	153.3	22.0	19506.4	118.5
12.1	3887.0	215.0	17.1	12923.5	152.3	22.1	19624.9	118.0
12.2	4102.0	213.3	17.2	13075.8	151.5	22.2	19742.9	117.4
12.3	4315.3	211.6	17.3	13227.3	150.6	22.3	19860.3	116.9
12.4	4526.9	209.8	17.4	13377.9	149.7	22.4	19977.2	116.4
12.5	4736.7	208.2	17.5	13527.6	148.9	22.5	20093.6	115.8
12.6	4944.9	206.5	17.6	13676.5	148.0	22.6	20209.4	115.4
12.7	5151.4	205.0	17.7	13824.5	147.2	22.7	20324.8	114.8
12.8	5356.4	203.3	17.8	13971.7	146.3	22.8	20439.6	114.4
12.9	5559.7	201.7	17.9	14118.0	145.6	22.9	20554.0	113.8
13.0	5761.4	200.2	18.0	14263.6	144.7	23.0	20667.8	113.3
13.1	5961.6	198.7	18.1	14408.3	144.0	23.1	20781.1	112.9
13.2	6160.3	197.2	18.2	14552.3	143.1	23.2	20894.0	112.4
13.3	6357.5	195.7	18.3	14695.4	142.4	23.3	21006.4	111.9
13.4	6553.2	194.3	18.4	14837.8	141.6	23.4	21118.3	111.4
13.5	6747.5	192.8	18.5	14979.4	140.9	23.5	21229.7	110.9
13.6	6940.3	191.4	18.6	15120.3	140.0	23.6	21340.6	110.5
13.7	7131.7	190.0	18.7	15260.3	139.4	23.7	21451.1	110.0
13.8	7321.7	188.6	18.8	15399.7	138.6	23.8	21561.1	109.5
13.9	7510.3	187.3	18.9	15538.3	137.9	23.9	21670.6	109.1
14.0	7697.6	186.0	19.0	15676.2	137.1	24.0	21779.7	108.7
14.1	7883.6	184.6	19.1	15813.3	136.5	24.1	21888.4	108.2
14.2	8068.2	183.3	19.2	15949.8	135.7	24.2	21996.6	107.7
14.3	8251.5	182.1	19.3	16085.5	135.0	24.3	22104.3	107.3
14.4	8433.6	180.9	19.4	16220.5	134.3	24.4	22211.6	106.8
14.5	8614.4	179.6	19.5	16354.8	133.7	24.5	22318.4	106.4
14.6	8794.0	178.3	19.6	16488.5	132.9	24.6	22424.8	106.0
14.7	8972.3	177.2	19.7	16621.4	132.3	24.7	22530.8	105.6
14.8	9149.5	176.0	19.8	16753.7	131.6	24.8	22636.4	105.1
14.9	9325.5	174.8	19.9	16885.3	131.0	24.9	22741.5	104.8
15.0	9500.3	173.5	20.0	17016.3	130.3	25.0	22846.3	104.3
15.1	9673.8	172.4	20.1	17146.6	129.7	25.1	22950.6	103.8
15.2	9846.2	171.3	20.2	17276.3	129.0	25.2	23054.4	103.5
15.3	10017.5	170.2	20.3	17405.3	128.4	25.3	23157.9	103.1
15.4	10187.7	169.1	20.4	17533.7	127.7	25.4	23261.0	102.6
15.5	10356.8	168.0	20.5	17661.4	127.2	25.5	23363.6	102.3
15.6	10524.8	167.0	20.6	17788.6	126.5	25.6	23465.9	101.8
15.7	10691.8	165.9	20.7	17915.1	125.9	25.7	23567.7	101.5
15.8	10857.7	164.8	20.8	18041.0	125.3	25.8	23669.2	101.1
15.9	11022.5	163.8	20.9	18166.3	124.7	25.9	23770.3	100.7
16.0	11186.3		21.0	18291.0		26.0	23871.0	

TABLE I.

PART I.—HEIGHTS BY MERCURIAL BAROMETER—*Continued.*

Inches.	Feet.	Diff.	Inches.	Feet.	Diff.	Inches.	Feet.	Diff.
26.0	23871.0		27.8	25619.9		29.4	27081.9	
		100.3			93.8			88.7
26.1	23971.3		27.9	25713.7		29.5	27170.6	
		99.9			93.4			88.4
26.2	24071.2		28 0	25807.1		29.6	27259.0	
		99.5			93.2			88.1
26.3	24170.7		28.1	25900.3		29.7	27347.1	
		99.1			92.8			87.8
26.4	24269.8		28.2	25993.1		29.8	27434.9	
		98.8			92.5			87.6
26.5	24368.6		28.3	26085.6		29.9	27522.5	
		98.4			92.1			87.2
26.6	24467.0		28.4	26177.7		30.0	27609.7	
		98.1			91.9			86.9
26.7	24565.1		28.5	26269.6		30.1	27696.6	
		97.6			91.5			86.7
26.8	24662.7		28.6	26361.1		30.2	27783.3	
		97.3			91.2			86.4
26.9	24760.0		28.7	26452.3		30.3	27869.7	
		97.0			90.9			86.0
27.0	24857.0		28.8	26543.2		30.4	27955.7	
		96.6			90.5			85.8
27.1	24953.6		28.9	26633.7		30.5	28041.5	
		96.2			90.3			85.6
27.2	25049.8		29.0	26724.0		30.6	28127.1	
		95.9			89.9			85.2
27.3	25145.7		29.1	26813.9		30.7	28212.3	
		95.5			89.6			85.0
27.4	25241.2		29.2	26903.5		30.8	28297.3	
		95.2			89.3			84.7
27.5	25336.4		29.3	26992.8		30.9	28382.0	
		94.8			89.1			84.4
27.6	25431.2		29.4	27081.9		31.0	28466.4	
		94.5						
27.7	25525.7							
		94.2						
27.8	25619.9							

TABLE I.

PART II.—CORRECTION DUE TO T - T', OR THE DIFFERENCE OF THE TEMPERATURES OF THE BAROMETERS THEMSELVES (NOT FOR THAT OF THE INTERMEDIATE AIR) AT THE TWO STATIONS.

This correction is negative when the temperature at the upper station is lowest, and *vice versa*.

T - T'	Correction.	T - T'	Correction.	T - T'	Correction.	T - T'	Correction.	T - T'	Correction.
Deg. F.	Feet.	Deg. F.	Feet.	Deg. F.	Feet.	Deg. F.	Feet.	Deg. F.	Feet.
1	2.3	17	39.8	33	77.3	49	114.7	64	149.8
2	4.7	18	42.1	34	79.6	50	117.0	65	152.2
3	7.0	19	44.5	35	81.9	51	119.4	66	154.5
4	9.4	20	46.8	36	84.3	52	121.7	67	156.8
5	11.7	21	49.2	37	86.6	53	124.1	68	159.2
6	14.0	22	51.5	38	89.0	54	126.4	69	161.5
7	16.4	23	53.8	39	91.3	55	128.7	70	163.9
8	18.7	24	56.2	40	93.6	56	131.1	71	166.2
9	21.1	25	58.5	41	96.0	57	133.4	72	168.6
10	23.4	26	60.9	42	98.3	58	135.8	73	170.9
11	25.8	27	63.2	43	100.7	59	138.1	74	173.3
12	28.1	28	65.5	44	103.0	60	140.4	75	175.6
13	30.4	29	67.9	45	105.3	61	142.8	76	177.9
14	32.8	30	70.2	46	107.7	62	145.1	77	180.3
15	35.1	31	72.6	47	110.0	63	147.5	78	182 6
16	37.5	32	74.9	48	112.4				

App. Alt.	Part III.—Correction due to Change of Gravity from the Latitude of 45° to the Latitude of the Place of Observation. *Positive from Lat. 0° to 45°. Negative from Lat. 45° to 90°.* Latitude.						Part IV.—Correction for Decrease of Gravity on a Vertical. *Always positive.*	Part V.—Correction due to the Height of the Lower Station. *Always positive.* Height of Barometer at Lower Station.							App. Alt.
	0°	10°	20°	30°	40°	45°		16 in.	18 in.	20 in.	22 in.	24 in.	26 in.	28 in.	
	90°	80°	70°	60°	50°										
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
1,000	2.6	2.5	2.0	1.3	0.5	0	2.5	1.6	1.3	1.0	0.8	0.6	0.4	0.2	1,000
2,000	5.3	5.0	4.1	2.6	0.9	0	5.2	3.1	2.5	2.0	1.5	1.1	0.7	0.3	2,000
3,000	7.9	7.5	6.1	4.0	1.4	0	7.9	4.7	3.8	3.0	2.3	1.7	1.1	0.5	3,000
4,000	10.6	10.0	8.1	5.3	1.8	0	10.8	6.3	5.1	4.0	3.1	2.2	1.4	0.7	4,000
5,000	13.2	12.4	10.1	6.6	2.3	0	13.7	7.8	6.4	5.0	3.8	2.8	1.8	0.8	5,000
6,000	15.9	14.9	12.2	7.9	2.8	0	16.7	9.4	7.6	6.0	4.6	3.3	2.1	1.0	6,000
7,000	18.5	17.4	14.2	9.3	3.2	0	19.9	11.0	8.9	7.1	5.4	3.9	2.5	1.2	7,000
8,000	21.2	19.9	16.2	10.6	3.7	0	23.1	12.5	10.2	8.1	6.2	4.4	2.8	1.3	8,000
9,000	23.8	22.4	18.3	11.9	4.1	0	26.4	14.1	11.4	9.1	6.9	5.0	3.2	1.5	9,000
10,000	26.5	24.9	20.3	13.2	4.6	0	29.8	15.7	12.7	10.1	7.7	5.5	3.5	1.7	10,000
11,000	29.1	27.4	22.3	14.6	5.1	0	33.3	17.2	14.0	11.1	8.5	6.1	3.9	1.8	11,000
12,000	31.8	29.9	24.4	15.9	5.5	0	36.9	18.8	15.3	12.1	9.2	6.6	4.2	2.0	12,000
13,000	34.4	32.4	26.4	17.2	6.0	0	40.6	20.4	16.5	13.1	10.0	7.2	4.6	2.2	13,000
14,000	37.1	34.9	28.4	18.5	6.4	0	44.4	21.9	17.8	14.1	10.8	7.7	4.9	2.3	14,000
15,000	39.7	37.3	30.4	19.9	6.9	0	48.3	23.5	19.1	15.1	11.5	8.3	5.3	2.5	15,000
16,000	42.4	39.8	32.5	21.2	7.4	0	52.3	25.1	20.3	16.1	12.3	8.8	5.6	2.7	16,000
17,000	45.0	42.3	34.5	22.5	7.8	0	56.4	26.6	21.6	17.1	13.1	9.4	6.0	2.8	17,000
18,000	47.7	44.8	36.5	23.8	8.3	0	60.5	28.2	22.9	18.1	13.8	9.9	6.3	3.0	18,000
19,000	50.3	47.3	38.6	25.2	8.7	0	64.8	29.8	24.1	19.2	14.6	10.5	6.7	3.2	19,000
20,000	53.0	49.8	40.6	26.5	9.2	0	69.2	31.3	25.4	20.2	15.4	11.0	7.0	3.3	20,000
21,000	55.6	52.3	42.6	27.8	9.7	0	73.6	32.9	26.7	21.2	16.1	11.6	7.4	3.5	21,000
22,000	58.3	54.8	44.7	29.1	10.1	0	78.2	34.5	28.0	22.2	16.9	12.1	7.7	3.7	22,000
23,000	60.9	57.3	46.7	30.5	10.6	0	82.9	36.0	29.2	23.2	17.7	12.7	8.1	3.8	23,000
24,000	63.6	59.8	48.7	31.8	11.0	0	87.6	37.6	30.5	24.2	18.5	13.2	8.4	4.0	24,000
25,000	66.2	62.2	50.7	33.1	11.5	0	92.5	39.1	31.8	25.2	19.2	13.8	8.8	4.1	25,000

TABLE II.

PART I.—HEIGHTS BY ANEROID BAROMETER.

60384.3 × log H or *h*.

Barometer in English Inches.	HUNDREDTHS OF AN INCH.									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
17.0	13915.5	13930.9	13946.3	13961.7	13977.1	13992.5	14007.9	14023.3	14038.6	14054.0
17.1	14069.3	14084.6	14100.0	14115.3	14130.6	14145.9	14161.2	14176.4	14191.7	14207.0
17.2	14222.2	14237.5	14252.7	14267.9	14283.1	14298.3	14313.5	14328.7	14343.9	14359.1
17.3	14374.2	14389.4	14404.5	14419.7	14434.8	14449.9	14465.0	14480.1	14495.2	14510.3
17.4	14525.4	14540.5	14555.5	14570.6	14585.6	14600.6	14615.7	14630.7	14645.7	14660.7
17.5	14675.7	14690.7	14705.6	14720.6	14735.6	14750.5	14765.4	14780.4	14795.3	14810.2
17.6	14825.1	14840.0	14854.9	14869.8	14884.6	14899.5	14914.4	14929.2	14944.0	14958.9
17.7	14973.7	14988.5	15003.3	15018.1	15032.9	15047.7	15062.4	15077.2	15092.0	15106.7
17.8	15121.4	15136.2	15150.9	15165.6	15180.3	15195.0	15209.7	15224.4	15239.0	15253.7
17.9	15268.4	15283.0	15297.6	15312.3	15326.9	15341.5	15335.1	15370.7	15385.3	15399.9
18.0	15414.5	15429.0	15443.6	15458.1	15472.7	15487.2	15501.7	15516.2	15530.7	15545.2
18.1	15559.7	15574.2	15588.7	15603.2	15617.6	15632.1	15646.5	15661.0	15675.4	15689.8
18.2	15704.2	15718.6	15733.0	15747.4	15761.8	15776.2	15790.5	15804.9	15819.2	15833.6
18.3	15847.9	15862.3	15876.6	15890.9	15905.2	15919.5	15933.8	15948.0	15962.3	15976.6
18.4	15990.8	16005.1	16019.3	16033.6	16047.8	16062.0	16076.2	16090.4	16104.6	16118.8
18.5	16133.0	16147.1	16161.3	16175.5	16189.6	16203.8	16217.9	16232.0	16246.1	16260.2
18.6	16274.4	16288.4	16302.5	16316.6	16330.7	16344.8	16358.8	16372.9	16386.9	16400.9

18.7	16415.0	16429.0	16443.0	16457.0	16471.0	16485.0	16499.0	16512.9	16526.9	16540.9
18.8	16554.8	16568.8	16582.7	16596.6	16610.6	16624.5	16638.4	16652.3	16666.2	16680.1
18.9	16694.0	16707.8	16721.7	16735.5	16749.4	16763.2	16777.1	16790.9	16804.7	16818.5
19.0	16832.3	16846.1	16859.9	16873.7	16887.5	16901.3	16915.0	16928.8	16942.5	16956.3
19.1	16970.0	16983.7	16997.4	17011.2	17024.9	17038.6	17052.3	17065.9	17079.6	17093.3
19.2	17106.9	17120.6	17134.2	17147.9	17161.5	17175.2	17188.8	17202.4	17216.0	17229.6
19.3	17243.2	17256.8	17270.3	17283.9	17297.5	17311.0	17324.6	17338.1	17351.7	17365.2
19.4	17378.7	17392.2	17405.7	17419.2	17432.7	17446.2	17459.7	17473.2	17486.6	17500.1
19.5	17513.5	17527.0	17540.4	17553.9	17567.3	17580.7	17594.1	17607.5	17620.9	17634.3
19.6	17647.7	17661.1	17674.4	17687.8	17701.1	17714.5	17727.8	17741.2	17754.5	17767.8
19.7	17781.1	17794.4	17807.7	17821.0	17834.3	17847.6	17860.9	17874.2	17887.4	17900.7
19.8	17913.9	17927.2	17940.4	17953.6	17966.8	17980.1	17993.3	18006.5	18019.7	18032.9
19.9	18046.0	18059.2	18072.4	18085.5	18098.7	18111.8	18125.0	18138.1	18151.2	18164.4
20.0	18177.5	18190.6	18203.7	18216.8	18229.9	18243.0	18256.0	18269.1	18282.2	18295.2
20.1	18308.3	18321.3	18334.4	18347.4	18360.4	18373.4	18386.4	18399.5	18412.5	18425.4
20.2	18438.4	18451.4	18464.4	18477.3	18490.3	18503.3	18516.2	18529.1	18542.1	18555.0
20.3	18567.9	18580.8	18593.8	18606.7	18619.6	18632.4	18645.3	18658.2	18671.1	18683.9
20.4	18696.8	18709.7	18722.5	18735.3	18748.2	18761.0	18773.8	18786.6	18799.4	18812.2
20.5	18825.0	18837.8	18850.6	18863.4	18876.2	18888.9	18901.7	18914.4	18927.2	18939.9
20.6	18952.7	18965.4	18978.1	18990.8	19003.5	19016.2	19028.9	19041.6	19054.3	19067.0
20.7	19079.6	19092.3	19105.0	19117.6	19130.3	19142.9	19155.6	19168.2	19180.8	19193.4
20.8	19206.0	19218.6	19231.2	19243.8	19256.4	19269.0	19281.6	19294.1	19306.7	19319.3
20.9	19331.8	19344.4	19356.9	19369.4	19382.0	19394.5	19407.0	19419.5	19432.0	19444.5
21.0	19457.0	19469.5	19482.0	19494.4	19506.9	19519.4	19531.8	19544.3	19556.7	19569.1
21.1	19581.6	19594.0	19606.4	19618.8	19631.2	19643.6	19656.0	19568.4	19680.8	19693.2
21.2	19705.6	19717.9	19730.3	19742.6	19755.0	19767.3	19779.7	19792.0	19804.3	19816.7
21.3	19829.0	19841.3	19853.6	19865.9	19878.2	19890.5	19902.7	19915.0	19927.3	19939.5
21.4	19951.8	19964.1	19976.3	19988.5	20000.8	20013.0	20025.2	20037.4	20049.7	20061.9

TABLE II.

PART I.—HEIGHTS BY ANEROID BAROMETER—*Continued.*

$60384.3 \times \log H$ or h.

Barometer in English Inches.	HUNDREDTHS OF AN INCH.									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
21.5	20074.1	20086.3	20098.4	20110.6	20122.8	20135.0	20147.1	20159.3	20171.5	20183.6
21.6	20195.8	20207.9	20220.1	20232.2	20244.3	20256.4	20268.5	20280.6	20292.7	20304.8
21.7	20316.9	20329.0	20341.0	20353.1	20365.2	20377.2	20389.3	20401.3	20413.4	20425.4
21.8	20437.5	20449.5	20461.5	20473.5	20485.5	20497.5	20509.5	20521.5	20533.5	20545.5
21.9	20557.5	20569.5	20581.4	20593.4	20605.3	20617.3	20629.2	20641.2	20653.1	20665.0
22.0	20677.0	20688.9	20700.8	20712.7	20724.6	20736.5	20748.4	20760.3	20772.1	20784.0
22.1	20795.9	20807.7	20819.6	20831.5	20843.3	20855.2	20867.0	20878.8	20890.6	20902.5
22.2	20914.3	20926.1	20937.9	20949.7	20961.5	20973.3	20985.1	20996.8	21008.6	21020.4
22.3	21032.1	21043.9	21055.7	21067.4	21079.1	21090.9	21102.6	21114.3	21126.1	21137.8
22.4	21149.5	21161.2	21172.9	21184.6	21196.3	21208.0	21219.6	21231.3	21243.0	21254.6
22.5	21266.3	21277.9	21289.6	21301.2	21312.9	21324.5	21336.1	21347.8	21359.4	21371.0
22.6	21382.6	21394.2	21405.8	21417.4	21429.0	21440.5	21452.1	21463.7	21475.3	21486.8
22.7	21498.4	21509.9	21521.5	21533.0	21544.5	21556.1	21567.6	21579.1	21590.6	21602.1
22.8	21613.6	21625.1	21636.6	21648.1	21659.6	21671.1	21682.6	21694.0	21705.5	21717.0
22.9	21728.4	21739.9	21751.3	21762.7	21774.2	21785.6	21797.0	21808.5	21819.9	21831.3
23.0	21842.7	21854.1	21865.5	21876.9	21888.3	21899.6	21911.0	21922.4	21933.7	21945.1
23.1	21956.5	21967.8	21979.2	21990.5	22001.8	22013.2	22024.5	22035.8	22047.1	22058.4

23.2	22069.7	22081.0	22092.3	22103.6	22114.9	22126.2	22137.5	22148.7	22160.0	22171.3
23.3	22182.5	22193.8	22205.0	22216.3	22227.5	22238.7	22250.0	22261.2	22272.4	22283.6
23.4	22294.8	22306.0	22317.2	22328.4	22339.6	22350.8	22362.0	22373.2	22384.3	22395.5
23.5	22406.7	22417.8	22429.0	22440.1	22451.3	22462.4	22473.5	22484.7	22495.8	22506.9
23.6	22518.0	22529.1	22540.2	22551.3	22562.4	22573.5	22584.6	22595.7	22606.8	22617.8
23.7	22628.9	22640.0	22651.0	22662.1	22673.1	22684.2	22695.2	22706.3	22717.3	22728.3
23.8	22739.3	22750.4	22761.4	22772.4	22783.4	22794.4	22805.4	22816.4	22827.3	22838.3
23.9	22849.3	22860.3	22871.2	22882.2	22893.1	22904.1	22915.0	22926.0	22936.9	22947.9
24.0	22958.8	22969.7	22980.6	22991.6	23002.5	23013.4	23024.3	23035.2	23046.1	23056.9
24.1	23067.8	23078.7	23089.6	23100.5	23111.3	23122.2	23133.0	23143.9	23154.7	23165.6
24.2	23176.4	23187.3	23198.1	23208.9	23219.7	23230.5	23241.4	23252.2	23263.0	23273.8
24.3	23284.6	23295.4	23306.1	23316.9	23327.7	23338.5	23349.2	23360.0	23370.8	23381.5
24.4	23392.3	23403.0	23413.7	23424.5	23435.2	23445.9	23456.7	23467.4	23478.1	23488.8
24.5	23499.5	23510.2	23520.9	23531.6	23542.3	23553.0	23563.7	23574.3	23585.0	23595.7
24.6	23606.3	23617.0	23627.7	23638.3	23648.9	23659.6	23670.2	23680.9	23691.5	23702.1
24.7	23712.7	23723.3	23734.0	23744.6	23755.2	23765.8	23776.4	23786.9	23797.5	23808.1
24.8	23818.7	23829.3	23839.8	23850.4	23861.0	23871.5	23882.1	23892.6	23903.1	23913.7
24.9	23924.2	23934.7	23945.3	23955.8	23966.3	23976.8	23987.3	23997.8	24008.3	24018.8
25.0	24029.3	24039.8	24050.3	24060.8	24071.3	24081.7	24092.2	24102.7	24113.1	24123.6
25.1	24134.0	24144.5	24154.9	24165.3	24175.8	24186.2	24196.6	24207.1	24217.5	24227.9
25.2	24238.3	24248.7	24259.1	24269.5	24279.9	24290.3	24300.7	24311.0	24321.4	24331.8
25.3	24342.2	24352.5	24362.9	24373.2	24383.6	24393.9	24404.3	24414.6	24424.9	24435.3
25.4	24445.6	24455.9	24466.2	24476.6	24486.9	24497.2	24507.5	24517.8	24528.1	24538.4
25.5	24548.6	24558.9	24569.2	24579.5	24589.7	24600.0	24610.3	24620.5	24630.8	24641.0
25.6	24651.3	24661.5	24671.8	24682.0	24692.2	24702.5	24712.7	24722.9	24733.1	24743.3
25.7	24753.5	24763.7	24773.9	24784.1	24794.3	24804.5	24814.7	24824.9	24835.0	24845.2
25.8	24855.4	24865.5	24875.7	24885.8	24896.0	24906.1	24916.3	24926.4	24936.6	24946.7
25.9	24956.8	24966.9	24977.1	24987.2	24997.3	25007.4	25017.5	25027.6	25037.7	25047.8

TABLE II.

PART I.—HEIGHTS BY ANEROID BAROMETER—*Continued.*

$60384.3 \times \log H$ or h.

Barometer in English Inches.	HUNDREDTHS OF AN INCH.									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
26.0	25057.9	25068.0	25078.0	25088.1	25098.2	25108.3	25118.3	25128.4	25138.4	25148.5
26.1	25158.5	25168.6	25178.6	25188.7	25198.7	25208.7	25218.8	25228.8	25238.8	25248.8
26.2	25258.8	25268.8	25278.8	25288.8	25298.8	25308.8	25318.8	25328.8	25338.8	25348.8
26.3	25358.7	25368.7	25378.7	25388.6	25398.6	25408.5	25418.5	25428.4	25438.4	25448.3
26.4	25458.3	25468.2	25478.1	25488.0	25498.0	25507.9	25517.8	25527.7	25537.6	25547.5
26.5	25557.4	25567.3	25577.2	25587.1	25597.0	25606.8	25616.7	25626.6	25636.5	25646.3
26.6	25656.2	25666.0	25675.9	25685.7	25695.6	25705.4	25715.3	25725.1	25734.9	25744.8
26.7	25754.6	25764.4	25774.2	25784.0	25793.8	25803.6	25813.4	25823.2	25833.0	25842.8
26.8	25852.6	25862.4	25872.2	25882.0	25891.7	25901.5	25911.3	25921.0	25930.8	25940.5
26.9	25950.3	25960.0	25969.8	25979.5	25989.3	25999.0	26008.7	26018.4	26028.2	26037.9
27.0	26047.6	26057.3	26067.0	26076.7	26086.4	26096.1	26105.8	26115.5	26125.2	26134.9
27.1	26144.5	26154.2	26163.9	26173.6	26183.2	26192.9	26202.5	26212.2	26221.8	26231.5
27.2	26241.1	26250.8	26260.4	26270.0	26279.7	26289.3	26298.9	26308.5	26318.2	26327.8
27.3	26337.4	26347.0	26356.6	26366.2	26375.8	26385.4	26394.9	26404.5	26414.1	26423.7
27.4	26433.3	26442.8	26452.4	26462.0	26471.5	26481.1	26490.6	26500.2	26509.7	26519.3
27.5	26528.8	26538.3	26547.9	26557.4	26566.9	26576.4	26586.0	26595.5	26605.0	26614.5
27.6	26624.0	26633.5	26643.0	26652.5	26662.0	26671.5	26680.9	26690.4	26699.9	26709.4

27.7	26718.8	26728.3	26737.8	26747.2	26756.7	26766.1	26775.6	26785.0	26794.5	26803.9
27.8	26813.3	26822.8	26832.2	26841.6	26851.0	26860.5	26869.9	26879.3	26888.7	26898.1
27.9	26907.5	26916.9	26926.3	26935.7	26945.1	26954.5	26963.8	26973.2	26982.6	26992.0
28.0	27001.3	27010.7	27020.0	27029.4	27038.8	27048.1	27057.5	27066.8	27076.1	27085.5
28.1	27094.8	27104.1	27113.5	27122.8	27132.1	27141.4	27150.8	27160.1	27169.4	27178.7
28.2	27188.0	27197.3	27206.6	27215.9	27225.1	27234.4	27243.7	27253.0	27262.3	27271.5
28.3	27280.8	27290.1	27299.3	27308.6	27317.8	27327.1	27336.3	27345.6	27354.8	27364.1
28.4	27373.3	27382.5	27391.8	27401.0	27410.2	27419.4	27428.7	27437.9	27447.1	27456.3
28.5	27465.5	27474.7	27483.9	27493.1	27502.3	27511.5	27520.6	27529.8	27539.0	27548.2
28.6	27557.3	27566.5	27575.7	27584.8	27594.0	27603.2	27612.3	27621.5	27630.6	27639.7
28.7	27648.9	27658.0	27667.2	27676.3	27685.4	27694.5	27703.6	27712.8	27721.9	27731.0
28.8	27740.1	27749.2	27758.3	27767.4	27776.5	27785.6	27794.7	27803.8	27812.8	27821.9
28.9	27831.0	27840.1	27849.1	27858.2	27867.3	27876.3	27885.4	27894.4	27903.5	27912.5
29.0	27921.6	27930.6	27939.6	27948.7	27957.7	27966.8	27975.8	27984.8	27993.8	28002.8
29.1	28011.9	28020.9	28029.9	28038.9	28047.9	28056.9	28065.9	28074.9	28083.8	28092.8
29.2	28101.8	28110.8	28119.8	28128.7	28137.7	28146.7	28155.6	28164.6	28173.6	28182.5
29.3	28191.5	28200.4	28209.4	28218.3	28227.3	28236.2	28245.1	28254.1	28263.0	28271.9
29.4	28280.8	28289.7	28298.7	28307.6	28316.5	28325.4	28334.3	28343.2	28352.1	28361.0
29.5	28369.9	28378.8	28387.6	28396.5	28405.4	28414.3	28423.2	28432.0	28440.9	28449.8
29.6	28458.6	28467.5	28476.3	28485.2	28494.0	28502.9	28511.7	28520.6	28529.4	28538.2
29.7	28547.1	28555.9	28564.7	28573.5	28582.4	28591.2	28600.0	28608.8	28617.6	28626.4
29.8	28635.2	28644.0	28652.8	28661.6	28670.4	28679.2	28688.0	28696.7	28705.5	28714.3
29.9	28723.1	28731.8	28740.6	28749.4	28758.1	28766.9	28775.6	28784.4	28793.1	28801.9
30.0	28810.6	28819.4	28828.1	28836.8	28845.6	28854.3	28863.0	28871.8	28880.5	28889.2
30.1	28897.9	28906.6	28915.3	28924.0	28932.7	28941.4	28950.1	28958.8	28967.5	28976.2
30.2	28984.9	28993.6	29002.2	29010.9	29019.6	29028.3	29036.9	29045.6	29054.3	29062.9
30.3	29071.6	29080.2	29088.9	29097.5	29106.2	29114.8	29123.5	29132.1	29140.7	29149.4
30.4	29158.0	29166.6	29175.2	29183.8	29192.5	29201.1	29209.7	29218.3	29226.9	29235.5

TABLE II.

PART I.—HEIGHTS BY ANEROID BAROMETER—*Continued.*

$60384.3 \times \log H$ or h.

Barometer in English Inches.	HUNDREDTHS OF AN INCH.									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
30.5	29244.1	29252.7	29261.3	29269.9	29278.5	29287.1	29295.6	29304.2	29312.8	29321.4
30.6	29329.9	29338.5	29347.1	29355.6	29364.2	29372.8	29381.3	29389.9	29398.4	29407.0
30.7	29415.5	29424.1	29432.6	29441.1	29449.7	29458.2	29466.7	29475.2	29483.7	29492.3
30.8	29500.8	29509.3	29517.8	29526.3	29534.8	29543.3	29551.8	29560.3	29568.8	29577.3
30.9	29585.8	29594.3	29602.8	29611.2	29619.7	29628.2	29636.7	29645.1	29653.6	29662.1

TABLE II.

Part II.—Heights by Aneroid Barometer.

$t+t'$	$\frac{t+t'-64}{900}$	$t+t'$	$\frac{t+t'-64}{900}$	$t+t'$	$\frac{t+t'-64}{900}$	$t+t'$	$\frac{t+t'-64}{900}$
30°	−0.0378	70°	+0.0067	110°	+0.0511	150°	+0.0956
31°	−0.0367	71°	+0.0078	111°	+0.0522	151°	+0.0967
32°	−0.0356	72°	+0.0089	112°	+0.0533	152°	+0.0978
33°	−0.0344	73°	+0.0100	113°	+0.0544	153°	+0.0989
34°	−0.0333	74°	+0.0111	114°	+0.0556	154°	+0.1000
35°	−0.0322	75°	+0.0122	115°	+0.0567	155°	+0.1011
36°	−0.0311	76°	+0.0133	116°	+0.0578	156°	+0.1022
37°	−0.0300	77°	+0.0144	117°	+0.0589	157°	+0.1033
38°	−0.0289	78°	+0.0156	118°	+0.0600	158°	+0.1044
39°	−0.0278	79°	+0.0167	119°	+0.0611	159°	+0.1056
40°	−0.0267	80°	+0.0178	120°	+0.0622	160°	+0.1067
41°	−0.0256	81°	+0.0189	121°	+0.0633	161°	+0.1078
42°	−0.0244	82°	+0.0200	122°	+0.0644	162°	+0.1089
43°	−0.0233	83°	+0.0211	123°	+0.0656	163°	+0.1100
44°	−0.0222	84°	+0.0222	124°	+0.0667	164°	+0.1111
45°	−0.0211	85°	+0.0233	125°	+0.0678	165°	+0.1122
46°	−0.0200	86°	+0.0244	126°	+0.0689	166°	+0.1133
47°	−0.0189	87°	+0.0256	127°	+0.0700	167°	+0.1144
48°	−0.0178	88°	+0.0267	128°	+0.0711	168°	+0.1156
49°	−0.0167	89°	+0.0278	129°	+0.0722	169°	+0.1167
50°	−0.0156	90°	+0.0289	130°	+0.0733	170°	+0.1178
51°	−0.0144	91°	+0.0300	131°	+0.0744	171°	+0.1189
52°	−0.0133	92°	+0.0311	132°	+0.0756	172°	+0.1200
53°	−0.0122	93°	+0.0322	133°	+0.0767	173°	+0.1211
54°	−0.0111	94°	+0.0333	134°	+0.0778	174°	+0.1222
55°	−0.0100	95°	+0.0344	135°	+0.0789	175°	+0.1233
56°	−0.0089	96°	+0.0356	136°	+0.0800	176°	+0.1244
57°	−0.0078	97°	+0.0367	137°	+0.0811	177°	+0.1256
58°	−0.0067	98°	+0.0378	138°	+0.0822	178°	+0.1267
59°	−0.0056	99°	+0.0389	139°	+0.0833	179°	+0.1278
60°	−0.0044	100°	+0.0400	140°	+0.0844	180°	+0.1289
61°	−0.0033	101°	+0.0411	141°	+0.0856	181°	+0.1300
62°	−0.0022	102°	+0.0422	142°	+0.0867	182°	+0.1311
63°	−0.0011	103°	+0.0433	143°	+0.0878	183°	+0.1322
64°	0.0000	104°	+0.0444	144°	+0.0889	184°	+0.1333
65°	+0.0011	105°	+0.0456	145°	+0.0900	185°	+0.1344
66°	+0.0022	106°	+0.0467	146°	+0.0911	186°	+0.1356
67°	+0.0033	107°	+0.0478	147°	+0.0922	187°	+0.1367
68°	+0.0044	108°	+0.0489	148°	+0.0933	188°	+0.1378
69°	+0.0056	109°	+0.0500	149°	+0.0944	189°	+0.1389

TABLE III.

PART I.—HEIGHTS BY BOILING POINT THERMOMETER.

Boiling Point, Fahr.	Altitude above level at which Water boils at 212° (temp. of Intermediate Air being 32° F.).	Approximate corresponding Height of Aneroid or Barometer.	Boiling Point, Fahr.	Altitude above level at which Water boils at 212° (temp. of Intermediate Air being 32° F.).	Approximate corresponding Height of Aneroid or Barometer.
185.0	14,698	17.048	189.6	12,096	18.833
185.1	14 641	17.085	189.7	12,040	18.874
185.2	14,584	17.122	189.8	11,984	18.914
185.3	14,528	17.160	189.9	11,928	18.955
185.4	14,471	17.197	190.0	11,872	18.996
185.5	14,414	17.235	190.1	11,816	19.036
185.6	14,357	17.272	190.2	11,760	19.077
185.7	14,300	17.310	190.3	11,704	19.118
185.8	14,244	17.348	190.4	11,648	19.159
185.9	14,187	17.385	190.5	11,592	19 200
186.0	14,130	17.423	190.6	11,536	19.241
186.1	14,073	17.461	190.7	11,480	19.283
186.2	14,017	17.499	190.8	11,424	19.324
186.3	13,960	17.537	190.9	11,368	19.365
186.4	13,903	17.575	191.0	11,312	19.407
186.5	13,857	17.614	191.1	11,257	19.448
186.6	13,790	17.652	191.2	11,201	19.490
186.7	13,733	17.690	191.3	11,146	19.532
186.8	13,676	17.729	191.4	11,090	19.573
186.9	13,620	17.767	191.5	11,034	19.615
187.0	13,563	17.806	191.6	10,978	19.657
187.1	13,506	17.844	191.7	10,922	19.699
187.2	13,450	17.883	191.8	10,867	19.741
187.3	13,394	17.922	191.9	10,811	19.783
187.4	13,337	17.961	192.0	10,755	19.825
187.5	13,281	18.000	192.1	10,699	19.868
187.6	13,224	18.039	192.2	10,644	19.910
187.7	13,167	18.078	192.3	10,588	19.952
187.8	13,111	18.117	192.4	10,533	19.995
187.9	13,054	18.156	192.5	10,477	20.037
188.0	12,998	18.195	192.6	10,422	20.080
188.1	12,942	18.235	192.7	10,366	20.123
188.2	12,885	18.274	192.8	10,310	20.166
188.3	12,829	18.314	192.9	10,255	20.208
188.4	12,772	18.353	193.0	10,199	20.251
188.5	12,716	18.393	193.1	10,144	20.294
188.6	12,660	18.432	193.2	10,088	20.338
188.7	12,603	18.472	193.3	10 033	20.381
188.8	12,547	18.512	193.4	9,978	20.424
188.9	12,490	18.552	193.5	9,923	20.467
189.0	12,434	18.592	193.6	9,867	20.511
189.1	12,377	18.632	193.7	9,812	20.554
189.2	12,321	18.672	193.8	9.757	20.598
189.3	12,265	18.712	193.9	9,701	20.641
189.4	12,209	18.753	194.0	9,646	20.685
189.5	12,153	18.793	194.1	9,591	20.729

TABLE III.

PART I.—HEIGHTS BY BOILING POINT THERMOMETER—*Continued.*

Boiling Point, Fahr.	Altitude above level at which Water boils at 212° (temp. of Intermediate Air being 32° F.).	Approximate corresponding Height of Aneroid or Barometer.	Boiling Point, Fahr.	Altitude above level at which Water boils at 212° (temp. of Intermediate Air being 32° F.).	Approximate corresponding Height of Aneroid or Barometer.
194.2	9,536	20.773	199.4	6,691	23.163
194.3	9,481	20.817	199.5	6,637	23.211
194.4	9,426	20.861	199.6	6,583	23.259
194.5	9,371	20.905	199.7	6,529	23.308
194.6	9,315	20.949	199.8	6,474	23.356
194.7	9,260	20.993	199.9	6,420	23.405
194.8	9,205	21.038	200.0	6,366	23.453
194.9	9,150	21.082	200.1	6,312	23.502
195.0	9,095	21.126	200.2	6,258	23.550
195.1	9,040	21.171	200.3	6,203	23.599
195.2	8,985	21.216	200.4	6,149	23.648
195.3	8,930	21.260	200.5	6,095	23.697
195.4	8,875	21.305	200.6	6,041	23.746
195.5	8,820	21.350	200.7	5,987	23.795
195.6	8,765	21.395	200.8	5,933	23.845
195.7	8,710	21.440	200.9	5,879	23.894
195.8	8,655	21.485	201.0	5,825	23.943
195.9	8,600	21.530	201.1	5,771	23.993
196.0	8,545	21.576	201.2	5,717	24.042
196.1	8,490	21.621	201.3	5,663	24.092
196.2	8,435	21.666	201.4	5,609	24.142
196.3	8,381	21.712	201.5	5,556	24.191
196.4	8,326	21.751	201.6	5,502	24.241
196.5	8,271	21.803	201.7	5,448	24.291
196.6	8,216	21.849	201.8	5,394	24.341
196.7	8,161	21.895	201.9	5,340	24.391
196.8	8,107	21.941	202.0	5,286	24.442
196.9	8,052	21.987	202.1	5,232	24.492
197.0	7,997	22.033	202.2	5,178	24.542
197.1	7,942	22.079	202.3	5,124	24.593
197.2	7,888	22.125	202.4	5,070	24.644
197.3	7,833	22.172	202.5	5,017	24.694
197.4	7,779	22.218	202.6	4,964	24.745
197.5	7,724	22.264	202.7	4,910	24.796
197.6	7,669	22.311	202.8	4,856	24.847
197.7	7,615	22.358	202.9	4,802	24.898
197.8	7,560	22.404	203.0	4,749	24.949
197.9	7,506	22.451	203.1	4,695	25.000
198.0	7,451	22.498	203.2	4,641	25.051
198.1	7,397	22.545	203.3	4,588	25.103
198.2	7,343	22.592	203.4	4,535	25.154
198.3	7,289	22.639	203.5	4,482	25.206
198.4	7,234	22.686	203.6	4,428	25.257
198.5	7,180	22.734	203.7	4,375	25.309
198.6	7,125	22.781	203.8	4,322	25.361
198.7	7,071	22.829	203.9	4,268	25.413
198.8	7,016	22.876	204.0	4,215	25.465
198.9	6,962	22.924	204.1	4,161	25.517
199.0	6,908	22.971	204.2	4,107	25.569
199.1	6,854	23.019	204.3	4,053	25.621
199.2	6,800	23.067	204.4	4,000	25.674
199.3	6,745	23.115	204.5	3,947	25.726

PART I.—HEIGHTS BY BOILING POINT THERMOMETER—*Continued.*

Boiling Point, Fahr.	Altitude above level at which Water boils at 212° (temp. of Intermediate Air being 32° F.).	Approximate corresponding Height of Aneroid or Barometer.	Boiling Point, Fahr.	Altitude above level at which Water boils at 212° (temp. of Intermediate Air being 32° F.).	Approximate corresponding Height of Aneroid or Barometer.
204.6	3,894	25.779	209.8	1,149	28.636
204.7	3,841	25.831	209.9	1,096	28.693
204.8	3,788	25.884	210.0	1,044	28.751
204.9	3,735	25.937	210.1	992	28.809
205.0	3,682	25.990	210.2	939	28.866
205.1	3,625	26.043	210.3	887	28.924
205.2	3,574	26.096	210.4	835	28.982
205.3	3,521	26.149	210.5	783	29.040
205.4	3,468	26.202	210.6	730	29.098
205.5	3,416	26.255	210.7	678	29.156
205.6	3,363	26.309	210.8	626	29.215
205.7	3,310	26.362	210.9	573	29.273
205.8	3,256	26.416	211.0	521	29.331
205.9	3,203	26.470	211.1	469	29.390
206.0	3,151	26.523	211.2	417	29.449
206.1	3,098	26.577	211.3	365	29.508
206.2	3,045	26.631	211.4	313	29.566
206.3	2,992	26.685	211.5	261	29.625
206.4	2,939	26.740	211.6	208	29.684
206.5	2,886	26.794	211.7	156	29.744
206.6	2,833	26.848	211.8	104	29.803
206.7	2,780	26.903	211.9	52	29.862
206.8	2,727	26.957	212.0	0	29.922
206.9	2,674	27.012	212.1	− 52	29.981
207.0	2,622	27.066	212.2	− 104	30.041
207.1	2,569	27.121	212.3	− 155	30.101
207.2	2,516	27.179	212.4	− 207	30.161
207.3	2,464	27.231	212.5	− 259	30.221
207.4	2,411	27.286	212.6	− 311	30.281
207.5	2,358	27.341	212.7	− 363	30.341
207.6	2,305	27.397	212.8	− 414	30.401
207.7	2,252	27.452	212.9	− 466	30.461
207.8	2,199	27.507	213.0	− 518	30.522
207.9	2,146	27.563	213.1	− 570	30.583
208.0	2,094	27.618	213.2	− 621	30.644
208.1	2,041	27.674	213.3	− 673	30.705
208.2	1,989	27.730	213.4	− 724	30.766
208.3	1,936	27.786	213.5	− 776	30.827
208.4	1,884	27.842	213.6	− 828	30.888
208.5	1,831	27.898	213.7	− 880	30.949
208.6	1,778	27.954	213.8	− 932	31.010
208.7	1,726	28.011	213.9	− 983	31.071
208.8	1,673	28.067	214.0	− 1,035	31.132
208.9	1,621	28.123	214.1	− 1,086	31.194
209.0	1,568	28.180	214.2	− 1,138	31.256
209.1	1,516	28.237	214.3	− 1,189	31.318
209.2	1,463	28.293	214.4	− 1,241	31.380
209.3	1,411	28.350	214.5	− 1,293	31.442
209.4	1,358	28.407	214.6	− 1,344	31.504
209.5	1,306	28.464	214.7	− 1,396	31.566
209.6	1,254	28.521	214.8	− 1,447	31.628
209.7	1,201	28.579	214.9	− 1,549	31.690

TABLE III.

PART II.—CORRECTION FOR TEMPERATURE OF INTERMEDIATE AIR.

Mean Temperature of Intermediate Air.	Multiplier.	Mean Temperature of Intermediate Air.	Multiplier.	Mean Temperature of Intermediate Air.	Multiplier.
Degrees.		Degrees.		Degrees.	
20	0.9734	42	1.0222	64	1.0711
21	0.9756	43	1.0244	65	1.0733
22	0.9778	44	1.0266	66	1.0755
23	0.9801	45	1.0288	67	1.0777
24	0.9823	46	1.0311	68	1.0799
25	0.9845	47	1.0333	69	1.0822
26	0.9867	48	1.0355	70	1.0844
27	0.9889	49	1.0377	71	1.0866
28	0.9912	50	1.0399	72	1.0888
29	0.9934	51	1.0422	73	1.0911
30	0.9956	52	1.0444	74	1.0933
31	0.9978	53	1.0466	75	1.0955
32	1.0000	54	1.0488	76	1.0977
33	1.0022	55	1.0511	77	1.0999
34	1.0044	56	1.0533	78	1.1022
35	1.0066	57	1.0555	79	1.1044
36	1.0088	58	1.0577	80	1.1066
37	1.0111	59	1.0599	81	1.1088
38	1.0133	60	1.0622	82	1.1111
39	1.0155	61	1.0644	83	1.1133
40	1.0177	62	1.0666	84	1.1156
41	1.0199	63	1.0688	85	1.1178

TABLE IV.

TACHEOMETER OR STADIA TABLE FOR THE REDUCTION OF INCLINED SIGHTS.*

Minutes.	0°		1°		2°		3°	
	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.
0	1.00000	.00000	.99969	.01745	.99878	.03488	.99726	.05227
2	1.00000	.00058	.99967	.01803	.99874	.03546	.99720	.05285
4	1.00000	.00116	.99965	.01862	.99870	.03604	.99714	.05342
6	.99999	.00175	.99963	.01920	.99866	.03662	.99708	.05400
8	.99999	.00233	.99961	.01978	.99862	.03720	.99702	.05458
10	.99999	.00291	.99959	.02036	.99857	.03778	.99695	.05516
12	.99999	.00349	.99956	.02094	.99853	.03836	.99689	.05573
14	.99999	.00407	.99953	.02152	.99849	.03894	.99683	.05631
16	.99998	.00465	.99950	.02210	.99844	.03952	.99676	.05689
18	.99997	.00524	.99948	.02268	.99839	.04010	.99669	.05746
20	.99996	.00582	.99946	.02326	.99834	.04068	.99662	.05804
22	.99996	.00640	.99943	.02384	.99829	.04125	.99656	.05863
24	.99995	.00698	.99941	.02442	.99825	.04184	.99649	.05921
26	.99994	.00756	.99938	.02500	.99820	.04242	.99642	.05978
28	.99993	.00814	.99935	.02559	.99815	.04300	.99635	.06036
30	.99992	.00873	.99931	.02617	.99810	.04358	.99627	.06094
32	.99991	.00931	.99929	.02675	.99805	.04416	.99620	.06151
34	.99990	.00989	.99926	.02733	.99800	.04474	.99613	.06209
36	.99989	.01047	.99923	.02791	.99795	.04531	.99606	.06267
38	.99988	.01105	.99919	.02850	.99789	.04589	.99599	.06324
40	.99986	.01164	.99915	.02907	.99784	.04648	.99591	.06382
42	.99985	.01222	.99912	.02966	.99778	.04706	.99588	.06440
44	.99984	.01280	.99909	.03024	.99772	.04764	.99575	.06497
46	.99983	.01338	.99906	.03082	.99767	.04821	.99568	.06555
48	.99981	.01396	.99902	.03139	.99762	.04879	.99560	.06612
50	.99979	.01454	.99898	.03197	.99756	.04937	.99558	.06670
52	.99977	.01513	.99894	.03255	.99750	.04995	.99545	.06728
54	.99975	.01571	.99890	.03314	.99744	.05053	.99537	.06786
56	.99973	.01629	.99886	.03372	.99738	.05110	.99529	.06844
58	.99971	.01687	.99882	.03430	.99732	.05169	.99521	.06901
60	.99969	.01745	.99878	.03488	.99726	.05227	.99513	.06959
$f+d=0.75$	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
$f+d=1.00$	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
$f+d=1.25$	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

* This table was calculated by Mr A. Winslow of the Geological Survey of Pennsylvania, and is given by Professor J. B. Johnson in his "Theory and Practice of Surveying." It is here calculated to an additional place of decimals and rearranged so as to be suitable for an instrument in which the constant $\frac{f}{i}$ may have any value.

TABLE IV.

TACHEOMETER OR STADIA TABLE FOR THE REDUCTION OF INCLINED SIGHTS—*Continued.*

Minutes.	4°		5°		6°		7°	
	Hor. Distance Cos².	Height Cos × Sin	Hor. Distance Cos².	Height Cos × Sin.	Hor. Distance Cos².	Height Cos × Sin.	Hor. Distance Cos².	Height Cos × Sin
0	.99513	.06959	.99240	.08683	.98907	.10396	.98515	.12096
2	.99505	.07017	.99230	.08740	.98895	.10453	.98501	.12153
4	.99497	.07074	.99220	.08796	.98883	.10510	.98487	.12209
6	.99489	.07132	.99210	.08854	.98871	.10566	.98472	.12265
8	.99481	.07192	.99200	.08911	.98859	.10623	.98458	.12322
10	.99472	.07247	.99189	.08968	.98846	.10680	.98443	.12379
12	.99464	.07305	.99179	.09026	.98834	.10737	.98429	.12434
14	.99456	.07362	.99169	.09083	.98822	.10794	.98415	.12491
16	.99447	.07419	.99158	.09140	.98809	.10851	.98400	.12547
18	.99438	.07477	.99147	.09197	.98796	.10907	.98386	.12603
20	.99429	.07534	.99136	.09255	.98783	.10964	.98371	.12660
22	.99421	.07592	.99126	.09312	.98770	.11021	.98356	.12716
24	.99412	.07649	.99115	.09369	.98758	.11078	.98342	.12773
26	.99402	.07707	.99104	.09426	.98745	.11134	.98327	.12828
28	.99393	.07764	.99092	.09484	.98732	.11191	.98312	.12885
30	.99384	.07822	.99081	.09541	.98718	.11247	.98296	.12941
32	.99375	.07879	.99070	.09597	.98706	.11304	.98281	.12997
34	.99366	.07937	.99059	.09654	.98693	.11361	.98266	.13053
36	.99357	.07994	.99048	.09711	.98680	.11418	.98251	.13110
38	.99348	.08052	.99037	.09769	.98666	.11475	.98235	.13165
40	.99338	.08109	.99025	.09826	.98652	.11531	.98220	.13222
42	.99328	.08166	.99014	.09883	.98639	.11587	.98205	.13278
44	.99318	.08224	.99002	.09940	.98625	.11644	.98189	.13334
46	.99309	.08281	.98990	.09997	.98611	.11701	.98174	.13390
48	.99299	.08339	.98979	.10054	.98598	.11757	.98158	.13446
50	.99290	.08396	.98967	.10111	.98584	.11814	.98142	.13502
52	.99280	.08453	.98955	.10167	.98570	.11870	.98127	.13558
54	.99270	.08511	.98943	.10225	.98557	.11927	.98111	.13614
56	.99260	.08568	.98931	.10282	.98543	.11983	.98095	.13670
58	.99250	.08626	.98919	.10339	.98529	.12039	.98079	.13726
60	.99240	.08683	.98907	.10396	.98515	.12096	.98063	.13782
$f+d=0.75$	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
$f+d=1.00$	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
$f+d=1.25$	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

TABLE IV.

TACHEOMETER OR STADIA TABLE FOR THE REDUCTION OF INCLINED SIGHTS—*Continued.*

Minutes.	8°		9°		10°		11°	
	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$
0	.98063	.13782	.97553	.15450	.96985	.17101	.96359	.18730
2	.98047	.13838	.97535	.15506	.96965	.17156	.96337	.18784
4	.98031	.13895	.97517	.15562	.96945	.17210	.96315	.18838
6	.98015	.13949	.97499	.15617	.96925	.17265	.96293	.18892
8	.97999	.14006	.97481	.15672	.96905	.17320	.96271	.18946
10	.97982	.14061	.97462	.15728	.96884	.17374	.96249	.18999
12	.97966	.14117	.97444	.15782	.96864	.17428	.96227	.19053
14	.97950	.14172	.97426	.15838	.96844	.17483	.96205	.19108
16	.97933	.14229	.97407	.15893	.96823	.17538	.96182	.19162
18	.97916	.14285	.97389	.15948	.96803	.17592	.96160	.19215
20	.97899	.14340	.97370	.16003	.96782	.17646	.96138	.19269
22	.97883	.14396	.97352	.16058	.96762	.17701	.96115	.19322
24	.97867	.14451	.97334	.16114	.96741	.17755	.96093	.19376
26	.97849	.14507	.97315	.16168	.96720	.17810	.96071	.19430
28	.97832	.14563	.97295	.16223	.96700	.17864	.96048	.19483
30	.97815	.14619	.97276	.16279	.96679	.17919	.96025	.19537
32	.97798	.14674	.97257	.16333	.96658	.17973	.96002	.19590
34	.97781	.14730	.97238	.16389	.96637	.18027	.95979	.19644
36	.97763	.14786	.97219	.16444	.96616	.18081	.95956	.19697
38	.97746	.14841	.97200	.16498	.96595	.18135	.95934	.19751
40	.97729	.14897	.97180	.16554	.96574	.18189	.95911	.19804
42	.97711	.14952	.97161	.16608	.96553	.18244	.95888	.19858
44	.97695	.15008	.97142	.16663	.96532	.18298	.95865	.19911
46	.97678	.15063	.97123	.16718	.96510	.18352	.95842	.19965
48	.97660	.15119	.97103	.16773	.96488	.18406	.95818	.20018
50	.97642	.15174	.97083	.16827	.96467	.18460	.95795	.20071
52	.97625	.15230	.97064	.16883	.96445	.18514	.95771	.20124
54	.97607	.15285	.97044	.16937	.96424	.18569	.95747	.20177
56	.97589	.15341	.97024	.16991	.96402	.18623	.95725	.20230
58	.97571	.15396	.97005	.17047	.96381	.18676	.95701	.20283
60	.97553	.15450	.96985	.17101	.96359	.18730	.95677	.20337
$f+d=0.75$	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
$f+d=1.00$	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
$f+d=1.25$	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

TABLE IV.

TACHEOMETER OR STADIA TABLE FOR THE REDUCTION OF INCLINED SIGHTS—*Continued.*

Minutes.	12°		13°		14°		15°	
	Hor. Distance Cos².	Height Cos×Sin.	Hor. Distance Cos².	Height Cos×Sin.	Hor. Distance Cos².	Height Cos×Sin	Hor. Distance Cos².	Height Cos×Sin.
0	.95677	.20337	.94940	.21918	.94147	.23473	.93301	.25000
2	.95653	.20390	.94915	.21971	.94120	.23525	.93271	.25050
4	.95629	.20443	.94889	.22023	.94093	.23576	.93242	.25100
6	.95606	.20496	.94863	.22075	.94066	.23627	.93214	.25150
8	.95582	.20549	.94837	.22128	.94038	.23679	.93185	.25202
10	.95558	.20603	.94811	.22179	.94010	.23730	.93155	.25252
12	.95534	.20655	.94786	.22232	.93983	.23782	.93125	.25302
14	.95510	.20708	.94760	.22284	.93955	.23832	.93096	.25352
16	.95486	.20761	.94734	.22336	.93927	.23884	.93067	.25402
18	.95462	.20814	.94708	.22388	.93899	.23935	.93037	.25452
20	.95438	.20867	.94682	.22440	.93871	.23985	.93007	.25502
22	.95413	.20920	.94655	.22492	.93843	.24037	.92977	.25553
24	.95389	.20973	.94629	.22544	.93815	.24088	.92947	.25603
26	.95365	.21025	.94603	.22595	.93787	.24138	.92918	.25652
28	.95340	.21078	.94576	.22648	.93759	.24190	.92888	.25702
30	.95315	.21131	.94550	.22700	.93731	.24241	.92858	.25752
32	.95290	.21184	.94523	.22751	.93702	.24291	.92828	.25802
34	.95266	.21235	.94497	.22804	.93674	.24343	.92798	.25852
36	.95242	.21289	.94471	.22855	.93646	.24393	.92768	.25901
38	.95217	.21342	.94444	.22907	.93617	.24443	.92738	.25951
40	.95192	.21394	.94417	.22958	.93589	.24495	.92708	.26001
42	.95166	.21447	.94390	.23010	.93560	.24545	.92677	.26050
44	.95141	.21499	.94364	.23061	.93532	.24596	.92647	.26100
46	.95116	.21552	.94338	.23113	.93503	.24646	.92616	.26150
48	.95091	.21604	.94311	.23164	.93475	.24697	.92586	.26199
50	.95066	.21657	.94283	.23217	.93446	.24748	.92556	.26249
52	.95041	.21709	.94255	.23268	.93417	.24798	.92525	.26298
54	.95016	.21762	.94228	.23319	.93388	.24849	.92494	.26348
56	.94990	.21814	.94201	.23371	.93359	.24900	.92463	.26397
58	.94965	.21866	.94175	.23423	.93330	.24950	.92432	.26448
60	.94940	.21918	.94147	.23473	.93301	.25000	.92402	.26496
$f+d=0.75$	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
$f+d=1.00$	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
$f+d=1.25$	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE IV.

TACHEOMETER OR STADIA TABLE FOR THE REDUCTION OF INCLINED SIGHTS—*Continued.*

Minutes.	16°		17°		18°		19°	
	Hor. Distance $\cos^2$.	Height Cos × Sin.	Hor. Distance $\cos^2$.	Height Cos × Sin	Hor. Distance $\cos^2$.	Height Cos × Sin	Hor. Distance $\cos^2$.	Height Cos × Sin
0	.92402	.26496	.91452	.27959	.90451	.29390	.89401	.30783
2	.92370	.26546	.91419	.28008	.90417	.29436	.89365	.30829
4	.92340	.26595	.91387	.28056	.90382	.29483	.89329	.30875
6	.92310	.26643	.91354	.28104	.90348	.29531	.89293	.30921
8	.92279	.26693	.91321	.28153	.90314	.29577	.89257	.30966
10	.92248	.26742	.91288	.28200	.90279	.29624	.89221	.31012
12	.92217	.26791	.91256	.28249	.90244	.29670	.89184	.31058
14	.92186	.26840	.91223	.28296	.90210	.29718	.89148	.31103
16	.92154	.26890	.91190	.28344	.90175	.29765	.89112	.31149
18	.92122	.26939	.91157	.28392	.90140	.29811	.89076	.31194
20	.92091	.26988	.91124	.28440	.90106	.29857	.89040	.31239
22	.92060	.27037	.91091	.28488	.90071	.29905	.89003	.31285
24	.92029	.27085	.91058	.28536	.90036	.29951	.88967	.31330
26	.91997	.27134	.91024	.28584	.90001	.29998	.88930	.31376
28	.91965	.27183	.90991	.28631	.89967	.30044	.88894	.31421
30	.91934	.27232	.90958	.28679	.89932	.30090	.88857	.31466
32	.91902	.27280	.90924	.28726	.89897	.30137	.88821	.31512
34	.91870	.27329	.90890	.28775	.89862	.30184	.88784	.31556
36	.91838	.27378	.90857	.28822	.89826	.30230	.88748	.31601
38	.91806	.27427	.90823	.28869	.89791	.30276	.88711	.31646
40	.91774	.27475	.90790	.28917	.89756	.30322	.88674	.31692
42	.91742	.27524	.90756	.28964	.89721	.30368	.88637	.31738
44	.91710	.27573	.90722	.29012	.89686	.30414	.88599	.31781
46	.91678	.27621	.90689	.29059	.89650	.30461	.88562	.31826
48	.91646	.27669	.90655	.29107	.89615	.30508	.88525	.31871
50	.91614	.27718	.90621	.29153	.89579	.30554	.88489	.31916
52	.91581	.27767	.90587	.29200	.89544	.30600	.88452	.31961
54	.91549	.27814	.90553	.29248	.89508	.30646	.88415	.32006
56	.91516	.27863	.90519	.29295	.89473	.30692	.88377	.32050
58	.91484	.27912	.90485	.29342	.89437	.30737	.88339	.32094
60	.91452	.27959	.90451	.29390	.89401	.30783	.88302	.32139
$f+d=0.75$	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
$f+d=1.00$	0.86	0.28	0.95	0.30	0.95	0.32	0.94	0.33
$f+d=1.25$	1.20	0.35	1.19	0.38	1.19	0.40	1.18	0.42

TABLE IV.

TACHEOMETER OR STADIA TABLE FOR THE REDUCTION OF INCLINED SIGHTS—*Continued.*

Minutes.	20°		21°		22°		23°	
	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.
0	.88302	.32139	.87157	.33457	.85967	.34733	.84733	.35967
2	.88264	.32184	.87118	.33500	.85927	.34775	.84691	.36008
4	.88227	.32228	.87079	.33542	.85887	.34817	.84649	.36047
6	.88190	.32273	.87040	.33586	.85846	.34858	.84607	.36088
8	.88152	.32318	.87001	.33629	.85805	.34900	.84565	.36128
10	.88115	.32361	.86962	.33672	.85764	.34941	.84523	.36169
12	.88077	.32406	.86923	.33715	.85724	.34983	.84481	.36209
14	.88039	.32450	.86884	.33758	.85683	.35025	.84439	.36249
16	.88001	.32494	.86844	.33801	.85642	.35066	.84397	.36289
18	.87963	.32539	.86804	.33844	.85601	.35108	.84355	.36329
20	.87926	.32583	.86765	.33886	.85560	.35150	.84312	.36369
22	.87888	.32627	86726	.33930	.85520	.35190	.84270	.36408
24	.87850	.32671	.86687	.33973	.85479	.35232	.84228	.36448
26	.87812	.32715	.86647	.34015	.85438	.35273	.84185	.36488
28	.87774	.32759	.86607	.34057	.85397	.35314	.84143	.36528
30	.87736	.32803	.86568	.34100	.85355	.35355	.84100	.36568
32	.87697	.32847	.86528	.34142	.85314	.35396	.84056	.36607
34	.87659	.32891	.86488	.34185	.85273	.35438	.84014	.36648
36	.87621	.32934	.86448	.34227	.85232	.35479	.83972	.36686
38	.87582	.32978	.86408	.34270	.85191	.35519	.83930	.36726
40	.87544	.33022	.86369	.34313	.85149	.35560	.83887	.36766
42	.87506	.33065	.86329	.34355	.85107	.35602	.83843	.36805
44	.87467	.33109	.86289	.34397	.85065	.35642	.83800	.36844
46	.87428	.33152	.86248	.34439	.85025	.35683	.83758	.36883
48	.87390	.33197	.86208	.34481	.84983	.35724	.83715	.36923
50	.87351	.33240	.86168	.34523	.84942	.35764	.83672	.36962
52	.87312	.33283	.86128	.34565	.84900	.35805	.83631	.37001
54	.87273	.33327	.86088	.34608	.84858	.35845	.83587	.37040
56	.87234	.33370	.86047	.34649	.84817	.35886	.83542	.37079
58	.87196	.33413	.86007	.34691	.84775	.35927	.83499	.37119
60	.87157	.33457	.85967	.34733	.84733	.35967	.83457	.37158
$f+d=0.75$	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
$f+d=1.00$	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
$f+d=1.25$	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

TABLE IV.

TACHEOMETER OR STADIA TABLE FOR THE REDUCTION OF INCLINED SIGHTS—*Continued.*

Minutes.	24°		25°		26°		27°	
	Hor. Distance Cos².	Height Cos × Sin.	Hor. Distance Cos².	Height Cos × Sin	Hor. Distance Cos².	Height Cos × Sin.	Hor. Distance Cos².	Height Cos × Sin
0	.83457	.37158	.82139	.38302	.80783	.39400	.79389	.40451
2	.83413	.37196	.82095	.38340	.80737	.39436	.79342	.40485
4	.83367	.37235	.82050	.38377	.80691	.39472	.79295	.40520
6	.83326	.37274	.82006	.38414	.80645	.39508	.79248	.40553
8	.83283	.37313	.81961	.38452	.80599	.39543	.79201	.40587
10	.83240	.37351	.81916	.38488	.80553	.39579	.79153	.40621
12	.83196	.37390	.81870	.38526	.80507	.39615	.79106	.40655
14	.83153	.37428	.81824	.38563	.80461	.39650	.79059	.40689
16	.83109	.37467	.81780	.38600	.80415	.39685	.79011	.40722
18	.83066	.37505	.81736	.38637	.80369	.39721	.78964	.40757
20	.83022	.37543	.81692	.38673	.80323	.39756	.78917	.40790
22	.82978	.37582	.81648	.38710	.80276	.39791	.78869	.40823
24	.82935	.37620	.81602	.38748	.80230	.39827	.78822	.40857
26	.82890	.37659	.81557	.38784	.80184	.39862	.78774	.40891
28	.82846	.37697	.81512	.38821	.80137	.39897	.78726	.40924
30	.82803	.37735	.81466	.38857	.80091	.39932	.78679	.40958
32	.82759	.37773	.81420	.38894	.80044	.39967	.78631	.40990
34	.82715	.37812	.81375	.38930	.79998	.40002	.78583	.41024
36	.82671	.37850	.81330	.38967	.79952	.40036	.78536	.41058
38	.82627	.37888	.81284	.39003	.79905	.40071	.78488	.41090
40	.82583	.37926	.81239	.39039	.79858	.40106	.78440	.41124
42	.82539	.37964	.81194	.39076	.79812	.40141	.78392	.41156
44	.82494	.38002	.81149	.39112	.79764	.40176	.78344	.41190
46	.82450	.38039	.81103	.39149	.79718	.40210	.78296	.41223
48	.82406	.38077	.81057	.39185	.79671	.40245	.78248	.41256
50	.82362	.38114	.81012	.39221	.79624	.40279	.78200	.41288
52	.82318	.38152	.80966	.39257	.79576	.40314	.78152	.41322
54	.82273	.38190	.80920	.39293	.79531	.40348	.78104	.41354
56	.82228	.38227	.80875	.39329	.79484	.40382	.78056	.41386
58	.82183	.38265	.80829	.39365	.79436	.40416	.78008	.41419
60	.82139	.38302	.80783	.39400	.79389	.40451	.77960	.41452
$f+d=0.75$	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
$f+d=1.00$	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
$f+d=1.25$	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

TABLE IV.

TACHEOMETER OR STADIA TABLE FOR THE REDUCTION OF INCLINED SIGHTS—*Continued.*

Minutes.	28°		29°		30°	
	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.	Hor. Distance $\cos^2$.	Height $\cos \times \sin$.
0	.77960	.41452	.76496	.42402	.75000	.43302
2	.77911	.41485	.76446	.42433	.74950	.43330
4	.77863	.41517	.76397	.42464	.74899	.43359
6	.77815	.41549	.76348	.42495	.74849	.43388
8	.77766	.41582	.76298	.42525	.74798	.43417
10	.77718	.41615	.76249	.42556	.74748	.43446
12	.77670	.41646	.76199	.42586	.74697	.43475
14	.77621	.41678	.76150	.42617	.74647	.43503
16	.77574	.41711	.76100	.42647	.74596	.43533
18	.77524	.41743	.76051	.42677	.74545	.43561
20	.77475	.41774	.76001	.42708	.74494	.43589
22	.77427	.41806	.75951	.42738	.74444	.43618
24	.77378	.41838	.75901	.42768	.74393	.43646
26	.77329	.41870	.75852	.42798	.74342	.43675
28	.77281	.41902	.75802	.42829	.74291	.43703
30	.77232	.41934	.75752	.42858	.74240	.43731
32	.77183	.41965	.75702	.42888	.74190	.43759
34	.77134	.41997	.75652	.42918	.74139	.43787
36	.77085	.42028	.75602	.42948	.74089	.43815
38	.77036	.42059	.75552	.42978	.74037	.43843
40	.76988	.42091	.75502	.43007	.73986	.43871
42	.76939	.42122	.75452	.43037	.73935	.43899
44	.76889	.42154	.75402	.43066	.73883	.43927
46	.76840	.42185	.75352	.43096	.73832	.43955
48	.76791	.42216	.75302	.43126	.73780	.43982
50	.76742	.42247	.75251	.43155	.73730	.44010
52	.76693	.42279	.75201	.43184	.73679	.44037
54	.76644	.42309	.75151	.43214	.73628	.44065
56	.76594	.42341	.75101	.43243	.73576	.44093
58	.76545	.42371	.75050	.43273	.73525	.44120
60	.76496	.42402	.75000	.43302	.73474	.44148
$f+d=0.75$	0.66	0.36	0.65	0.37	0.65	0.38
$f+d=1.00$	0.88	0.48	0.87	0.49	0.86	0.51
$f+d=1.25$	1.10	0.60	1.09	0.62	1.08	0.64

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